Fast Skeletons of Handwritten Texts in Digital Images

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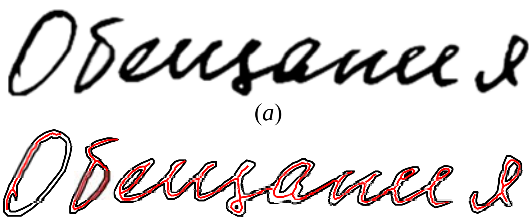
Abstract: The article considers the problem of constructing a Voronoi Diagram (VD) of a polygonal figure - a polygon with polygonal holes. A planar sweeping algorithm is proposed for constructing the VD of the interior of a polygonal figure with n vertices, which has complexity $O(n \, log \, n)$. Two factors provide a reduction in the amount of calculations and an increase in robustness compared to known solutions. This is the direct construction of only the inner part of the VD, as well as the use of the pairwise incidence property of linear segments formed by the sides of a polygonal figure. The proposed algorithm has been implemented and practically tested for polygonal figures of dimension $n \sim 10^5$ in studies on the analysis and recognition of handwriting. Computational experiments illustrate the robustness and efficiency of the proposed method.

1 INTRODUCTION

A polygonal figure (PF) is a closed area on a plane, the boundary of which consists of non-intersecting simple polygons, i.e. it is a polygon with polygonal holes. The skeleton or median axis of such a figure is the set of points of the centers of the maximum circles contained within the figure. Skeletons are used in shape recognition tasks, in particular, for recognizing handwritten text from digital images. The image of the text is approximated by polygonal figures, and then a skeleton is built for this set of figures (Fig. 1) (Mestetskiy, 2008).



(*a*) (*b*)

Figure 1: (*a*) the binary bitmap, (*b*) polygonal figure and its skeleton.

Figures and skeletons in practical problems of the analysis of archival handwritten documents have large number of vertices of the order of 10^5 (Fig. 2).

The skeleton of PF is a subgraph of the generalized Voronoi diagram (VD) of the set of site-points and site-segments created from the vertices and sides of the figure. Algorithms for constructing VD and image skeletons of handwritten documents are subject to high requirements for robustness and computational efficiency for application in large archives of digitized manuscripts. Therefore, when choosing algorithms for practical use, their real speed and robustness are important.

The computational efficiency of algorithms for constructing skeletons and VD has always been in the spotlight. The algorithm (Lee, 1982) builds the inner skeleton of a simple n-gon in $O(n \, log \, n)$ time. Later, a solution appeared for multiply connected polygonal regions (polygons with holes) with complexity $O(n(log \, n + m))$, where n is the number of polygon vertices and m is the number of holes (Srinivasan, 1987). However, this algorithm for such images as in Fig.2, in which $m = O(n)$, spends $O(n^2)$ time. Algorithms for constructing a VD of a set of n line segments (Fortune,1987), having complexity $O(n \, log \, n)$, are a guideline for developing more efficient solutions. Reducing the PF to a finite set of linear segments can be done in $O(n)$ time. Later works develop the topic of constructing theoretically efficient algorithms (Bae, 2015). However, the practical implementation of these approaches faces the problem of algorithm robustness.

The problem of robustness arises in solving many problems of computational geometry. It consists of the gap between theoretically correct geometric algorithms and practical computer

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programs. This is mainly due to the fact that the actual calculations contain numerical errors; these errors sometimes cause inconsistencies in the topology and thus lead to program crashes (Imai, 1996, Sugihara, 2000). When constructing a VD, the source of such errors is the calculation of large inscribed circles. The circle incident to the triple of almost collinear sites has a very large radius, since its center is the intersection point of the "almost parallel" bisectors. To ensure the practical robustness of algorithms, it is necessary to apply various heuristic techniques focused on the features of specific software solutions (Sugihara, 2000, Held, 2001, Menelaos, 2004). In this case, the theoretical computational efficiency $O(n \log n)$ is not achieved.

