An Interval Distribution Analysis for RTI+

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Abstract: The algorithm RTI+ learns a Probabilistic Deterministic Real-Time Automaton (PDRTA) from unlabeled timed sequences. RTI+ is an efficient algorithm that runs in polynomial time and can be applied to a variety of real-world behavior identification problems. Nevertheless, we uncover a lack of accuracy in identifying the intervals (or time guards) of the PDRTA. This inaccuracy can lead to wrong predictions of timed sequences in the learned model. We show by example that segments in intervals that are not covered by training data are responsible for this effect. We call those segments gaps and name three types of gaps that can appear. Two of the types cause wrong predictions of sequences and should thus be removed from the model. Therefore, we introduce our novel Interval Distribution Analysis (IDA) which utilizes statistical outlier detection to identify and remove gaps. In the context of ATM fraud detection, we show that IDA can improve the results of RTI+ in a real-world scenario.

1 INTRODUCTION

The algorithm *Real-Time Identification from Positive Data* (RTI+) was developed by Verwer et al. (Verwer et al., 2010) to identify real-time behavioral models from positive data. RTI+ runs in polynomial time and can identify sufficiently large models to be able to learn the behavior of real-world systems (Verwer et al., 2010). Hence, it is a practically valuable algorithm which can be applied to a wide range of behavioral scenarios of log-emitting systems, e.g. ATM fraud detection or elevator breakdown prediction.

In this paper, we reveal a weakness of RTI+. The models learned by RTI+ may include a wider variety of time values than the training data yields. Hence, it underfits the data regarding the time values. This effect is reflected in widened intervals which are part of the model. We call the segments of the intervals, that are not covered by the training data, gaps. To overcome this imprecision of RTI+, we introduce our novel Interval Distribution Analysis (IDA). IDA utilizes statistical outlier detection to detect and remove the gaps and, thus, improves the model quality.

The paper is structured as follows: We start with a description of the algorithm RTI+ which our research is based on in Section 2. In Section 3 we go into details of RTI+ and analyze the origin of gaps in intervals. Furthermore, we describe the negative impact of gaps in assigning probabilities to sequences. Our consequent next step is to introduce our IDA procedure for detecting and removing those gaps in Section 4. There we describe the metrics that IDA uses and how RTI+ and IDA collaborate. Subsequently, we apply RTI+ and IDA to the problem of ATM fraud detection in Section 5 to point out the impact of IDA in a practical use case. Moreover, in Section 6 we give an outlook of the future development of IDA by describing alternative algorithms to solve the problem. This is followed by a review of related work in Section 7. In the end, we summarize the main points of the paper and conclude on IDA in Section 8.

2 BASICS: RTI+

In this section we describe the algorithm RTI+ (introduced by Verwer et al. in (Verwer et al., 2010)). This algorithm identifies real-time behavior models in polynomial time.

RTI+ is based on the Evidence-Driven State Merging (EDSM) algorithm (Lang et al., 1998). It computes a model out of a finite multiset of unlabeled timed sequences. A timed sequence is a chain of symbols extended by the time delays between them. As a model,

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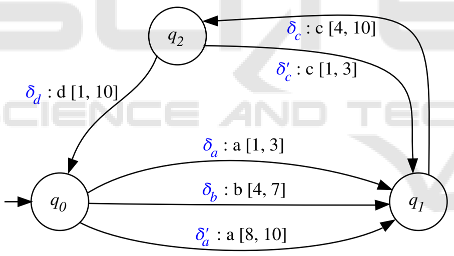
Verwer et al. use a Probabilistic Deterministic Real-Time Automaton (PDRTA) which is an extended Deterministic Real-Time Automaton (DRTA) (cf. (Dima, 2001)). A PDRTA is defined as follows:

Definition 1 (PDRTA)**.** *A Probabilistic Deterministic Real-Time Automaton (PDRTA) is a four-tuple* $A = \langle A', H, \mathcal{S}, \mathcal{T} \rangle$*, where*

