

Model Order in Sugiyama Layouts

Sören Domrös^a, Max Riepe^b and Reinhard von Hanxleden^c

Department of Computer Science, Kiel University, Kiel, Germany

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Abstract: Graph drawing algorithms traditionally consider a graph to consist of unordered sets of nodes and edges, which may disregard information already provided by the developer. In practice, as recently argued by (Domrös and von Hanxleden, 2022), a graph often consists of *ordered* sets, which have an intended *model order* of nodes and edges. We present how this model order can be enforced or used as a tie-breaker, while optimizing common aesthetic criteria. This allows the developer to control the layout of layered graphs via the model order. On the example of SCCharts, we show that the order of nodes and edges does indeed correlate with the way people think about a model, and how that order can be used to emphasize the semantics of a sensibly designed model. Moreover, we suggest model order strategies to be used for control-flow and data-flow diagrams based on expert developer feedback on SCCharts and Lingua Franca.

1 INTRODUCTION

In previous work, (Domrös and von Hanxleden, 2022) argued to use *model order*—the order of the input model—to influence the layout since common algorithms usually only optimize for geometric criteria such as backward edges, edge length, edge straightness, or edge crossings.

As introductory example, Figure 1a and 1b show two different drawings of the same small graph G , which in the common mathematical unordered set notation consists of nodes $N = \{n1, n2, n3\}$ and edges $E = \{(n1, n2), (n2, n3), (n3, n2)\}$. Throughout this paper, we assume nodes to be ordered according to their numbering, e. g., that $n1$ is “before” $n2$.

In Figure 1a the number of backward edges is the same as in Figure 1b. Figure 1a is the result of a random decision, which places $n3$ as a *dangling node*—a node that goes against the main layout direction, which we assume without loss of generality to be left-to-right. Thus, we consider Figure 1a to be Obviously Non-Optimal (ONO) violating the Nothing is Obviously Non-Optimal (NONO) principle (Kieffer et al., 2016). Using model order, the desired Obvious Yet Easily Superior (OYES) solution in Figure 1b emphasizes the control-flow and prevents the dangling node, as now $n3$ is to the right of $n2$.

As the main layout direction is left-to-right, the layer-based Sugiyama algorithm forms vertical layers. In Figure 1c, $n2$ and $n4$ are in the same vertical layer; in Figure 1d, $n3$ and $n4$ share the same layer. The node $n4$ can be put in the same layer as $n2$ or $n3$ without increasing the edge length or size of the drawing. The ordering suggests that $n4$ should be “after” $n3$, which is not clearly represented by the layout in Figure 1c in which the order of $n3$ and $n4$ is ambiguous since $n3$ is above $n4$ but $n4$ is left of $n3$. Thus, we consider Figure 1c to be ONO. Figure 1d solves this problem by moving $n4$ to the same layer as $n3$.

Figures 1e and 1f also show the same graph. Here, the vertical ordering inside a layer differs. Figure 1e is ONO since it does not respect the order of $n2$, $n3$, and $n4$ and orders them differently, although there is no aesthetic criterion such as edge crossings that would justify it. The OYES solution in Figure 1f places the nodes by their ordering in the input model.

ONO drawings are a practical obstacle to the adaptation of automatic layout and with it modeling pragmatics (Fuhrmann and von Hanxleden, 2010), which embraces the best of textual and graphical modeling. In the past, Figure 1a and 1e have been the standard layout for SCCharts (von Hanxleden et al., 2014), a control-flow based state-chart dialect, as a result of a specific random seed. Since the order was scrambled by previous phases before crossing minimization started and was randomized during crossing minimization, the developer had very limited control over

