






# Deep Minutiae Fingerprint Extraction Using Equivariance Priors

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**Abstract:** Currently, fingerprints are one of the most explored characteristics in biometric systems. These systems typically rely on minutiae extraction, a task highly dependent on image quality, orientation, and size of the fingerprint images. In this paper, a U-Net model capable of performing minutiae extraction is proposed (position, angle, and type). Based on this model, we explore two different ways of regularizing the model based on equivariance priors. First, we adapt the model architecture so that it becomes equivariant to rotations. Second, we use a multi-task learning approach in order to extract a more comprehensive set of information from the fingerprints (binary images, segmentation, frequencies, and orientation maps). The two approaches improved accuracy and generalization capability in comparison with the baseline model. On the 16 test datasets of the Fingerprint Verification Competition, we obtained an average Equal-Error Rate (EER) of 2.26, which was better than a well-optimized commercial product.


## 1 INTRODUCTION


A fingerprint is an impression formed by the contact of the friction ridges on the fingertips with a surface (Adiga V and Sivaswamy, 2019). Friction ridges are described as “three-dimensional surfaces with irregular structures separated by narrow furrows valleys” (Hicklin, 2009). A detailed and distinct pattern of ridges and valleys exists for each human, which cannot be changed during the individual’s life (Adiga V and Sivaswamy, 2019). These characteristics explain why fingerprints are useful as a primary biometric modality in our daily lives (Adiga V and Sivaswamy, 2019; Peralta et al., 2015; Joshi et al., 2019).


Experts can recognize fingerprints by comparing two fingerprint images and matching the structures present in them. One of the most important steps


is minutiae extraction, in which friction ridge skin features are detected. The combination of these features results in different minutiae types in different positions and orientations, which are unique to each individual (Hicklin, 2009). Minutiae extraction is influenced by conditions such as humidity or skin dryness, the presence of dirt, or the existence of wounds (Adiga V and Sivaswamy, 2019). Also, the variety of sensors available to capture fingerprints results in images with different characteristics (Adiga V and Sivaswamy, 2019). As a result, fingerprint images may have different levels of noise, orientations, and scale. To address this variability, various deep learning methods, such as Convolutional Neural Networks (CNNs), have been tested, for minutiae extraction and pre-processing or matching fingerprints.


Minutiae can be extracted using deep learning methods that learn feature representations directly from data (Rebelo et al., 2019; Tang et al., 2017). The result is a probability map of minutiae positions, that needs to be post-processed to precise positions and orientations of minutiae (Rebelo et al., 2019). These methods have shown promise in tackling fingerprint image complexity, especially artefacts. Despite this,

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the use of CNNs for minutiae extraction is restricted by the scarcity of real fingerprint datasets with adequate minutiae marking, as well as privacy concerns. Alternatively, synthetic fingerprint generators capable of creating large databases that mimic inter-class and intra-class variations can be used. Although helpful, mimicking the natural degradation that occurs in naturally enrolled fingerprints or naturally acquired latent fingerprints is difficult. This emphasizes the importance of developing models with strong generalization capabilities, even when trained with small amounts of data. The field of Geometric Deep Learning proposes the use of equivariant CNNs to obtain networks robust to input transformations, and with a greater capacity for generalization to unseen data. The equivariance priors used are motivated by prior knowledge of the data and task at hand. In the case of fingerprint images, due to the importance of minutiae orientation, rotation-equivariant CNNs are worth investigating.

The aim of this paper is to investigate how a CNN model can be used to extract minutiae (position, angle, and type) from various types of fingerprint images. Specifically, how the model optimization can be regularized by incorporating rotation equivariance properties into the model architecture, and how the use of additional ground-truth information can impact network performance in the main task of minutiae extraction for fingerprint images from various sources. The main contributions of our paper are the following:

- We propose a deep learning approach for minutiae extraction trained on synthetic data but with excellent performance in real fingerprint images.
- We demonstrate how regularization with additional ground-truth information results in a better performance in the minutiae extraction task and allows also the recovery of extra fingerprint information.
- We demonstrate how equivariance priors can be incorporated into the model architecture and training to improve generalization.

## 2 RELATED WORK

### 2.1 CNNs for Minutiae Extraction

Deep learning for minutiae extraction can take two forms: building a neural network from scratch or combining domain knowledge with neural networks to improve structure design (Rebelo et al., 2019). The FingerNet (Tang et al., 2017) and the MinutiaeNet (Nguyen et al., 2017) are two of those exam-

ples where domain knowledge is combined with deep learning.

As a rule, the methods first generate a proposal for the minutiae points and then extract the exact position, orientation, or type of minutiae (Jiang et al., 2016; Tang et al., 2017; Zhou et al., 2020; Jiang and Liu, 2017). Other methods consider the minutiae extraction problem to be a segmentation task where a U-Net model encodes the minutiae positions using a binary mask (Pinetz et al., 2017) or the positions and orientations using a mask with multiple classes, according to the angle intervals (Nguyen et al., 2020). In addition to minutiae information, multi-task learning and transfer learning can be used to extract frequency and texture information (Takahashi et al., 2020; Zhang et al., 2021).