- $A' = \langle Q, \Sigma, \Delta, q_0 \rangle$ *is a DRTA without accepting states, where*
  - Q *is a finite set of states*
  - Σ *is a finite set of symbols*
  - Δ *is a finite set of timed transitions*
  - q_0 *is the start state*
- H *is a finite set of histogram bins* $h = [v, v']$ *with* $v, v' \in \mathbb{N}$
- $\mathcal{S}$ *is a finite set of symbol probability distributions* $S_q = \{Pr(s \mid q) \mid s \in \Sigma, q \in Q\}$
- $\mathcal{T}$ *is a finite set of histogram bin probability distributions* $T_q = \{Pr(t \in h \mid q) \mid h \in H, q \in Q\}$

The DRTA is extended by probability distributions for symbols and time delays which are modeled by global histogram bins. These probability distributions are used for learning the model and later also for predicting sequences.



$H = \{\ [1,3],\ [4,7],\ [8,10]\ \}$

Figure 1: Example PDRTA with three states and six transitions.

An example PDRTA is shown in Fig. 1. This PDRTA consists of three states where q_0 is the start state indicated by the sourceless transition on the left. q_0 has three outgoing transitions which all have state q_1 as target. For example when in state q_0, the symbol a with a time delay of 2 time units will use transition δ_a. The symbol a can also take transition δ'_a when it is delayed with a time in $[8, 10]$. The three histogram bins for time delays are shown below the automaton structure. For reasons of clarity we do not show the probability distributions $\mathcal{S}$ and $\mathcal{T}$. But there exist probabilities for each state to be left with a certain symbol and a time delay in a certain histogram bin. Both probability distributions are independent of each other. In the following, we describe the routine and methods of RTI+ to construct a PDRTA from a multiset of training sequences.

The goal of RTI+ is to find a small PDRTA that describes the training sequences sufficiently well. RTI+ starts with generating a Prefix Tree Acceptor from training sequences. Initially the intervals (time guards) of the transitions are chosen as the global minimum and maximum of time delays in the training sequences. Hence, the structure of the tree is equal to the untimed version of the tree (when time delays are ignored). This initial model may be missing time-dependent substructures and may contain multiple similar substructures. The task of RTI+ is to identify these two types of substructures. In case of a missing time-dependent substructure, the PDRTA is specialized (SPLIT), while in case of multiple similar substructures, the PDRTA is generalized to compress the model (MERGE).

To improve the initial tree, RTI+ iterates within the red-blue coloring framework (inherited from EDSM) over the tree and manipulates it with a MERGE operation for compression (inherited from EDSM) and a newly introduced SPLIT operation for specialization. The pseudo code of this routine can be found in Algorithm 3 in Section 4 alongside with the routine for our novel distribution analysis.

The MERGE operation is used to generalize the model by combining two similar substructures into a new one with a preferably small loss of model quality regarding the training sequences. The merging process of the substructures starts by merging two corresponding states in each substructure into one state. All transitions of the two states are preserved which can cause non-determinism in the outgoing transitions of the merged state. To solve this problem, all target states of non-deterministic transitions are merged recursively, too, until the model is deterministic again. Note that a MERGE operation can also create cycles or combine two subtrees.

The SPLIT operation has a different goal than a MERGE. Instead of generalizing the model, SPLITS aim to specialize by creating time-dependent substructures that model the training sequences more precisely. This is done by splitting an interval of a transition at a certain time value. For the two resulting transitions, both appending subtrees are recomputed on the basis of the training sequences. Since a wrong SPLIT can be made undone by the more powerful MERGE operation, SPLITS will always be preferred to MERGES.

Usually, many possibilities to perform MERGES and SPLITS exist in every iteration of RTI+. For determining useful operations and for deciding which of these operations to perform, Verwer et al. introduce a statistical Likelihood Ratio Test (LRT) for PDRTAs.

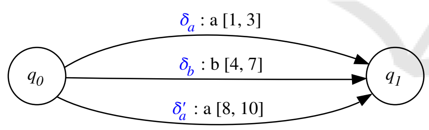
In each iteration RTI+ tests all possible SPLITS and MERGES before greedily performing the operation that results in a PDRTA which should model the training sequences best. RTI+ may also perform no operation at all if the actual PDRTA is the best option. Before presenting our approach to improve SPLITS of time values we describe and identify the reason for improvement in the next section.