^a <https://orcid.org/0000-0002-8011-8484>

^b <https://orcid.org/0000-0001-6779-2207>

^c <https://orcid.org/0000-0001-5691-1215>

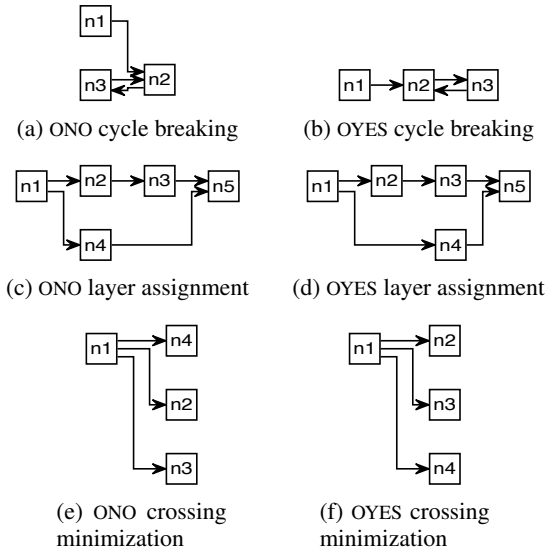


Figure 1: All drawings are optimal in terms of number of backward edges for cycle breaking, edge length for layer assignment, and edge crossings for crossing minimization. However, (a), (c), (e) violate the model order.

the layout. To nevertheless influence the layout, developers renamed or reordered the states and introduced constraints to force nodes in the first or last layer, which did not always work if multiple edges were involved. The same applies for the data-flow based polyglot coordination language Lingua Franca (Lohstroh et al., 2021). Lingua Franca allows specifying deterministic actors called reactors mainly consisting of actions and reactions, which are event triggered and execute their body consisting of code in the desired target language.

1.1 Contribution & Outline

Section 2 briefly reviews the Sugiyama algorithm. This paper extends the model order approach for Sugiyama layouts proposed by (Domrös and von Hanxleden, 2022) by adding strategies for cycle breaking, layer assignment, and crossing minimization to the already existing strategies for tie-breaking model order crossing minimization. The main contributions covered in the next sections are as follows:

- We propose two strategies for cycle breaking, a tie-breaking and an enforcing model order cycle breaking strategy (Section 3);
- We present a node promotion strategy for model order layer assignment (Section 4);
- We present a new tie-breaking strategy and enforced model order strategies for crossing minimization (Section 5).

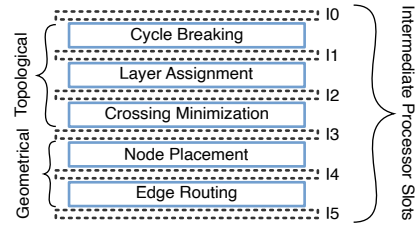


Figure 2: Structure of the layered algorithm.

Section 6 evaluates the enforced cycle breaking and crossing minimization strategies and discusses potential use cases based on developer feedback for SC-Charts and Lingua Franca in Section 6. Section 7 presents further related work and Section 8 concludes this paper.

2 THE LAYERED ALGORITHM

The Sugiyama algorithm, or *layered algorithm*, handles the inherent complexity of assigning coordinates to nodes and routes to edges by dividing the layout problem of directed graphs into five phases: The topological phases cycle breaking, layer assignment, and crossing minimization compute the relative positioning between the nodes and edges. The geometrical phases node placement and edge routing set x and y -coordinates of nodes and edge routes. In its implementation in the Eclipse Layout Kernel (ELK) the algorithm is further divided, as seen in Figure 2, by adding intermediate processors between the phases that handle pre- and post-processing (Schulze et al., 2014).

A directed ordered graph $G = (V, E)$ consists of an ordered set of nodes $V = \langle n_1, \dots, n_k \rangle$ and an ordered set of p_i outgoing edges for each node n_i with $E = \langle \langle e_{1,1}, \dots, e_{1,p_1} \rangle, \dots, \langle e_{k,1}, \dots, e_{k,p_k} \rangle \rangle$. As explained by (Domrös and von Hanxleden, 2022), the edges implicitly define the ordered set of ports, which are the anchor points on the edges on the nodes and which define the ordering between the edges. However, for simplicity we here only use the edge order, which we use as an equivalent to the order of the outgoing ports, since incoming ports can be ordered by their incoming connections. A layered ordered graph $G = (V, E, L)$ additionally has a layering $L = \langle L_1, \dots, L_m \rangle$ of m layers of r_i nodes with $L_i = \langle n_{L_i,1}, \dots, n_{L_i,r_i} \rangle$, which assigns a node to exactly one layer. In-layer edges, i. e. edges between nodes of the same layer, are forbidden.