In our proposal, the minutiae information (position, angle, and type) is encoded in different target masks, with the U-Net model being trained in a multi-task approach. The model's outputs are then post-processed to produce the typical minutiae template. Our work is most similar to the approaches based on the use of the U-Net (Pinetz et al., 2017; Nguyen et al., 2020). We encode the minutiae positions and orientations in different masks and obtain also the minutiae type. Our work differs from the state-of-the-art since we only use synthetic data in the training instead of real fingerprints (Tang et al., 2017) to obtain a larger training set. In addition, our proposed models were tested in diverse datasets to test the generalization capability of the models.

### 2.2 Equivariant CNNs

A CNN is equivariant when a specific transformation in the network's input results in a predictable change in network output.

For instance, traditional CNNs are translation equivariant: shifting the input of a layer yields the same feature map as feeding the original image (without shifting) to that layer and shifting the feature map. This results from the weight sharing and local connectivity properties of these models, which are contributing factors to the high performance when processing natural images (Castro et al., 2020). A CNN can be classified as translation equivariant when feature extraction is identical regardless of the image region being processed (Castro et al., 2020; Cohen and Welling, 2016). The importance of equivariance to translations in traditional CNNs motivates the study of additional types of equivariances that could be used to enhance the accuracy of these models in other computer vision tasks. In other words, the application of symmetries was a fundamental design principle for

network architectures and can result in more precise models (Gerken et al., 2021). For the specific case of minutiae extraction, fingerprint image characteristics motivate the search for rotation-equivariant CNNs.

The most common way to address scale and rotation equivariance in CNNs is to use data augmentation, which involves rotating the data or changing the scale (Castro et al., 2020; Naderi et al., 2020; Worrall et al., 2017). This method ensures the capability of generalization of CNNs but does not ensure equivariance at all network layers (Worrall et al., 2017). Data augmentation results in a heavy training cost and complex model parameters, which may reduce network performance (Naderi et al., 2020). In this way, several methods have been proposed to incorporate geometric transformation information into network architectures.

One approach is to use four operations (slice, pool, roll, and stack) to allow parameter sharing between different orientations via feature map rotations to obtain equivariance to rotations (Dieleman et al., 2016). The same property is obtained in the Deep Rotation Equivariant Networks (DRENs) by replacing the rotations of the features maps with rotations of the filters (Li et al., 2018). Another proposal is the Group Equivariant Convolutional Neural Networks (G-CNNs), which allows the rotation equivariance by defining the operation G-convolution (Group-equivariant convolution) (Cohen and Welling, 2016). This operation exploits the known symmetries of the data and is responsible for the generalization of the common convolution operator to deal with rotations and reflections. The limitation presented in this method is the capability of dealing only with discrete groups, and only rotations of multiple of 90° are considered. The Harmonic Networks (H-Nets) generalize the rotation equivariance for continuous groups of 360° rotations by replacing regular CNN filters with circular harmonics (Worrall et al., 2017). This property is the result of the replacement of regular CNN filters with circular harmonics that return a maximal response and orientation. Circular harmonics are steerable filters as the ones used to obtain scale equivariance in the Scale Equivariant Convolutional Neural Networks (SE-CNNs) (Naderi et al., 2020). The Group Equivariant Capsule Networks also achieve rotation equivariance by using group equivariant capsule layers (Lenssen et al., 2018).

The aforementioned proposals use diverse techniques to address the problem of scale and/or rotation equivariance, and all demonstrated superior performance in classification tasks compared to baseline models. This indicates that the integration of scale and/or rotation equivariance priors should be consid-

ered when designing different networking architectures. The purpose of introducing these equivariances was to improve the robustness and generalization of the networks to unseen data. In this paper, the rotation equivariance was embedded using G-convolutions to replace the conventional convolutions of the U-net model and obtain a Group Equivariant U-Net that can be used for minutiae extraction.

## 3 METHODS

### 3.1 U-Net Architecture

To allow an image-to-image mapping between a fingerprint image and a map with the same dimensions of the minutiae positions, a U-Net model was employed (Ronneberger et al., 2015). It consists of an encoder-decoder structure that uses as the main block (ConvBlock) a sequence of a 3 by 3 Convolutional layer (3x3 Conv), a Batch Normalization layer (BN) and a Rectified Linear Unit (ReLU) as the activation function. The Encoding Layer consists of four repetitions of two blocks ConvBlocks followed by a max-pooling operation of stride 2. The Decoder consists of four repetitions of two ConvBlocks followed by an up-sampling operation with stride 2. The final output is obtained by a 1 by 1 Convolutional layer. Another important detail is the use of skip connections, which provide the decoder with information typically lost during down-sampling. These consist of concatenating the feature maps after up-sampling in the decoder with feature maps before down-sampling in the encoder.