3 GAPS IN INTERVALS

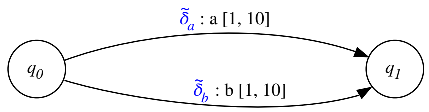
In this section we first describe the problem of RTI+ when creating time intervals of timed transitions, followed by two reasons why intervals are identified incorrectly.

The intervals of the transitions of PDRTAs that are learned by RTI+ may contain segments that are not covered by sequences from the training set. In the following we refer to these segments as gaps.

For demonstrating the influence of RTI+ in creating intervals with gaps, we use the PDRTA from Fig. 1 to sample a set of training sequences. Afterwards we let RTI+ recreate the automaton. For our example we only consider the transitions between state q_0 and state q_1 which are shown in Fig. 2(a). There exist three transitions δ_b, δ_a and δ'_a that go from state q_0 to state q_1. Transition δ_b is used for symbol b and time delays in interval $[4,7]$. In addition, transition δ_a (resp. δ'_a) is used for symbol a and interval $[1,3]$ (resp. $[8,10]$).



(a) Transitions between state q_0 and state q_1 in the original PDRTA shown in Fig. 1.



(b) Transitions between state q_0 and state q_1 in the PDRTA generated by RTI+.

Figure 2: Differences for an excerpt of the PDRTA in Fig. 1.

The transitions of the PDRTA trained with RTI+ are shown in Fig. 2(b). In contrast to the original automaton the transitions δ_a and δ'_a are now modeled by a single transition $\widetilde{\delta}_a$. Furthermore, the interval of $\widetilde{\delta}_b$ is extended compared to the original δ_b. This is caused by the disability of RTI+ to detect gaps in intervals. For $\widetilde{\delta}_b$ these gaps are in $[1,3]$ and $[8,10]$ while for $\widetilde{\delta}_a$ the gap is in $[4,7]$.

After exemplarily showing the existence of gaps in intervals, we investigate the reasons for those gaps to appear in learned PDRTAs. Fig. 3 shows a visualization of the three different types of gaps in an interval. The blue bars mark segments of time delays that are covered by sequences of the training set. The gaps are caused by one of the following reasons:

Symbol Diversity. Multiple symbols have different domains of time delays. Since the initial tree is constructed with intervals defined by the global minimum and maximum of delays in the training set, large gaps may appear at the beginning or the end of intervals (cf. Fig. 3). The gaps in $\widetilde{\delta}_b$ are caused by this effect.

Time Delay Partitioning. Time delays of a symbol are partitioned into several disconnected subdomains. Intervals then contain gaps between the subdomains, e.g. the subdomains $[x,x']$ and $[y,y']$ with $x \leq x' \ll y \leq y'$ surround a gap between x' and y. This applies for the gap in the interval of $\widetilde{\delta}_a$. In this example we say $3 \ll 8$ related to the global maximum delay of 10. These gaps can be a lot larger in real-world scenarios. We visualized this gap type in Fig. 3.

Training Set Incompleteness. The finite set of training sequences usually does not contain all possible sequences of the (infinite) model space. Therefore, some time delays within the domain(s) of a symbol may not be included in the training set for some states in the PDRTA. These missing time delays also form gaps. One of those gaps is shown in Fig. 3. The gaps of this type may not only appear in between covered segments but also next to gaps of the other two types.

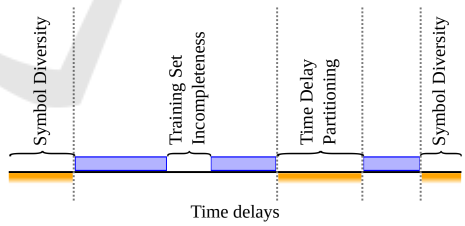


Figure 3: Visualization of the different gap types *Symbol Diversity*, *Time Delay Partitioning* and *Training Set Incompleteness* in an interval.