The layered algorithm assigns nodes to vertical layers, as seen in the graph in Figure 1c, which has four layers $L = \langle \langle n1 \rangle, \langle n2, n4 \rangle, \langle n3 \rangle, \langle n5 \rangle \rangle$. Such

a layered graph is *proper*, if edges only occur between neighboring layers. To create a proper layering, dummy nodes are added to break edges that span multiple layers, e. g., by adding a dummy node to $\langle n3 \rangle$ to break the edge from $n4$ to $n5$ into two. We define $\text{dummy} : V \rightarrow \{\text{true}, \text{false}\}$ as the dummy node predicate that returns true if a node is a dummy node. Furthermore, we call nodes that do occur in the model *real nodes*; dummy nodes that are placeholders for edge labels are called *label dummy nodes*.

Furthermore, let $s : E \rightarrow V$ with $e = (u, v) \in E$ and $s(e) = u$ be the *source function* and $t : E \rightarrow V$ with $e = (u, v) \in E$ and $t(e) = v$ be the *target function*. Let $o : V \cup E \rightarrow \mathbb{N} \cup \perp$ be the *order function* that outputs the model order value for edges and nodes, and returns \perp for any dummy node or node without a model order. Let $\text{indegree} : V \rightarrow \mathbb{N}$ and $\text{outdegree} : V \rightarrow \mathbb{N}$ be the functions that return the number of incoming and outgoing connections respectively.

There already exist several heuristics that optimize aesthetic criteria for the different phases. We argue that model order should be considered in the first three topological phases, since they establish the relative position between nodes and edges. Since all phases should work independently of each other, we cannot guarantee that nodes are ordered before each phase. Therefore, the nodes and edges should be sorted before using model order strategies, which is the main concept of model order crossing minimization by (Domrös and von Hanxleden, 2022).

Cycle breaking makes the input graph acyclic by reversing edges. The problem of minimizing the number of edges that need reversing is called *feedback arc set problem* (Eades et al., 1993) and is NP-hard. Therefore, heuristics are used to reverse edges, which sometimes rely on randomization to make decisions if no unique optimal alternative exists.

Layer assignment creates a (proper) layered graph from a given acyclic digraph by assigning nodes to layers. Beginning from the sources (or sinks), nodes are assigned to layers. These can be optimized for aesthetic criteria such as edge length (Gansner et al., 1993) or total layer width (Nikolov and Tarassov, 2006). If the graph is built with a node model order that emphasizes the layers, model order layer assignment can form semantic grouping, as detailed in Section 4.

Crossing minimization minimizes edge crossings by changing the order of nodes and ports in the same vertical layer. Crossing minimization may utilize model order either as a tie-breaker, as discussed by (Domrös and von Hanxleden, 2022), or to constrain nodes and edges, as proposed in Section 5.

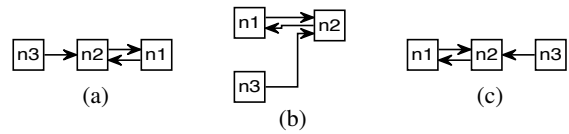


Figure 3: (a) The greedy cycle breaker tries to optimize the number of backward edges and uses random decisions if no unique solution exists. (b) The greedy model order cycle breaker tries to optimize the number of backward edges and uses model order if multiple optimal solutions exist. (c) The model order cycle breaker reverses edges to nodes with a lower model order.

3 CYCLE BREAKING

There are two basic concepts of model order during cycle breaking. Figure 3 showcases how model order can be ignored, be used as a tie-breaker, or be enforced.

In Figure 3a the greedy cycle breaker optimizes the number of backward edges. However, reversing the edge from $n1$ to $n2$ instead of $n2$ to $n1$ results in the same number of backward edges but violates the model order.

Figure 3b solves this by using model order as a tie-breaker. As a result $n1$ is placed in the layer before $n2$ and not the other way around.

In Figure 3c we assume that the backward edge aesthetic criterion is disregarded in favor of order. Such assumptions depend on the specific graphical language in use, as seen in Section 6.2 and 6.4.

3.1 Model Order as Tie-Breaker

Many cycle breaking algorithms, such as depth-first (Gansner et al., 1993) or breadth-first, already implicitly use model order. A depth-first cycle breaker begins with the sources. Next, all nodes on the sources are visited recursively, and edges that point to already visited nodes are reversed. If model order is not taken into account, there might be multiple solutions based on the traversal order of the graph. Even though it is not explicitly stated by Gansner et al., it is assumed that the next nodes are visited based on the edge model order or the node model order.