### 3.2 Multi-Task Learning

We modify the U-Net model to perform multiple tasks concurrently via a hard parameter-sharing strategy, i.e., different tasks share hidden layers and have distinct output layers (Ruder, 2017). Through the use of these shared representations, the model focuses on the most crucial aspects. The optimization of the multi-task model is performed using a linear combination of multiple loss functions for each task, following the general equation presented in Equation 1.  $L_{total}$  corresponds to the total value of the loss function with  $N$  partial losses,  $L_i$  corresponds to the value of each partial loss, and  $\lambda_i$  to the respective weight.

$$L_{total} = \sum_{i=1}^N (\lambda_i \cdot L_i) \quad (1)$$

### 3.3 Baseline Model

The baseline model was designed to extract only information regarding the minutiae: position  $(x_0, y_0)$ , angle  $(\theta_0)$  and type (type 1 for a termination point or type 2 for a bifurcation). The model uses the above-mentioned U-Net architecture with four output channels that encode the minutiae information.

The spatial information of each minutia is encoded using a 2D Gaussian centered on the minutia's position. Two channels were used for encoding, one for endpoints (Mask XY\_1) and one for bifurcations (Mask XY\_2). A  $21 \times 21$  Gaussian window, with a standard deviation of 2 was used and normalized so that the maximum value is 1. These two channels can be seen as probability maps where the maximum probability of 1, corresponds to the exact position of the minutiae. For this reason, the sigmoid was used as the output activation function.

For encoding the angle of each minutia, two more channels were used. A box centered in each minutia with the values of the sine (Mask A\_Sen) and the cosine (Mask A\_Cos) encode the minutia angle. Using the trigonometric functions ensures there are no discontinuities. For instance, when the model's prediction is  $360^\circ$  and the actual value of the angle is  $0^\circ$ , the difference would be high even though the model is accurate. Due to the range of these trigonometric functions  $([-1, 1])$  the Hyperbolic Tangent (Tanh) was used as the output activation function.

The loss function ( $L_{minutiae}$ ) for the baseline model is presented in Equation 2, and consists of a sum of four partial losses. The first two relate to the minutiae position encoding and the two relate to orientation encoding were based on the Mean-Squared Error (MSE). Notice that for the angle encoding losses, a mask is used to ensure only regions around minutiae ( $21 \times 21$  boxes) contribute to the loss since other regions do not have a defined minutiae orientation. The same weight is attributed to each partial loss. The combination of the partial loss functions used for each output channel of the model, the respective activation function, and the range of output values are presented in Table 1.

$$L_{minutiae} = MSE_{XY\_1} + MSE_{XY\_2} + MSE_{A\_Sen} + MSE_{A\_Cos} \quad (2)$$

### 3.4 Regularization with Additional Ground-Truth Information

Five additional output channels were added to the baseline model to act as model regularization. These

outputs are used to learn additional tasks, such as the position, orientation, and frequency of the ridge patterns, as well as a binary segmentation of the fingerprint. These are not required for minutiae extraction but serve as a regularization of the learning process. The proposed loss based on these extra channels ( $L_{extra}$ ) is combined with the loss of the main task of minutiae extraction ( $L_{minutiae}$ ), using the parameter  $\lambda_{total}$  to weight the sum (Equation 3). The parameter  $\lambda_{total}$  was tuned to provide the best performance in the task of minutiae extraction.

$$L_{total} = \lambda_{total} \cdot L_{minutiae} + (1 - \lambda_{total}) \cdot L_{extra} \quad (3)$$

The first two channels added correspond to the masks containing information about the binary ridge pattern (Mask Bin) and the segmentation map (Mask Seg). These two channels allow obtaining additional information about different regions in the image: foreground/fingerprint, background, and ridge lines. A sigmoid activation is used in each of these channels, and the BCE serves as the loss function for the corresponding additional tasks.

Two more channels were added to reconstruct the orientation map, sine (Mask O\_Sen) and cosine (Mask O\_Cos) masks. Due to the range of the sine and cosine functions  $([-1, 1])$ , we used a Tanh activation function and the MSE loss function for optimization. To recover the ridge pattern's frequency, another channel with normalized values between 0 and 1 was added (Mask Frq). Normalized frequency map limits allowed for Sigmoid activation and MSE loss. The total loss of these auxiliary tasks was calculated by adding the five individual losses, giving equal weight to each partial loss (Equation 4).

$$L_{extra} = BCE_{Bin} + BCE_{Seg} + MSE_{O\_Sen} + MSE_{O\_Cos} + MSE_{Frq} \quad (4)$$

### 3.5 Equivariant Model

Motivated by the importance of translation equivariance in CNNs and by the fingerprint characteristics, the goal was to obtain a Group Equivariant CNN (Cohen and Welling, 2016), equivariant to rotations and capable of performing the minutiae extraction. This led to the implementation of a Group Equivariant U-Net. The created model differs from the baseline model in the U-Net architecture, where G-convolutions replaced typical convolutions to create an equivariant model. To avoid duplicate output channels, the output layer convolutions were not replaced. This work used the group  $p4$  in the G-convolutions.