All of the gap types mentioned above are potentially included in PDRTAs that were learned by RTI+. But only *Training Set Incompleteness* is part of the normal behavior model. Hence, a PDRTA should return a probability of zero for test sequences that contain time delays lying in gaps of types *Symbol Diversity* and *Time Delay Partitioning*. This result cannot be guaranteed since Verwer et al. model the histogram bin probability distributions independently of the symbol probability distributions. In fact, such sequences can inherit a positive probability from other symbols' time delays. For example the pair of symbol and time delay $(a,5)$ is not part of the original PDRTA in Fig. 2(a). Nevertheless, it receives a positive probability for its time delay from the histogram bin $[4,7]$ in the reconstructed PDRTA in Fig. 2(b). This histogram bin is only used by pairs with symbol b in the original PDRTA and, hence, in the training sequences. Thus, the false probability assignment can cause false sequence predictions and should be avoided.

This problem could be easily overcome by using a symbol-dependent histogram bin probability distribution. But when designing RTI+, Verwer et al. decided to model the probabilities for symbols and histogram bins independently to avoid that the size of PDRTAs is increased by a polynomial factor (Verwer et al., 2010). This feature should be left untouched as the structural identification ability of RTI+ is recognizably good.

Our approach is to extend RTI+ by a new feature that simulates symbol-dependency for time delays. In contrast to the real symbol-dependency, our extension can be easily adapted and its impact on the resulting PDRTA can be scaled based on the needs for the current problem domain. In the following section we introduce the new feature called Interval Distribution Analysis (IDA) in detail.

4 INTERVAL DISTRIBUTION ANALYSIS (IDA)

In this section we describe the Interval Distribution Analysis (IDA) that removes segments of intervals that are not part of the normal behavior model.

Above we described how time delays that are not part of the model can inherit positive probabilities. In our approach, we try to avoid this wrong assignment of probabilities by limiting the paths in the resulting PDRTA. Thus, sequences which have time delays outside of the domains of the corresponding symbols do not have a path in the PDRTA after the paths have been limited. We limit the paths by removing gaps from the intervals of transitions during training. To realize this approach, we use the present SPLIT operation with a new heuristic to determine and remove empty segments in intervals.

First, reconsider which gaps should be removed and which should be kept according to the three cases described in the previous Section 3. The gaps from the first two types *Symbol Diversity* and *Time Delay Partitioning* are clearly not part of the normal behavior of the process to be modeled and should be removed. In Fig. 3, we underlined the segments that should be removed from the PDRTA in orange color. On the contrary the gaps of the third type *Training Set Incompleteness* are part of the normal behavior that is not covered by the training set and, hence, should be kept.

To be able to distinguish between gaps to remove and gaps to keep, we make an initial assumption for the training set: We assume that the gaps from the third case *Training Set Incompleteness* are smaller than those from the other two cases because the training sequences represent the model to be learned adequately. This transforms the problem to removing only larger gaps. For detecting the larger gaps, we use statistical methods for distribution analysis on the distances between covered elements in an interval. Therefore, we call our new feature Interval Distribution Analysis (IDA). IDA computes a maximally allowed gap $\widehat{g}$ for an interval by collecting and analyzing the distances between neighbored covered elements. For this analysis, we also consider the distances of size 0 between directly neighbored covered elements. Thus, we are able to take connected blocks of covered elements into account. To compute $\widehat{g}$ from the collected distances, we perform a statistical outlier detection. We use outlier-robust measures since the analyzed distributions of distances are unknown and our analysis should not be distorted by outliers. After the outlier detection, we remove the outlier distances that are larger than the expected distances and, thus, are gaps.

A suitable measure is the outlier detection with Median Absolute Derivation (MAD) (Leys et al., 2013). By using median and MAD, we define $\widehat{g}$ as the upper border for outliers as follows:

$$\widehat{g}_{MAD} = m + 2.5 \cdot \mathrm{MAD} = m + 2.5 \cdot \mathrm{median}\{|x_i - m|\}$$

where m is the median of the collected distances x_i. This formula is an outlier-robust and, thus, advisable replacement for the outlier detection with mean and standard deviation $\mu + 2.5 \cdot \sigma$ (Leys et al., 2013). We chose 2.5 as coefficient of the MAD which is moderately conservative according to Leys et al. To get $\widehat{g}_{MAD} = 0$, at least 50% of the distances need to have a length of 0.