The same applies for other layout algorithms such as the greedy cycle breaker presented by (Eades et al., 1993) and (Di Battista et al., 1999), seen in Algorithm 1, in which `removeFirst` is a function that removes and returns the first element of a list, `updateNeighbors` removes the edges to a node for a given graph and node, and `findMaxOutflow` returns the nodes with the highest out-in-degree difference. In addition to the traversal order, this algorithm makes random decisions in line 19 if the nodes have the same

Algorithm 1: greedy[ModelOrder]CB.

```

Input: A digraph  $G = (V, E)$ 
Output: An acyclic digraph
1  $G' := G$ , sources :=  $\emptyset$ , sinks :=  $\emptyset$ 
2 for ( $n \in V$ )
3   if ( $\text{indegree}(n) = 0$ )
4     sources  $\cup = n$ 
5   if ( $\text{outdegree}(n) = 0$ )
6     sinks  $\cup = n$ 
7 unprocessed :=  $|V|$ 
8 while ( $\text{unprocessed} > 0$ )
9   while ( $|\text{sinks}| > 0$ )
10    sink := removeFirst(sinks)
11    updateNeighbors( $G'$ , sink)
12    unprocessed --
13   while ( $|\text{sources}| > 0$ )
14    source := removeFirst(sources)
15    updateNeighbors( $G'$ , source)
16    unprocessed --
17   if ( $\text{unprocessed} > 0$ )
18     maxNodes := findMaxOutflow( $G'$ )
19      $n = \text{choose}[\text{ModelOrder}]Node(\text{maxNodes})$ 
20     // Reverse incoming edges
21     for ( $e = (v, n)$ )
22        $E := (E \setminus e) \cup (n, v)$ 
23     unprocessed --
24 return  $G$ 

```

outflow. Here chooseNode randomly chooses a node for which all incoming edges are reversed. Applied to real graphs, this can create ONO-cases such as Figure 1a. Here, a backward dangling node is produced since the algorithm randomly decides to reverse the edge from n_2 to n_3 and not the one from n_3 to n_2 as it is suggested by the model order.

We propose that in the *greedy model order cycle breaker*, chooseModelOrderNode returns the node with the minimal model order $o(n)$ instead.

In such a way every cycle breaker can use model order as a tie-breaker while traversing ordered sets or making random decisions.

3.2 Model Order as Constraint

If model order is enforced by reversing all edges to a node with a lower model order, the graph becomes acyclic. Moreover, any edge reversal that results in an acyclic graph can be enforced, e. g., by traversing the resulting tree-like structure breadth-first and ordering the nodes such that their model order corresponds the breadth-first visiting order. This gives the modeler full control over the cycle breaking step.

A model order cycle breaker that additionally allows constraining nodes to the first or last layer, which is common for e. g. SCCharts, can be seen in Algorithm 2, where $c : V \rightarrow \{-1, 0, 1\}$ is defined as follows:

$$c(n) = \begin{cases} -1, & n \text{ shall be in the first layer} \\ 1, & n \text{ shall be in the last layer} \\ 0, & \text{otherwise} \end{cases}$$

Algorithm 2: modelOrderCB.

```

Input: A digraph  $G = (V, E)$ 
Output: An acyclic digraph
1 for ( $e \in E$ )
2   if ( $c(s(e)) > c(t(e)) \vee (c(s(e)) = c(t(e)) \wedge o(s(e)) > o(t(e)))$ )
3      $E := (E \setminus e) \cup (t(e), s(e))$ 
4 return  $G$ 

```

The algorithm groups the nodes into nodes with first-layer constraint, nodes without constraints, and nodes with last-layer constraint and orders the nodes within these groups. The order of constraints and the order inside the constraint groups create a total order for all nodes, which identifies the edges to reverse.

The runtime of this algorithm is in $O(|E|)$, making it the fastest possible cycle breaking algorithm.

In Section 6.1 we evaluate the model order cycle breaker against the depth-first, the breadth-first, and the greedy cycle breaker.

4 LAYER ASSIGNMENT

The effect of model order layer assignment can be seen in Figure 1d compared to Figure 1c.