Table 1: Output channels from the U-Net model: task, activation function, loss function and channel range.

Task		Channel Name	Activation Function	Loss Function	Channel Range
Minutiae Extraction	Position and Type	XY_1	Sigmoid	MSE	[0 - 1]
		XY_2	Sigmoid	MSE	[0 - 1]
	Angle	A_Cos	Tanh	MSE *	[-1 - 1]
		A_Sen	Tanh	MSE *	[-1 - 1]
Binary Image		Bin	Sigmoid	BCE	[0 - 1]
Segmentation Map		Seg	Sigmoid	BCE	[0 - 1]
Orientation Map		O_Cos	Tanh	MSE **	[-1 - 1]
		O_Sen	Tanh	MSE **	[-1 - 1]
Frequency Map		Frq	Sigmoid	MSE **	[0 - 1]

\* Only computed in 21x21 boxes centered around each minutia.

\*\* Only computed in the fingerprint region from the segmentation map.

The G-convolution can be understood as the use of the same filter in different orientations such that a feature map is generated for each orientation. The collection of feature maps is concatenated such that a rotation on the input causes a rotation and a shift on the output (see Figure 1). The subsequent G-convolution layers account for this shift when transforming their filters, such that, when multiple layers are stacked, this input-output relationship is maintained. As such, the model can maintain equivariance to rotation. The G-convolutions can be defined according if they are in the first layer of a CNN being applied to an input image (Equation 5) (first-layer G-convolution) or if they are applied to a feature map in a hidden layer of the CNN (Equation 6) (full G-convolution) (Cohen and Welling, 2016). The convolution defined in Equation 5 receives an input image  $f$  and a filter  $\psi$ , both functions of the plane  $\mathbb{Z}^2$ , and outputs a feature map  $(f * \psi)$  function of the discrete group  $G$ . For the convolution defined in Equation 6, the filters  $\psi$  need also to be functions on  $G$ .

$$[f * \psi](g) = \sum_{y \in \mathbb{Z}^2} \sum_k f_k(y) \psi_k(g^{-1}y) \quad (5)$$

$$[f * \psi](g) = \sum_{h \in G} \sum_k f_k(h) \psi_k(g^{-1}h) \quad (6)$$

When replacing convolutions with their equivariant counterpart, we maintained the same number of channels. This ensures that the models are equivalent in terms of the number of operations resulting in a reduction of the network's trainable parameters. This reduction corresponds to using four rotated copies of each filter for the G-convolutions layers and is equivalent to a reduction of approximately one-quarter (from 17.3 million to 4.3 million trainable parameters). In addition, to ensure rotation-equivariance the batch normalization was implemented with a single scale and bias parameter per each  $p^4$  G-feature map.

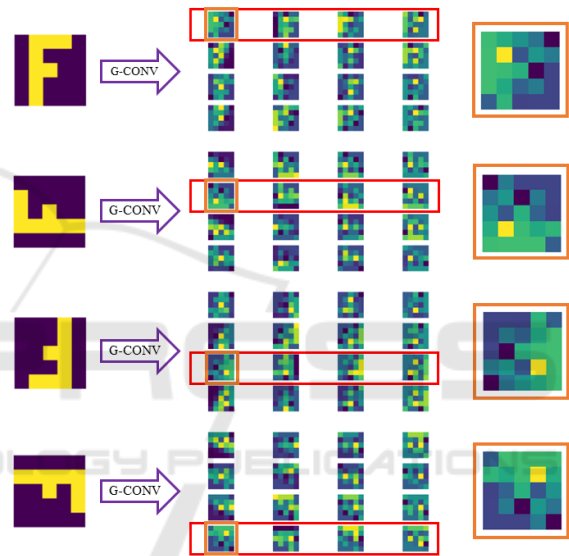


Figure 1: Graphical representation of the G-convolution equivariance, considering a convolution with 1 input channel and 16 output channels.