Another way to calculate the border for outliers is the Interquartile Range (IQR). This is an outlier-robust standard method in statistics for outlier detection which was proposed by Tukey (Tukey, 1977). $\widehat{g}$ is then defined as:

$$\widehat{g}_{IQR} = Q_3 + 1.5 \cdot \mathrm{IQR} = Q_3 + 1.5 \cdot (Q_3 - Q_1)$$

where Q_i are the quartiles. This method allows larger gaps than $\widehat{g}_{MAD}$ because more than 75% of the distances need to have a length of 0 to get $\widehat{g}_{IQR} = 0$.

After $\widehat{g}$ has been determined, the interval has to be segmented at the correct positions with SPLITS to remove the gaps. The borders of the resulting intervals should be chosen wisely, especially not directly after a

Input: A training set with timed sequences S_+, a set of histogram bins H and a significance level σ

Output: A small PDRTA A according to the input

1 Initially create PDRTA A as a tree from the input sequences in S_+
2 Color the start state q_0 of A red
3 **while** *A contains non-red states* **do**
4   Color blue all non-red target states of transitions with red source states
5   Let $\delta = \langle q_r, q_b, s, [n, n'] \rangle$ be the most visited transition from a red state q_r to a blue state q_b
6   **if** *$[n, n']$ is an initial interval* **then**
7   Check interval for empty spaces that should be removed by performing IDA
8   **if** *there exist such empty spaces* **then**
9   Remove these empty spaces with SPLITS
10   **if** *δ has not been modified by IDA* **then**
11   Evaluate all possible MERGES of q_b with red states
12   Evaluate all possible SPLITS of $[n, n']$
13   **if** *the lowest p-value of a* SPLIT *is less than σ* **then**
14   Perform this SPLIT
15   **else if** *the highest p-value of a* MERGE *is greater than σ* **then**
16   Perform this MERGE
17   **else**
18   Color q_b red

Algorithm 1: Extended RTI+ (eRTI+) (adapted version of the pseudo code in (Verwer et al., 2010)) which contains the new IDA routine (blue, lines 6-10).

covered element. This results from the gap type *Training Set Incompleteness* which says that some time delays may be missing in the training set. Hence, by splitting the interval not directly before or after a covered element, we want to create small gaps of type *Training Set Incompleteness*. For a margin between a SPLIT position and covered elements, we chose $\lceil \widehat{g}/2 \rceil$. Thus, a gap between two neighbored covered elements is at most of size $\widehat{g}$ and the distance from the interval borders to the first or last covered element is $\lceil \widehat{g}/2 \rceil$ after the interval has been split according to IDA.

Our new IDA procedure has now to be integrated into RTI+. We call this alternative procedure Extended RTI+ (eRTI+). The empty gaps should only be removed once from initial and untouched intervals. If IDA would be applied again to a resulting interval, IDA might lead to removing even more gaps unintentionally from this interval. Hence, the following change in the iteration routine of RTI+ will be applied (cf. Algorithm 3, lines 6-10). After identifying the most visited transition, RTI+ checks if the according interval borders are the global minimum and maximum of time delays in the training set which gives evidence whether IDA already processed this interval. If the interval is initial, IDA will be performed afterwards to determine whether or not gaps can be removed from the interval. If there are any gaps, they will be removed with SPLITS and the iteration ends because the determined most visited transition no longer exist in its original form and has to be recomputed. If no gaps can be removed, the original iteration routine with testing SPLITS and MERGES will be performed.