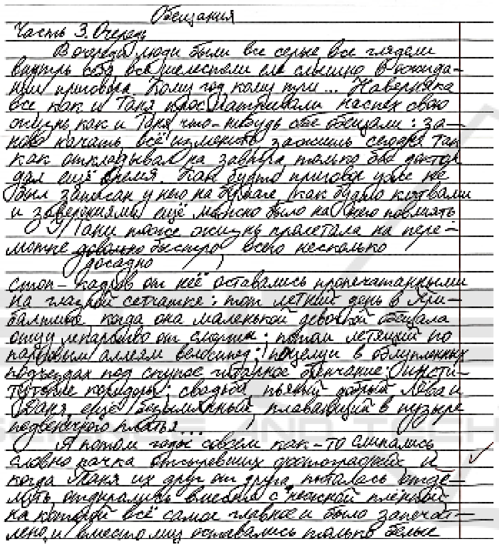

Figure 2: Text approximation by polygonal figures: 244 connected components, 1258 polygons, 63556 vertices. Skeleton: 95884 vertices, 96654 edges.

The solution proposed in this paper is based on the special feature of the VD PF problem, which allows one to reduce it to line segments VD in such a way as to avoid fatal numerical errors. This feature consists in the fact that it is necessary to find only the inner part of the VD that lies inside the PF. Therefore, if only inner parts are built, then such errors can be avoided, since all the inner inscribed circles of the figure do not exceed the size of the figure itself. But this requires a special algorithmic solution that will allow building only internal circles.

This feature can also be used to improve the computational efficiency of the algorithm. If the outer part of the VD is not needed, then it becomes possible to save on its calculations. Known algorithms for constructing a VD PF do not take this possibility into account. They build the VD of line segments on the entire plane, and then select the inner part in post-processing.

This article presents an algorithm for the direct construction of a VD for the internal part of the PF, based on the paradigm of sweepline. The proposed solution includes new elements that make it possible to build only the inner part of the VD PF. First, the concept of oriented site-segments is introduced, for which the inner side of the PF is defined. Secondly, the data structure Sweepline Status, which is traditionally present in sweepline algorithms, is built as an ordered set of site zones. Such a structure is an alternative to the traditionally used wave front or coastline.

In addition, the algorithm takes into account the specificity of the set of sites formed by the boundary of a polygonal figure, namely, the incidence of each point-site to two segment-sites and each segment-site to two point-sites. This property helps to perform a significant part of the operations in the status in constant time. To do this, the status is implemented as a combination of a balanced tree and a doubly linked list.

2 INNER VORONOY DIAGRAM

The boundary of a polygonal figure consists of one outer and several inner polygons. All of them are described by sequences of their vertices in such a way that the interior of the PF is to the left of the boundary. Each boundary polygon is broken down into subsets called sites. The vertices of the figure define a set of sites-points, and the sides of the figure define a set of sites-segments. Segment sites have a direction in accordance with the direction of the figure border, i.e. the interior of the shape lies to the left of the segment site.

The locus of a site is the set of points in a figure for which this site is the closest. This site is called the generating site of the locus. And the closest site for a point of a figure is determined by the position of its closest point on the border of the figure. If the closest point of the boundary coincides with the vertex of the figure, then the site corresponding to the vertex is the closest one. If the closest point is the orthogonal projection of the figure's point onto the segment site, then that site is the closest.

The boundary between a pair of loci consists of points equidistant from their generating sites, and is called the bisector of these sites. The set of bisectors of all loci is called the internal Voronoi diagram. It has the form of a geometric graph, the vertices of which are the points of the figure, and the edges are segments of straight lines and quadratic parabolas.

3 SECTIONS AND ZONES

The PF boundary can be represented as a finite set of monotone branches. Each branch is a polyline, the vertices of which are ordered lexicographically from left to right (Fig. 3).

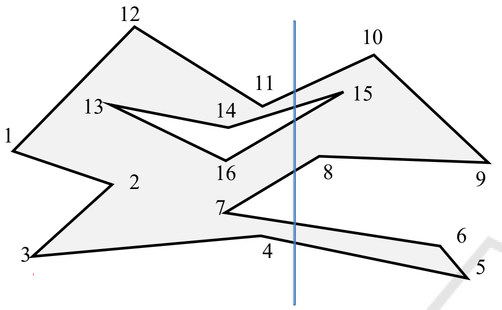

Figure 3: Monotone branches of the PF boundary: 1-12-11-10-9, 7-6-5, 7-8-9, 3-4-5, 1-2, 3-2, 13-14-15, 13-16-15.