### 3.6 Post-Processing

A correlation between the masks with the outputs and the Gaussian template is performed in the first step of the post-processing method. For the correlation, it was used a window with the same size and standard deviation as the one used for the target construction. The peaks of the correlation result with a value above the threshold of 1.0 were selected. In addition, the maximum number of peaks possible to extract was 100. When two peaks were very close (less than 5 pixels apart), the peak with the higher intensity was always chosen. The mask in which the peak was verified (Mask XY\_1 or Mask XY\_2) determined the minutia type (type 1 or type 2). After identifying the points with minutia, the angle sine and cosine outputs were used to calculate the angle. A circular

mean is computed for each minutia position to calculate the mean angle in a  $5 \times 5$  window around the minutia. The formula for the circular mean is presented in Equation 7, where  $\bar{a}$  corresponds to the average angle value calculated over  $N$  sine and cosine sample values. Each minutia was also assigned a relative quality based on the intensity of the correlation map.

$$\bar{a} = \arctan\left(\frac{1}{N} \sum_{i=1}^N \sin(a_i), \frac{1}{N} \sum_{i=1}^N \cos(a_i)\right) \quad (7)$$

## 4 EXPERIMENTAL SETUP

The work proposed in this paper began with the implementation of a basic U-Net model design to extract only the minutiae information from a grayscale fingerprint image, as described in Section 3.3. The model was then modified to use a more complex multi-task learning approach, in which the methodology extracts more information from the image, such as binary image, segmentation, frequency, and orientation maps (Section 3.4). Alternatively, the equivariant architecture described in Section 3.5 was implemented to obtain a Group Equivariant U-Net. Finally, we combine the two strategies in the same model. All the model’s outputs were post-processed using the same method (Section 3.6).

The optimization of the models followed the pipeline presented in Figure 2. The models were trained for 100 epochs, using the Adam optimizer algorithm, with a learning rate of  $1e^{-1}$  and using a batch size of 8. The datasets used for training, validation, and testing are described in Section 4.1 and the models were evaluated using the metrics presented in Section 4.2

### 4.1 Data

#### 4.1.1 Synthetic Fingerprint Databases

The training of CNNs, for minutiae extraction, demands a large number of fingerprint images with correct annotations for the minutiae information. Due to the difficulty in annotation and privacy concerns, synthetic databases were used. These databases were created using the *Synthetic Fingerprint Generator (SFinGe)* available in<sup>1</sup> (Maltoni et al., 2022). The generator tries to mimic inter-class and intra-class variations of real fingerprints, i.e., creates examples

of the “same individual” and from “different individuals”. This software generates:

1. Grayscale image of the fingerprint;
2. Binary image with the pattern of fingerprint’s ridges and valleys pattern.;
3. Frequency map with the frequency of the ridges and valleys pattern;
4. Orientations map with the angle of the ridges and valleys;
5. Segmentation map with information on the region of the fingerprint versus the background;
6. Minutiae template with information about the minutiae coordinates, angle and type (type 1 for a termination point or type 2 for a bifurcation point)

A total number of 21600 synthetic images (8 different images from 2700 different individuals) were used for training the models and 7200 synthetic images (8 different images from 900 different individuals) were used to validate the models.

#### 4.1.2 Real Fingerprint Databases

For testing the different models, the real fingerprint images provided by the Fingerprint Verification Competition (*FVC*) were chosen. These databases contain fingerprint images obtained with various sensors, resulting in different features, which are useful for testing the universality and robustness of the various steps of a Fingerprint Recognition System (Maio et al., 2002a; Maio et al., 2002b; Cappelli et al., 2005; Cappelli et al., 2007).

#### 4.1.3 Data Augmentation

Different data augmentation techniques were simultaneously applied to approximate the synthetic training images to real ones. The first set of transformations tried to mimic image textures when acquired by different sensors with low noise levels. They were only adopted in grayscale images. It includes contrast, brightness, saturation, blurring, sharpening, and inversion. The second set randomly crops and translates the image. It mimics the need to process non-centered partial fingerprints. The last set included random fingerprint rotations (from  $0^\circ$  and  $360^\circ$ ) and flips (horizontal and vertical). They were applied to provide the model fingerprints in diverse orientations, not just vertically, but also to provide minutiae in diverse orientations. Data augmentation was performed online and for each image was applied simultaneously one transformation from each set.