Model order layer assignment needs cycle breaking done by the model order cycle breaker and a breadth-first node model order. The strategy assigns nodes to layers such that no node has a node with a smaller model order in a higher layer and no node with a bigger model order in a smaller layer. The breath-first ordering of nodes is necessary to compare different control-flow *branches*. This may create additional layers, as seen in Figure 4a and 4b.

SCCharts have one initial state (see state without a name) and final states, hierarchy, and parallel regions of execution, which are not shown here. Priorities on the edges (see 1. on the edge from the initial state to s_1) express the evaluation order of their guards, which are omitted here, and derive from the order of the outgoing transitions of a state in the textual model, hence they represent the edge model order. The node order in the textual model has no semantics and could be arbitrary. However, control-flow branches are a common sight. Developers think one case of their program, e. g., branch b_1 , from start to finish before implementing a new one. The developer does, therefore, only intend that nodes in b_1 are before nodes in b_2 and does not intend the layering shown in Figure 4b. Thus, for SCCharts the nodes in b_1 and b_2 are not comparable by model order during layer assignment.

Model order layer assignment works by doing a trivial layer assignment beginning with the sources, which again implicitly utilizes the model order of the nodes or edges to traverse the nodes. As a second

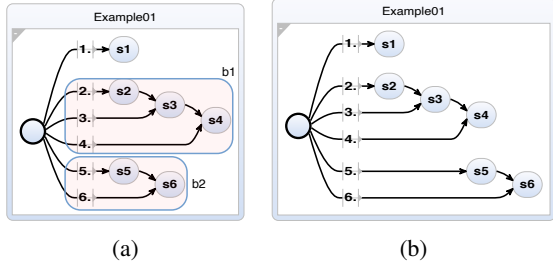


Figure 4: Drawing of an obfuscated SCCharts model. (a) Branches b_1 and b_2 are only used to infer on ordering between the nodes in them but not used to infer an alignment. (b) Interpreting the model order as a depth-first ordering creates an additional layer.

step in the l_2 intermediate processing slot, nodes are moved based on the model order of nodes in the next and the current layer, as seen in Algorithm 3.

Algorithm 3: modelOrderNodePromotion.

```

Input: A layered graph  $G = (V, E)$ 
Output: A layered ordered graph
1 somethingChanged := false
2 do
3   for ( $n \mid \neg \text{dummy}(n) \wedge \text{outdegree}(n) > 0$ )
4     if ( $o(n) = \perp$ )
5       continue
6     currentLayer :=  $L_{L(n)}$ 
7     if ( $|\text{currentLayer}| = 1 \wedge L(n) = 0$ )
8       continue
9     for ( $v \in \text{currentLayer}$ )
10      if ( $o(n) < o(v)$ )
11        continue
12    nextLayer :=  $L_{L(n)+1}$ 
13    allowsPromotion := false
14    dummyLayer := true
15    for ( $v \in \text{nextLayer}$ )
16      if ( $\neg \text{dummy}(v)$ )
17        allowsPromotion :=  $o(v) < o(n)$ 
18        dummyLayer := false
19      else if ( $\neg \text{allowsPromotion} \wedge \text{dummyLayer}$ )
20        if ( $(n, v) \in E \wedge L(\text{longEdgeTarget}(n)) - L(n) \leq 2$ )
21          dummyLayer := false
22    if ( $\text{allowsPromotion} \vee \text{dummyLayer}$ )
23      promoteNode( $G, n$ )
24      somethingChanged := true
25 while somethingChanged;
26 return  $G$ 

```

Here promoteNode moves a node to the next layer while also recursively promoting connected nodes that would otherwise end up in the same layer and would create in-layer edges.

A real node is promoted if (1) it has a model order (line 4-5), (2) it is not the only node in the first layer (line 7-8), (3) it has the highest model order in its layer (line 9-11), and (4) the next layer either has at least one real node with a lower model order (line 16-18) or (5) the next layer contains only dummy nodes while the current node has enough space to its con-

nected real node such that it can safely be moved in the next layer (line 19-21).

Constraint (1) is necessary since there might be nodes that are not dummy nodes that are created by the synthesis, which translates the model into a graph layout problem, as it is the case for Lingua Franca (von Hanxleden et al., 2022).