A sweeping line (SL) is a vertical line that moves from left to right parallel to itself.

The branches break the sweeping line into connected subsets, the so-called sections. The sections inside the figures are called internal, outside the figure - external (Fig. 3).

The internal sections, in turn, are divided into so-called site zones, defined as follows. For each point of the inner section, there is a maximum circle that lies inside the figure in the left half-plane relative to the sweeping line and touches it at this point. Since the circle is maximal, it also touches the inside of the figure's boundary at one or more points. Each touch point belongs to a site. These sites and the circle are called incident.

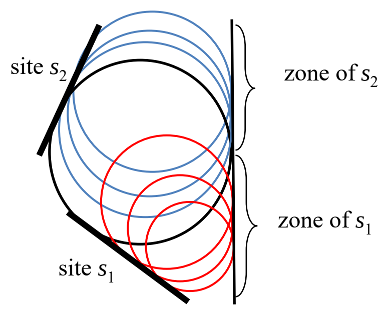

Figure 4: Zones of sites s_1, s_2. Circles of zone s_1, s_2 are red and blue. Common circle of two zones is black.

A site zone is a segment of a sweeping straight line, all points of which have tangent circles incident to this site. We will call this incident site the zone generator (Fig. 4). For terminology convenience, the external sections are also referred to as external zones.

Thus, site zones and external zones cover the entire SL. The order relation is defined on the set of zones – from bottom to top. As the SL moves, this set of zones changes: zones appear and disappear. The zone at the moment of generation has a length of 0, i.e. this is a point on the SL. As the straight line moves to the right, it turns into a segment, the size of the zone increases. Thereafter the size of the zone decreases to zero, it degenerates into a point and disappears.

The data structure that describes the ordered set of zones on the SL is called the "status of the SL", as in all sweepline algorithms (Preparata, 1985).

4 THE SWEEPING PROCESS

As the SL moves, all zones on it go through the same life cycle: generation - resizing (growth and contraction) - splitting (possible, but not mandatory) - disappearance.

The dynamically changing neighborhood of zones in the status of the SL completely determines the structure of the VD. Therefore, the task is to trace all status changes and identify all neighboring pairs of zones in the changing status structure.

The connection between the neighborhood of zones in the status and the edges and vertices of the VD is determined by the following statements.

1. If two sites have a common tangent inscribed circle, then there is such a position of the SL and such a pair of adjacent zones in status, for which these two sites are generators.

2. If two zones are adjacent in status, then the generator sites of these zones have adjacent loci in the VD, and the common boundary of these loci describes an edge in the VD.

3. If three sites have a common tangent inscribed circle, then there is such a position of the SL and such a triple of adjacent zones in status, for which these three sites are generators.

From Statements 1 and 2 it follows that each pair of adjacent internal zones in the status defines a Voronoi edge in the VD. This implies that if any two zones become adjacent in the process of sweeping, then the bisector of the sites-generators of these zones is an edge of VD.

The adjacency of zones changes during sweeping only at the points of events. A change in the adjacency of zones occurs when new zones are created and when existing ones disappear. The generation of zones occurs when the SL passes through the vertex of the PF. This position of the SL is called a vertex event.

The new zone included in the status forms two new adjacent pairs with the zones above and below it. When a zone disappears and is excluded from its status, two zones located above and below it form a new pair of adjacent zones. These new pairs automatically generate VD edges.

Statement 3 allows us to calculate the vertices of the VD. As soon as any three zones become neighbors in the status, we need to check if there is a common inscribed tangent circle for the site generators of these zones.

Thus, in order to find all vertices and edges of the VD, it is necessary to track all pairs and triples of adjacent zones in the status during sweeping.