<sup>1</sup><https://bit.ly/3ysNNej>

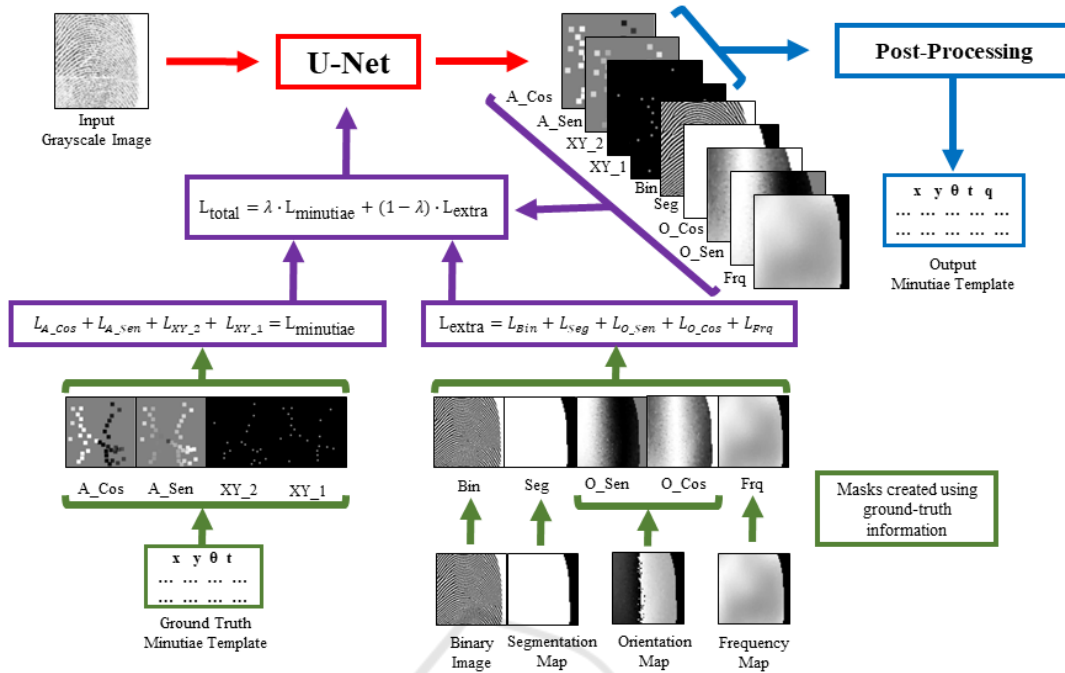


Figure 2: The complete pipeline used for the U-Net models training.

## 4.2 Minutiae Extraction Evaluation

The evaluation of the different models for minutiae extraction was centred on two main objectives: 1) the ability to obtain the correct information of each minutia - coordinates, angle, and type; and 2) the ability to distinguish fingerprints of different individuals when the extracted templates are used by a matching algorithm.

The Goodness Index (GI) was used to evaluate the proportion of the minutiae that were well detected, spurious, and not detected (Peralta et al., 2014; Bhattacharjee and Lee, 2010). To the GI calculus, a minutia was considered well-detected when inside a  $13 \times 13$  box centered on the closest ground-truth minutiae. In addition to the standard GI, more strict criteria to classify a minutia as well-detected were considered. It was also considered in addition to the position that the minutia angle must be within  $\pm 20^\circ$  of the ground truth minutiae to be considered as well detected (GI (w/a)). This metric measures the model's ability to identify the minutia angle. To evaluate the model's ability to extract the minutiae type correctly, the GI (w/t) was used, in this situation the minutia type was used in addition to the minutia position as criteria. The GI (w/at) considers all three criteria during the minutia classification.

The Equal Error Rate (EER) calculus in accordance with the FVC protocol (Maio et al., 2002a; Maio et al., 2002b; Cappelli et al., 2005; Cappelli

et al., 2007) was used to evaluate the quality of the model when used in conjunction with a matching algorithm for fingerprint verification. The matching algorithm was based on the Minutia Cylinder-Code (MCC) representation combined with the Local Similarity Assignment with Relaxation (LSA-R) was used (Cappelli et al., 2010; Rebelo et al., 2019).

The models were compared against two baseline minutiae extraction solutions: a well-optimized, traditional commercial method for minutiae extraction - the fingerIDALg (Rebelo et al., 2019); and a state-of-the-art method based on deep learning combined with domain knowledge- the FingerNet (Tang et al., 2017).

## 5 RESULTS AND DISCUSSION

The results for the validation set that contains only synthetic images were based on the GI values and are depicted in Table 2. To evaluate the performance of the models in test sets, due to the lack of ground-truth annotations, the results were based on EER values and are presented in Table 3.

### 5.1 Baseline Model

In the validation set with synthetic images, the non-equivariant model with  $\lambda_{total}$  of 1.0 performs better than the fingerIDALg (a GI of 0.01 versus -

Table 2: Results for the validation set of the reference system and all the four models: GI, Percentage missing minutiae (%M), percentage of spurious minutiae (%S), and the GI considering the angle (w/a), the type (w/t) and both (w/at) as a criterion for classification of a minutia as well detected.

Model		GI	%M*	%S**	GI (w/a)	GI (w/t)	GI (w/at)
Reference	fingerIDALg	-0.05	34.53	37.77	-0.24	-0.32	-0.47
Non-Equivariant	$\lambda_{total} = 1.0$	0.01	30.71	40.73	-0.19	-0.27	-0.43
	$\lambda_{total} = 0.8$	0.01	30.51	42.55	-0.21	-0.26	-0.43
Equivariant	$\lambda_{total} = 1.0$	0.02	30.27	41.18	-0.18	-0.26	-0.42
	$\lambda_{total} = 0.8$	0.02	30.40	40.21	-0.18	-0.23	-0.39

\* Calculated in relation to the ground truth minutiae set.

\*\* Calculated in relation to the extracted minutiae set.

Table 3: EER (%) values for the reference systems and the four models for the different FVC databases; and the average of the EER (%) for all the FVC databases.