To show the usefulness of our new feature, we rerun our experiment from the beginning of Section 3. But this time we deploy eRTI+ for reconstructing the PDRTA. Within IDA we use $\widehat{g}_{MAD}$ for determining the maximally allowed gap. With the help of IDA we are able to identify all transitions and intervals exactly. Hence, all gaps have been detected and removed correctly. Note that by using $\widehat{g}_{IQR}$, an exact reconstruction was not possible since $\widehat{g}_{IQR}$ allows larger gaps than $\widehat{g}_{MAD}$. Nevertheless, even with $\widehat{g}_{IQR}$ we achieved a PDRTA that is closer to the original one than the PDRTA generated by the original RTI+ (cf. Fig. 2). All in all, we are able to reduce or eliminate false probability assignments for sequences by using IDA in support of RTI+. With this result in mind, we evaluate IDA in experiments on ATM fraud detection with real-world data in the following section.

5 EXPERIMENTS WITH ATM LOG DATA

In this section we compare RTI+ and eRTI+ in the context of anomaly detection to detect ATM fraud. When withdrawing money from an Automated Teller Machine (ATM) the user usually inserts a card, enters a pin, chooses the amount of money, takes out the card again and finally the money. A delay is present between every of these steps. Additionally, some users e.g. just insert the card without entering the pin but pushing the cancel button and aborting the withdrawing. Hence, a PDRTA is a suitable behavior model for the withdrawing sequences of ATMs.

We use data from a publicly available ATM that was gathered in the period of nine months. In this time no attacks were registered, so the data set comprises normal sequences with 15 million events, resulting in size of 1.6 GB. To be able to measure the anomaly detection effectiveness of RTI+ and eRTI+ we need data containing fraud (attempts). As it is almost impossible to get real-world data of ATM fraud and since the simulation in a laboratory is very expensive, we choose to insert anomalies randomly into normal sequences. Hence, we modify a normal sequence by either switching two events in a sequence (anomalous event) or multiplying a single time value in a sequence (anomalous event timing). For a given PDRTA A (learned from normal sequences) we decide whether a given sequence s is normal by calculating the probability of

s given A. We do so by traversing the PDRTA A and computing the symbol and time probability in every state using the symbol and time probability distributions $\mathcal{S}$ and $\mathcal{T}$. Then, we multiply all probabilities and normalize by dividing by sequence length. Finally, we compare the result of these operations with a threshold and decide whether the given sequence s is abnormal with respect to the PDRTA A.
For testing, we implemented four variants of IDA:

v1: Use $\widehat{g}_{MAD}$ and remove all gaps discovered by IDA

v2: Use $\widehat{g}_{IQR}$ and remove all gaps discovered by IDA

v3: Use $\widehat{g}_{MAD}$ and remove only border gaps according to IDA

v4: Use $\widehat{g}_{IQR}$ and remove only border gaps according to IDA

We only present the results of IQR while removing only border gaps (v4) or all gaps (v2). We do not show the results of IDA with MAD because it always performed worse than IQR (opposed to experiments with artificial data). We measure the effectiveness of IDA in terms of precision, recall and F-measure. The precision is the ratio between correctly detected anomalies and all detected anomalies, while the recall is the ratio between detected anomalies and all anomalies. The F-measure (F1) is the harmonic mean between precision and recall. In real-world scenarios a high precision (low false positive rate) often improves existing business cases without any negative side effect. Nevertheless, we want to detect all anomalies, thus present the recall. The F-measure can be regarded as the trade-off between precision and recall. Figures 4 and 5 show the result of applying RTI+ with and without IDA when optimizing for F1 and precision.

In general, IDA with IQR improves the recall and often the F-measure. Additionally, IDA leads to more false positives because more normal sequences have no path in the inferred automaton as IDA removed gaps on purpose.

When optimizing for F1 (Fig. 4), IDA v4 improves the recall without a loss of precision while IDA v2 decreases recall and precision. The results in Fig. 5 (precision) are two fold. IDA variants 2 and 4 improve recall and F1 at the cost of lower precision. As in Fig. 4 IDA v4 achieves a similar precision as RTI+ but a higher recall while v2 drastically decreases the precision. In the scenario of ATM fraud detection a high precision is more important than a high recall because false positives (low precision) are costly causing technicians to examine the ATM, whereas a recall greater zero already yields an improvement.

Note that we applied eRTI+ with the same hyperparameter setting that we used for RTI+ to show the change caused by IDA. Maybe, we could achieve a better result when tuning the hyperparameters directly for eRTI+ because RTI+ is very sensitive with respect to the hyperparameter configuration. Additionally, the real-world data set may cause odd results because the data-generating process is not controllable.