5 INITIALIZATION OF ZONES

The general idea of the sweeping process is as follows. The sweep line moves from left to right discretely, occupying positions corresponding to the moments of events. Each event leads to a change in the status of the SL: the vertex event leads to the appearance of new zones, and the circle event leads to the exclusion of the zone from the status. At the same time, the set of zones in the status retains order in all changes.

Changes in the status lead to a change in the neighborhood of zones on the SL. New neighboring pairs and new triples of neighboring zones are formed. Tracking status changes, we get all the edges and vertices of the VD.

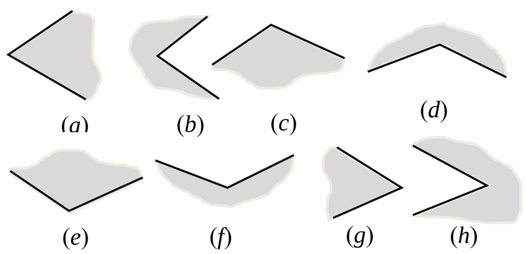

Figure 5: Types of vertices of a polygonal figure: left (*a*, *b*), through (*c*, *d*, *e*, *f*), right (*g*, *h*), convex (*a*, *c*, *e*, *g*), concave (*b*, *d*, *f*, *h*), lower (*d*, *e*) and upper (*c*, *f*). Interior polygon is marked in grey.

Thus, in the process of sweeping, it is necessary to make changes to the status for each event, identify all newly formed neighboring pairs and triplets of zones, and calculate the corresponding vertices and edges of the VD.

Changes in status upon the occurrence of a vertex event are determined by the vertex type. There are 8 types of vertices in total (Fig.5).

Denote:

v is a vertex site formed by the vertex of the polygon,

s_{pr}, s_{sc} are the segment sites formed by the sides of the polygon before and after the vertex site v in the cyclic list of polygon sites,

$z(s)$ is a zone that has site s as a generator,

z_* is an external zone that does not have a generator site.

The vertex event changes a set of zones in the status. These changes are uniquely determined depending on the type of vertex with which the event is associated. Variants of this status transformation are described below for all types of vertices shown in Fig.5. The "***Input***" line shows a fragment of the status before the event, and the "***Output***" line shows the same fragment after the event.

Left Convex Vertex

Site-point v falls into the outer zone z_*. Since the direction of traversal of the polygon is such that the interior of the PF lies to the left, in this case the segment site s_{pr} lies above the segment site s_{sc}. As a result of processing the event, the z_* zone is split into two outer zones, and a new inner section is generated between them, which consists of two new zones with generator sites s_{sc} and s_{pr}. The result of the status conversion looks like this (Fig.6*a*):

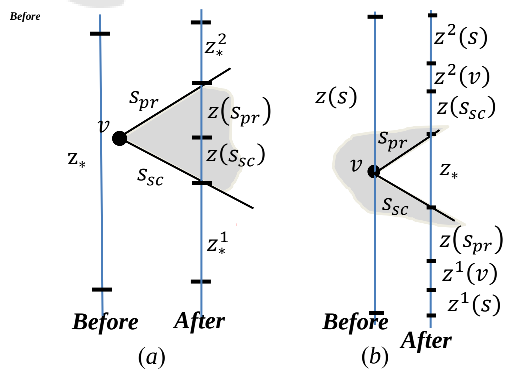

Figure 6: Left vertex event (*a*) convex, (*b*) concave.

***Input*:** $\ldots, z_*, \ldots$
***Output*:** $\ldots, z_*^1, z(s_{sc}), z(s_{pr}), z_*^2, \ldots$

One new pair of neighboring zones $\{z(s_{sc}), z(s_{pr})\}$ is formed.

Left Concave Vertex

The point site v falls into the inner section in one of the zones $z(s)$ with the site-generator s. As a result of processing the event, the zone $z(s)$ is split into two zones $z^1(s)$ and $z^2(s)$ having the same generator site s. And between $z^1(s)$ and $z^2(s)$ a chain of five new zones is generated and inserted. One of them is zone is external zone is z_*, it lies inside the hole of PF. Two zones have a generator site v and in two zones segment sites s_{pr} and s_{sc} are generators (Fig. 6*b*).