FVC Database		Reference		Model			
				Non-Equivariant		Equivariant	
		fingerIDALg	FingerNet	$\lambda_{total} = 1.0$	$\lambda_{total} = 0.8$	$\lambda_{total} = 1.0$	$\lambda_{total} = 0.8$
2000	DB1	1.01	<b>0.44</b>	0.36	<b>0.30</b>	0.36	0.40
	DB2	0.59	<b>0.55</b>	1.76	1.70	1.74	<b>0.44</b>
	DB3	2.44	<b>2.40</b>	3.84	4.08	4.65	<b>3.72</b>
	DB4*	<b>1.33</b>	1.84	0.79	0.79	<b>0.65</b>	0.69
2002	DB1	<b>0.65</b>	<b>0.65</b>	0.55	0.69	<b>0.44</b>	0.63
	DB2	<b>0.30</b>	0.32	0.30	0.40	0.26	<b>0.18</b>
	DB3	2.83	<b>1.29</b>	2.18	1.86	1.76	<b>1.58</b>
	DB4*	0.99	<b>0.85</b>	0.30	0.40	0.32	<b>0.18</b>
2004	DB1	3.94	<b>3.01</b>	3.15	3.37	3.19	<b>2.79</b>
	DB2	<b>3.01</b>	3.39	3.58	4.79	<b>3.09</b>	3.84
	DB3	2.73	<b>2.18</b>	3.92	<b>3.15</b>	3.39	3.23
	DB4*	2.08	<b>1.86</b>	1.01	0.97	<b>0.73</b>	0.79
2006	DB1	<b>11.41</b>	21.96	14.77	<b>12.00</b>	13.66	12.63
	DB2	0.55	<b>0.10</b>	1.37	1.70	<b>1.01</b>	1.25
	DB3	5.25	<b>3.76</b>	4.26	4.22	3.98	<b>3.29</b>
	DB4*	2.91	<b>1.29</b>	0.83	0.73	0.69	<b>0.59</b>
Average EER(%)		<b>2.63</b>	2.87	2.69	2.57	2.49	<b>2.26</b>

\* Synthetic Databases

0.05). The GI (w/a) results show that the baseline model misses some minutia angles, but it outperforms the commercial solution. The model fails to identify some minutia types but performs better than the reference system. GI (w/at) results show that the model can fail in minutia type and predict minutia orientation and vice versa.

Table 3 shows the EER (%) values for all the 16 FVC test sets using the fingerIDALg, the FingerNet, and the models proposed in this work. Comparing both reference systems, one based on traditional methods and the other on deep learning combined with domain knowledge, the traditional method has a lower average EER value: 2.63% versus 2.87%. The fingerIDALg had the lowest EER for the 2002.2

database and the highest for the 2006.1 database. This results in a range from 0.30% to 11.41% which shows a high-performance range for the test datasets.

The baseline model's lower EER value was 0.30% in the 2002.2 database and 14.77% in the 2006.1 database. EER averaged 2.69%. These results show that the baseline model is competitive in comparison with a well-optimized commercial system, surpassing it in 10 of 16 test databases: 4 synthetic databases (2000.4; 2002.4; 2004.4; and 2006.4) and 6 databases with real images (2000.1; 2002.1; 2002.2; 2002.3; 2004.1; and 2006.3). The baseline model outperforms the state-of-the-art reference, the FingerNet, in 8 test sets (4 synthetic and 4 real test sets). Notice that 12 real test sets contain images out-



domain in comparison with the 4 synthetic test sets that are from a domain similar to the training domain. This explains the high variability in results and the higher accuracy in the synthetic sets, as expected.

## 5.2 Regularization with Additional Ground-Truth Information

Considering the  $\lambda_{total}$  optimization, the results indicate that the performance of the validation set models was independent of  $\lambda_{total}$ . Using a  $\lambda_{total}$  value of 0.8 resulted in a similar GI value than using a  $\lambda_{total}$  value of 1.0, and when the percentage of missing and suspicious data is considered, the performance of both models is also comparable.

The EER values for the test set shown in Table 3, indicate that a  $\lambda_{total}$  value of 0.8 can reduce the average EER value to 2.57%. This value falls below the average value of the reference systems. In 11 of the test sets, the EER value was decreased due to the reduction of  $\lambda_{total}$ .

These results show that adding extra tasks to the model can act as regularization during optimization, leading to better minutiae extraction. This model can also extract more information from a greyscale fingerprint image, which can be useful for some more sophisticated matching algorithms.

## 5.3 Equivariant Models

Regarding the introduction of equivariance, the equivariant model with a  $\lambda_{total}$  of 1.0 has a slightly higher GI value (0.02) than the baseline model, but their performance in the validation set is very similar (see Table 2). The same occurs for the models with a  $\lambda_{total}$  of 0.8.