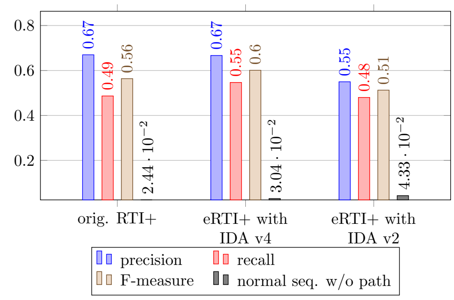


Figure 4: Precision, recall, F-measure and relative frequency of normal sequences without path for original RTI+ and eRTI+ with IDA variants v4 and v2. The results had the best F-measure for their RTI+ variant.

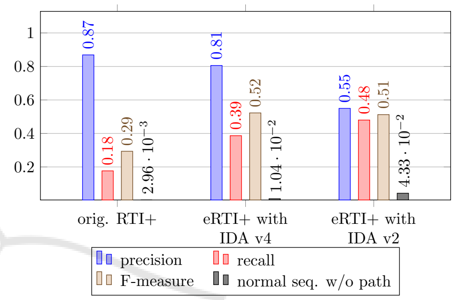


Figure 5: Precision, recall, F-measure and relative frequency of normal sequences without path for original RTI+ and eRTI+ with IDA variants v4 and v2. The results had the highest precision with recall greater zero for their RTI+ variant.

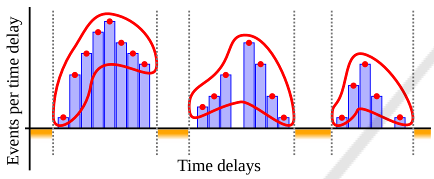
All in all, IDA usually improves the F-measure and recall compared to RTI+ in the ATM context. On the other hand the precision most often drops which is not desirable for ATM fraud detection. For other scenarios a high F-measure may be more important than a high precision, gaining a benefit from applying RTI+ with IDA.

6 FUTURE WORK

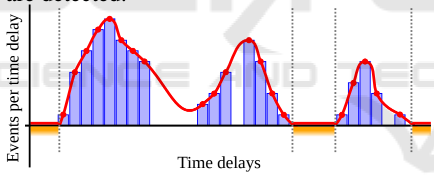
During the development of IDA, we already had some ideas to advance the IDA procedure. In this section, we describe how we can apply alternative algorithms to solve the IDA problem. Furthermore, we propose a new approach for IDA and RTI+ to work together.

First, we interpret IDA in a different way: From a more general perspective, the IDA procedure solves a (one-dimensional) density-based clustering problem without a predefined number of clusters. In the context of IDA, the unsplit segments of an interval form clusters and the gaps between the clusters are removed.

Therefore, we can use established clustering algorithms to identify gaps in an interval. Possible algorithm candidates are DBSCAN (Ester et al., 1996), OPTICS (Ankerst et al., 1999), and X-means (Pelleg and Moore, 2000). By considering the frequency besides the raw time values, we can transform IDA into a two-dimensional clustering problem. This extension to two dimensions might lead to better results. In Fig. 6(a) we show how IDA could look like with a clustering algorithm. The histogram values are combined into three clusters. The interval segments that are not covered by a cluster are removed (orange underlining), while the covered segments are kept. Since the density-based distance might be less important than the time-based distance, we can think of an additional weight parameter for eRTI+.



(a) The IDA problem solved with a two-dimensional clustering algorithm. Four gaps are detected.



(b) The IDA problem solved with a density function. Three gaps are detected.

Figure 6: Alternative approaches for solving the IDA problem.

Apart from clustering, a completely different approach would be to learn a density function of the interval where the parts without sequences will contain no data. The method we use for learning the density can be Kernel Density Estimation (Parzen, 1962) among others. The learned function will possibly approach zero for large gaps with high slope. We can split the interval at the points where the density function approaches zero. Fig. 6(b) shows how the IDA problem is solved with a density function. The density function is constructed from the histogram values and approaches zero four times. The interval segments where the density function is zero are removed (orange).