***Input*:** $\ldots, z(s), \ldots$
***Output*:**
$\ldots, z^1(s), z^1(v), z(s_{pr}), z_*, z(s_{sc}), z^2(v), z^2(s), \ldots$

Four new pairs of neighboring zones are formed here:
$\{z^1(s), z^1(v)\}$, $\{z^1(v), z(s_{pr})\}$, $\{z(s_{sc}), z^2(v)\}$, $\{z^2(v), z^2(s)\}$.

The zone $z(s)$ is split into two parts $z^1(s)$ and $z^2(s)$, which inherit the neighborhood of $z(s)$ from above and below.

Intermediate Convex Vertex

In the case of an intermediate vertex v, SL intersects the left site-segment incident to v, before v. This is one of the adjacent s_{pr} or s_{sc} segment sites, depending on the orientation of the polygon. The transformation depends on the location of the polygon relative to this vertex (higher or lower). Depending on these factors, two options for transforming a set of zones are obtained.

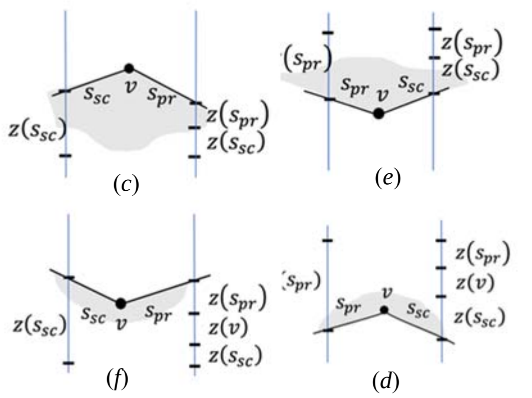

Figure 7: Intermediate vertex event: (*c*) upper convex, (*e*) lower convex, (*f*) upper concave, (*d*) lower concave.

Vertex v is upper convex (Fig. 7*c*):
***Input*:** $\ldots, z(s_{sc}), \ldots$
***Output*:** $\ldots, z(s_{sc}), z(s_{pr}), \ldots$

Vertex v is lower convex (Fig. 7*e*):
***Input*:** $\ldots, z(s_{pr}) \ldots$
***Output*:** $\ldots, z(s_{pr}), z(s_{sc}), \ldots$

In both cases shown in Fig. 7*c* and 7*e*, one new pair of neighboring zones $\{z(s_{pr}), z(s_{sc})\}$ is formed.

Intermediate Concave Vertex

Similar to the case of a convex vertex, this transformation depends on whether the polygon is above or below the vertex. There are two options for transforming the sequence of zones:

Vertex v is upper concave (Fig.7*f*):
***Input*:** $\ldots, z(s_{sc}), \ldots$
***Output*:** $\ldots, z(s_{sc}), z(v), z(s_{pr}), \ldots$

Vertex v lower concave (Fig. 7*d*):
***Input*:** $\ldots, z(s_{pr}), \ldots$
***Output*:** $\ldots, z(s_{sc}), z(v), z(s_{pr}), \ldots$

In both cases, in Fig. 7*f* and 7*d*, two new pairs of neighboring zones are formed: $\{z(s_{sc}), z(v)\}$, $\{z(v), z(s_{pr})\}$.

Right Convex Vertex

The transformation consists in deleting two zones of segment sites and "splicing" the two external zones (Fig. 8*g*).

***Input*:** $\ldots, z_*^1, z(s_{pr}), z(s_{sc}), z_*^2, \ldots$
***Output*:** $\ldots, z_*, \ldots$

In this case, no new pairs of adjacent zones are formed.

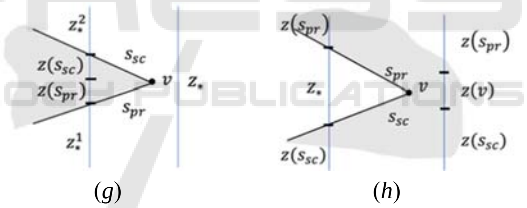

Figure 8: Right vertex events: (*g*) convex, (*h*) concave.