The use of an equivariant architecture leads to more robust models allowing both equivariant models to outperform the reference systems. In particular, the EER decreased from 2.69% to 2.49% when no additional regularization was used ( $\lambda_{total} = 1.0$ ) and was comparable for the scenario when  $\lambda_{total} = 0.8$  (reduction from 2.57% to 2.26%). These results showed that the best model performance occurs with the combination of an equivariant architecture with a regularization with extra ground-truth information.

We conducted an additional test to assess the impact of the proposed architectural change on the model's rotation equivariance property. For this, we rotated all the test images in the 2000.1 FVC dataset by a constant angle before being fed to the model. Then, the output minutiae were readjusted to the original position and compared to the ones obtained with

the initial images (with no rotation). The average difference between these outputs for different input angles is shown in Figure 3. The results show a lower average output angle deviation for the equivariant model. This is more meaningful for the input rotation angles multiple from  $90^\circ$ , as expected since the rotation equivariance of the network was assured for the  $p4$  group. Despite this, a deviation of  $0^\circ$  is only achieved to input rotations of  $0^\circ$  since the output layer of the Group Equivariant U-Net uses a typical convolution that does not assure rotation equivariance.

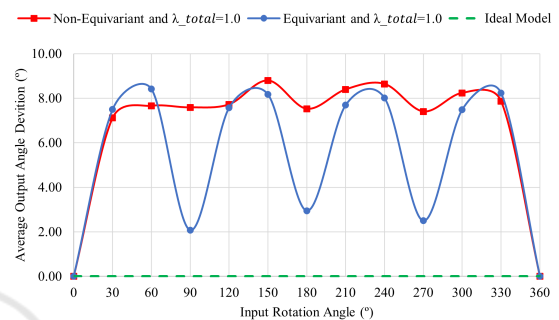


Figure 3: Average output angle deviation in the function of the fixated input rotation angle, for the 2000\_1 dataset, computed for the models with  $\lambda_{total} = 1$ , and for an ideal model with a constant angle deviation of  $0^\circ$ .

The final model (equivariant and with  $\lambda_{total}$  of 0.8) presented the lower average ERR with the value of 2.26%. However, a large interval of EER values from 0.18% (2002\_2) to 12.63% (2006\_1) was verified. Figure 4 shows one image from each database with the minutiae set extracted by the final model. The images are examples of the diversity of the fingerprint images used to test the models and how the acquisition conditions can change the characteristics of the fingerprint images and by that the fingerprint domain. This also shows that the synthetic generator even when combined with the data augmentation techniques during the training process, is not capable of mimicking, with the same quality the different fingerprints domain. In the case of Figure 4a, the pattern of ridges and valleys is well-defined which allows the correct minutiae extraction, this image belongs to a dataset with a domain close to the synthetic domain. For the case, of Figure 4b, the model partially fails to detect the fingerprint pattern with the quality needed for a correct minutiae extraction, the degradation, and the poor quality of this kind of images from this domain cannot be mimicked synthetically with the quality need to obtain a good generalization of the models to this domain.

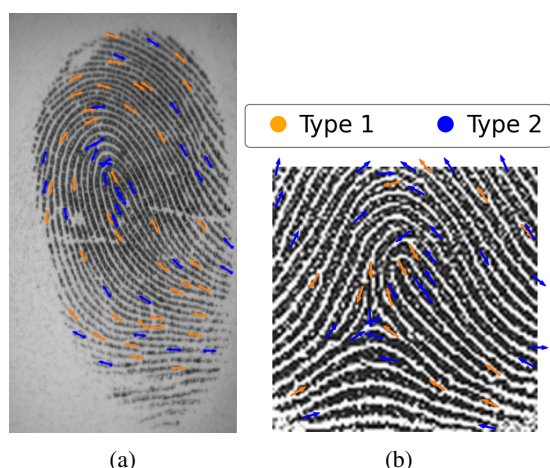


Figure 4: Test images with the minutiae sets extracted by the equivariant model with  $\lambda_{total}$  of 0.8: image 100.1 from the 2002.2 (a) and 2006.1 (b) datasets.

## 6 CONCLUSIONS

The model developed in this paper outperformed a well-optimized commercial solution, resulting in a lower EER average for fingerprint images from various sensors. Rotation equivariance improved results, especially model generalization. Furthermore, this paper demonstrated that including equivariance priors into the network architecture based on the fingerprint's prior knowledge can help deep learning methods match or exceed the performance of traditional systems. The introduction of extra tasks enabled regularization and knowledge transfer, resulting in better results. This allows the development of a model that extracts more information from the same fingerprint image without jeopardizing the main task of minutiae extraction.

Taking the final architecture into account, future research can be conducted to improve generalization. This paper focused on discrete rotation equivariance. A future model may include the incorporation of equivariance to a continuous range of rotations and equivariance to scale transformations. In addition, with the goal of achieving a more competitive model, the computational cost must be reduced, with special attention to the processing time.

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