Constraint (2) makes sure that the source node does not promote the whole graph over and over.

Constraint (3) makes sure that only the node with the highest model order can be promoted.

Constraints (4) and (5) describe the two different criteria for the next layer, the layer the current node might be moved to. Either a real node with a lower model order is in the next layer, as seen in Figure 4a for s_5 and s_3 , or the next layer has only dummy or dummy label nodes, as seen in Figure 5a, which shows an Systems Theoretic Process Analysis (STPA) control structure that is used to analyze hazardous scenarios. Here Engine System can be moved since Vehicle, its *long edge target*, which is the real node it is connected to, is far enough away to move the node and the label dummy without also moving Vehicle. Note that Safety System could be in the layer directly after Operator Safety, Safety System aligns itself with Telemetry based on the model order.

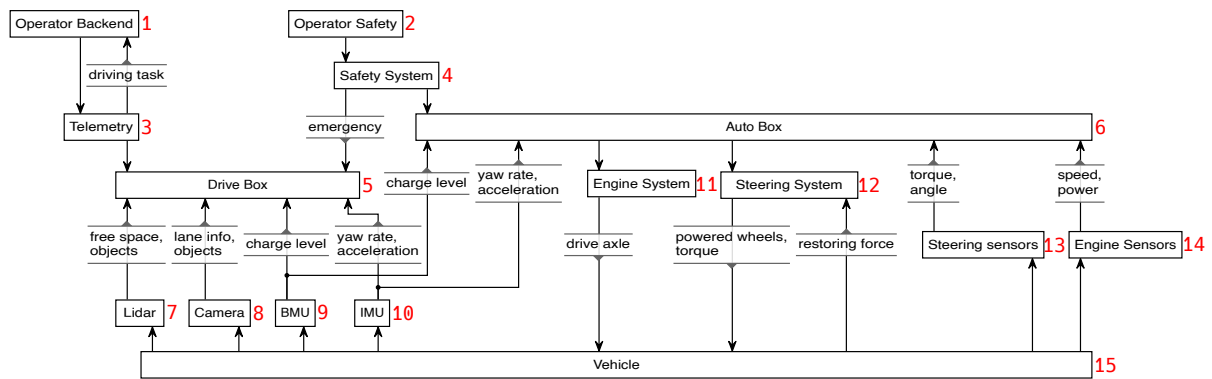
Each layering can be recreated by model order by ordering the nodes breadth-first based on their position in their intended layer, as seen in Figure 5b.

5 CROSSING MINIMIZATION

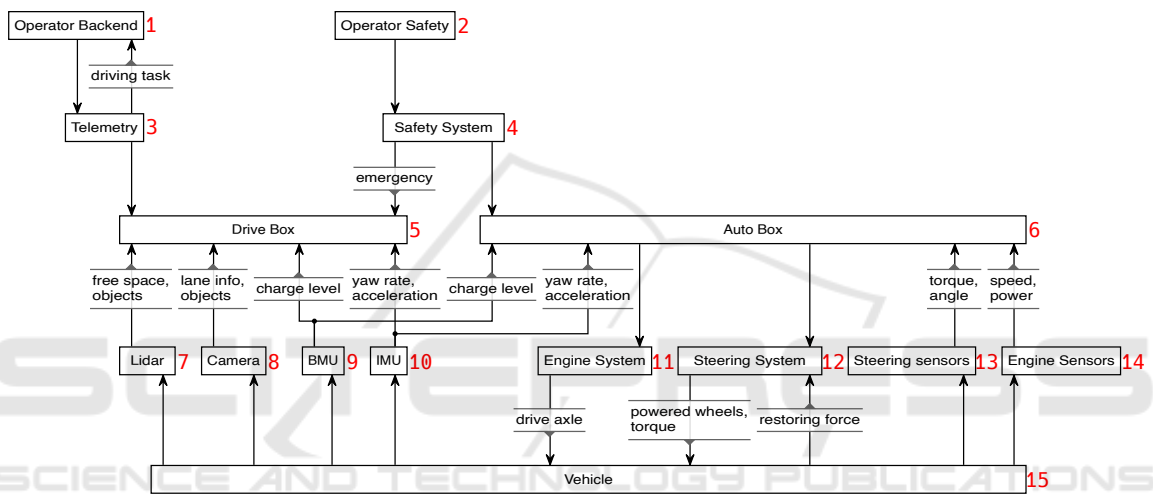
Crossing minimization determines the vertical order of nodes and ports inside a layer. This is done by *sweeping* back and forth through the layers and re-ordering all nodes and ports in a layer based on the previous layer. The outcome of a *layer sweep* depends on the starting configuration. Multiple *runs* and random starting configurations bounded by the *thoroughness* prevent local minima. As shown by (Domrös and von Hanxleden, 2022), it may need a high thoroughness to get the desired result if several starting configurations exist.