Right Concave Vertex

There is a “replacement” of the external zone by a zone with a generator site v (Fig. 8*h*).

***Input*:** $\ldots, z(s_{sc}), z_*, z(s_{pr}), \ldots$
***Output*:** $\ldots, z(s_{sc}), z(v), z(s_{pr}), \ldots$

Two new pairs of adjacent zones are formed: $\{z(s_{sc}), z(v)\}$, $z\{(v), z(s_{pr})\}$.

The analysis of vertex events shows that they lead to a correction of the status by the generation of up to five new zones. These zones are inserted into the status directly one after the other in a known sequence. In this case, up to four new pairs of adjacent zones are formed. Each pair of adjacent zones corresponds to a VD edge. It is necessary to calculate the endpoints of an edge in order to construct this edge explicitly. These endpoints are the vertices of the VD.

6 CLOSURE OF ZONES

In the geometric graph of the VD PF there are vertices of the first, second and third degree. Vertices of the first and second degree are points on the boundary of the PF. The convex PF vertices have degree 1 in the VD, and the concave vertices have degree 2. These vertices are formed when the vertex event occurs.

The VD vertex of the third degree is the internal point of the PF belonging to three loci. Such a point is the center of an inscribed circle tangent to the three sites. The generation of these VD vertices is associated with the so-called circle events.

A new zone after birth is placed in a status where it can form new triplets of neighboring zones with other zones. The number of such new triples can be from 0 to 3. If it is possible to build a tangent circle for the site-generators of a triple of neighboring zones, then its center can turn out to be the vertex point of the VD. But this will happen if this circle is "empty", i.e. PF sites lying to the right of the SL will not fall inside this circle. This cannot be checked at the moment the circle is formed, but when the SL moves to the right so that the entire circle is in the left half-plane, such a check should be made. The corresponding event, when the SL becomes tangent to the circle on the right, is called the circle event. When this event occurs, the middle zone from the three neighboring zones disappears and is excluded from the status.

When generating a new zone in the status, it is necessary to check the condition for the existence of common tangent circles for triples of site generators of the resulting new triples of neighboring zones. The calculation of the tangent circle of three sites is carried out by the methods of computational mathematics based on the solution of a system of equations or by geometric methods, for example, based on the intersection of bisectors (Marsden, 2020). If such a circle exists, then the center point of this circle is a candidate for declaring it the vertex of the VD. We need to schedule a circle event for this circle. Further, when this event occurs, the point center of the circle is declared the vertex of the VD.

The circle event leads to the disappearance and exclusion from the status of one zone. The middle zone is removed from the triple that defines the circle.

An illustration of the sequence of events and changes occurring in the status is shown in Fig.9. Here PF is a quadrilateral with segment sites s_1, s_2, s_3, s_4. The vertex events are associated with SL positions labeled *A*, *B*, *C*, *G*.

The inscribed circle of generator sites s_1, s_2, s_3 is formed for the three neighboring zones $z(s_1), z(s_2), z(s_3)$ at position *B*. This circle is shown in blue. When it is generated, a circle event is created, tied to position *F*. Further, in position *C* the zone $z(s_4)$ is included into the status. As a result, a new triplet of adjacent zones $z(s_2), z(s_3), z(s_4)$ is formed. There is an inscribed circle for sites s_2, s_3, s_4, for which a circle event *D* is created. This event is tied to the middle zone of the triple $z(s_3)$. When the event *D* occurs, the zone $z(s_3)$ degenerates into a point and is removed from the status. Then a new triplet of adjacent zones $z(s_1), z(s_2), z(s_4)$ appears after its removal. We build a circle for their site-generators and generate a circle event at time *E*. This event is tied to the middle zone $z(s_2)$ of the triple. Since this zone was previously tied to a blue circle, events are being rescheduled. Event *F* is deleted along with the circle attached to it, and a new circle event *E* is scheduled for the zone $z(s_2)$.