As an alternative to density functions, we can also fit general polynomials to the interval data. Accordingly, we can chose from a wider range of methods, e.g. Support Vector Regression (Drucker et al., 1997). In contrast to density functions, general polynomials can have values below zero where the slope is high. Hence, we can identify gaps where the polynomial is below zero and perform the necessary SPLITS at the zero-crossings. By considering a high slope of a learned function as an indicator for gaps, this approach might produce more natural results than the original IDA or the clustering approach.

Besides using alternative algorithms for IDA, we can also alternate the strategy of applying IDA. Instead of applying it actively during the training of a PDRTA, we could also apply IDA passively on the final PDRTA after RTI+ has terminated. We then iterate over all states of the PDRTA and apply IDA to all outgoing transitions of each state. If a transition between two states is split, the new transitions will be created in parallel between the same two states. Hence, the number of states in PDRTA is not increased by IDA and the basic structure is preserved. With this new strategy we want to avoid inconsistencies between the LRT and IDA. With our original strategy (cf. Algorithm 3) this inconsistency might occur when eRTI+ merges two states based on the LRT which have already been processed by IDA. Since the LRT cannot consider gaps, it is possible that sequences from one state are merged into the removed gaps of the other state. This will negatively affect the quality and size of the PDRTA. We can avoid this risk if we apply IDA on the PDRTA after RTI+ finished the identification.

Based on the various possibilities to create alternative IDA procedures and apply them in two different ways, we are confident to develop procedures that are applicable to many different types (e.g. crude or smooth) of training data. The validation of those procedures will be part of our future research.

7 RELATED WORK

Besides RTI+ other algorithms also infer automata inference from timed sequences. In this section we review other approaches and point out differences.

In (Verwer et al., 2012) Verwer et al. describe the Real-Time Identification (RTI) algorithm. It is based on the same idea as RTI+ but infers a DRTA with accepting and rejecting states. Therefore, it requires positive and negative labeled sequences for training.

The Bottom-Up Timed Learning Algorithm (BUTLA) (Maier, 2015) learns a Probabilistic Deterministic Timed Automaton (PDTA; very similar to a PDRTA) from positive timed sequences. Opposed to RTI+, BUTLA only performs a merge operation but no split. Instead, it performs a global preprocessing of all time values by fitting kernel density estimators and computing their local minima. For every local minimum BUTLA creates a subevent. This preprocessing shall remove the necessity of split operations.

In (Klerx et al., 2014) a Probabilistic Deterministic Timed-Transition Automaton (PDTTA) and an algorithm for learning PDTTAs are presented. The learning algorithm does not split events (like BUTLA) or transitions based on time values (like RTI+). Instead, it learns the event structure using any state-of-the-art algorithm (e.g. ALERGIA; (Carrasco and Oncina, 1994)) and approximates the time values per transition via kernel density estimators. Hence, it models the time behavior in more detail, is easier to learn, but cannot detect temporal substructures.

8 CONCLUSION

RTI+ is an efficient algorithm that learns PDRTAs from timed sequences. We have revealed a deficit of RTI+ in learning broadened intervals for time values. Combined with the independence of symbol and time probability distributions, this deficit leads to wrong predictions of sequences. We have investigated that two of three types of gaps cause the broadened intervals and developed our novel IDA procedure to remove those gaps. IDA has been integrated into the RTI+ algorithm, which we now call Extended RTI+ (eRTI+). We have shown that IDA is an effective way to eliminate the disadvantage of the independent time and symbol probability distributions used by RTI+. For our experiment with an artificial example PDRTA, IDA was able to identify and remove all gaps in intervals. IDA was also able to improve the results in the experiment with ATM fraud detection. Although IDA did not work optimal on this real-world data, we are confident that this result can be improved further. IDA is a very flexible and adaptable procedure. As mentioned in Section 6, we want to apply IDA after RTI+ has terminated instead of integrating IDA into the procedure in the future. Furthermore, we plan to replace our statistical outlier detection by established clustering algorithms and density estimation procedures.

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