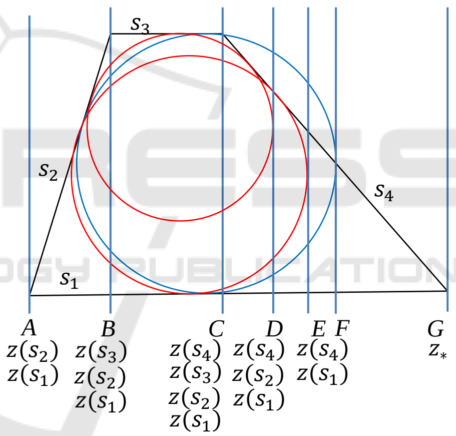

Figure 9: Quad sweeping, vertex events - *A*, *B*, *C*, *G*, circle events *D*, *E*, *F*. The composition of the zones is presented as a result for each event.

The vertex and circle events fully describe the process of obtaining edges and vertices of the VD during sweeping.

7 DATA STRUCTURES AND THEORETICAL COMPLEXITY

The algorithmic paradigm of SL is based on two data structures: List of events and Status of SL (Preparata, 1985). In the proposed algorithm, the list of events is presented in the form of a "priority queue" data structure, which is effectively implemented using an AVL tree (a self-balancing binary search tree). The list of events includes vertex events and circle events. The total number of events for an PF with n vertices is $O(n)$, and the complexity of processing one event is $O(log\ n)$.

A SL Status is a Dictionary structure that supports insertion, deletion, lookup, and neighbor selection for an ordered set of zones on a SL. The standard implementation of this structure based on the AVL tree is unable to take into account two features of the dynamic set of zones. Firstly, zones are inserted into the status in most cases not one at a time, but in batches of up to 5 zones at the same time. Secondly, all zones in one package have zero sizes at birth, i.e. are points on the SL that have the same ordinate. Therefore, it is necessary to insert the entire package of zones in one operation in a given sequence, as shown above in section 5. To solve this problem, it is proposed to implement the Status using a combination of a list of zones and an AVL tree of monotonous border branches described in section 2 and shown in Fig.3.

The list contains a set of zones ordered from bottom to top. The operations of inserting and deleting zones in the list are performed in constant time, if the place of insertion or deletion is known. In the case of the events "Passing vertex" and "Right vertex" this place is a zone of this vertex or of incident sites-segments. Thus, the processing of these events is performed in constant time.

For the "Left vertex" event, the insertion place is not known in advance, it must be determined based on the localization of this vertex in the current set of zones. The list makes it possible to do this only by sequential testing of zones. This operation has linear complexity in the number of zones. Since the number of zones in the state and the number of left vertices in the PF is $O(n)$, the localization of all left vertices in the list has complexity $O(n^2)$, which is unacceptable.

It is proposed to localize the left vertex in two stages: first, find the section in which the vertex is located, and then find the required zone in this section. The AVL-tree of monotone branches crossing the SL is used to search for a section (Fig. 3). The branches are ordered from bottom to top at the point of events, each pair of neighboring branches defines a section of the SL. This structure defines the section where the left vertex is located. Since the number of sections is $O(n)$, finding a section for a single vertex has $O(log\ n)$ complexity.

The search for the zone for the left vertex is performed on the found section. The section boundaries are defined by a pair of branches - upper and lower. And the branches have links to adjacent zones in the list, so the upper and lower zones of each section are known.

Further localization of the left vertex requires checking all zones of the section. For this purpose, the "fork" algorithm is proposed. The idea of the algorithm is to check the zones alternately from two edges inside the section.

Assume that the site contains k zones $z_1, \ldots, z_k$. The pair (z_{low}, z_{up}) defines the lower and upper zones of the search interval. First $z_{low} = z_1$, $z_{up} = z_k$. Let's also designate z.$above$ and $z.under$ the zones in the status, located above and below the zone z.

The search for the zone containing the left vertex v is carried out iteratively, each iteration includes two checks:

if the vertex v is located in the zone z_{low}, then z_{low} is returned, otherwise the search interval is compressed from below $z_{low} = z_{low}.above$;

if the vertex v is located in the zone z_{up}, then z_{up} is returned, otherwise the search interval is compressed from above, $z_{up} = z_{up}.under$.

This iterative process is guaranteed to end successfully, since the vertex v necessarily falls into one of the zones in this section. The computational complexity of the search will be $O(k)$. In this case, the maximum number of checks k will be required in the case when the vertex v lies in the median zone z_t, $t = \left\lceil \frac{k}{2} \right\rceil$, which occupies the middle position in the section.

Let us show that the localization of all left vertices in the zones of the SL has complexity $O(n\ log\ \ n)$. The number of zones generated during sweeping is *O(n)*. Without loss of generality, we assume that there are exactly n zones. Let m be the number of left vertices of the PF $m = O(n)$. Let us estimate the maximum number of checks for the entire time of operation. When the “left vertex” event occurs, the maximum number of checks will be required if two conditions are met:

- the vertex is localized in the section with the largest number of zones,
- when localized on the section, the vertex falls into the median zone, that is, k checks are performed.

Thus, if the maximum number of checks is always performed, then n checks are made for the first left vertex in a section of n zones. After the formation of new zones associated with the vertex event, two new sections are formed that will contain $\frac{n}{2}$ zones each. Two subsequent localizations with the maximum number of checks in segments of length $\frac{n}{2}$ will require $\frac{n}{2}$ checks each.

Inserting the corresponding zones into the status forms four sections of $\frac{n}{4}$ zones, and so on. At each j-th level, 2^{j-1} sections of $\frac{n}{2^{j-1}}$ zones each are formed. On these sections, 2^{j-1} localizations of left vertices occur with a total number of checks n. For j levels, $1+2+4+\cdots+2^{j-1}=2^j-1$ "left vertex" events are processed. With $j=\log(m+1)$ we get that all left vertices can be processed in j levels, since there are only n checks at each level, we get the total number of checks $O(n\,log\ m)$. Since $m=O(n)$, we get the final complexity $O(n\,log\ n)$.

Thus, the proposed algorithm using the two-stage hierarchical status of the SL has asymptotic complexity $O(n\,log\ n)$.

8 EXPERIMENTS

Practical verification of the algorithm for correctness, efficiency and reliability uses a set of polygonal figures of varying complexity: up to 6400 polygonal contours and up to 185,000 polygon vertices in one binary image. Working time on large examples is up to 1-1.5 sec.

Efficiency was tested by comparing the running time of the proposed algorithm and the Fortune's algorithm from the C++ Boost library https://www.boost.org/. This algorithm does not take into account the mutual arrangement of points and segments in a polygons, and also builds a VD on the entire plane, and not just for the inner part of the PF.

The comparison of the two algorithms was carried out in the same environment under the same conditions. The time was averaged over 10 measurements. On images of a small size (up to 5000 vertices), the algorithm from Boost loses to the proposed algorithm by 2-4 times, on large images, with more than 100 thousand vertices, the running time of the algorithms differs by about 50 times.

Experiments with digital images of a handwritten text of the type presented in Fig. 2 required about 1 second to build the skeleton.

9 CONCLUSIONS

The article presents an algorithm for constructing an internal VD PF, focused on practical software implementation and applications to large-scale problems, in particular, to work with large images of handwritten documents. Due to the fact that the VD is built only for the internal part of the PF, several useful properties of the algorithm are achieved, which contribute to an increase in computational efficiency and numerical reliability.

The concept of the algorithm is based on the sweeping paradigm used in Fortune's algorithm. The proposed algorithm implements the reduction of the problem to the construction of VD linear segments, but at the same time includes new elements focused on using the properties of segments made up of polygons. Due to this, a significant part of the operations that have logarithmic complexity in the classical Fortune algorithm is implemented in constant time. In addition, the amount of calculations present in the known algorithms that build the redundant external part of the VD PF is reduced.

The algorithm has a high numerical reliability, since the internal VD PF does not require the calculation of large inscribed circles, as well as finding the VD vertices located at a very large distance from the figure.

The proposed algorithm is implemented in full and is used in solving practical problems of analysis and recognition of digital images of handwritten texts.

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