5.1 Pre-Ordering Nodes and Edges

Domrös and von Hanxleden introduced model order as a tie-breaker during crossing-minimization by beginning the first crossing minimization runs with a proper layered graph pre-ordered by model order in the l_2 intermediate processing slot. Moreover, they consider order violations in addition to edge crossings during crossing minimization.



(a) ONO decision during layer assignment. The number of layers is minimal, however, the relationship between the different states, which is partly expressed by model order, is lost.



(b) OYES decision during layer assignment. The actuators and sensors Lidar, Camera, BMU, IMU, Engine System, Steering System, Steering Sensors, and Engine Sensors are in the same layer.

Figure 5: STPA control structure with downward layout direction of an autonomous vehicle¹. The node model order is marked in red to the right of each node.

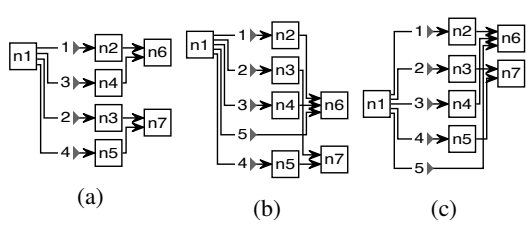


Figure 6: (a) Tie-breaking crossing minimization that pre-orders nodes and edges (no crossings). (b) The node order is enforced after pre-ordering (one crossing). (c) No crossing minimization, only model order (three crossings).

This algorithm configuration produces results such as Figure 6a. Since the ordered solution is checked first and order violations can be weighted against crossings, a solution is chosen from the crossing minimization runs that has minimal ordering violations as a secondary criterion.

(Domrös and von Hanxleden, 2022) introduced two different ordering strategies. These strategies serve as an initial order or as a tie-breaker during crossing-minimization. Subsequent runs might be randomized and are only taken if they are better than the ordered solution.

The *preferEdges* approach prefers the edge model order and only refers to the node model order if no clear solution could be found, as it is the case for dangling nodes such as n3 in Figure 1a or Figure 3b.

The *nodesAndEdges* approach considers nodes and edge order by ordering nodes and edges by their respective order and using the edge order of the connected source ports in the previous layer as a fallback if dummy nodes and real nodes are compared.

Additionally, we propose the *preferNodes* ap-

¹www.smartload-project.de

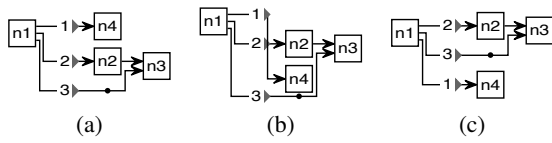


Figure 7: Different pre-ordering strategies for crossing minimization (l2 slot) visualized by omitting crossing minimization. (a) Edge order orders nodes and edges. Node order is used as a secondary criterion. (b) Edge order orders the edges and the dummy nodes. Node order orders the real nodes. (c) Edge order orders dummy nodes. Node order orders real nodes and edges.

proach that orders real nodes by model order, dummy nodes—as before—by their connected source ports, which are already ordered by node model order, and edges by their long edge target node’s model order. The different strategies can be seen in Figure 7. Figure 7a and Figure 7c do not change during crossing minimization, since they are already optimal in terms of edge crossings. Crossing minimization reduces the crossing in Figure 7b resulting in the ordering depicted in Figure 7a.

As already described in Section 3.2, the model order can also be interpreted as a constraint resulting in the following strategies.

5.2 Constraining Node Model Order

If we constrain the real nodes in each layer, many node positions for dummy nodes, which define the relative routes of edges, are still feasible without compromising the node model order. Crossing minimization is edge-centric and often disregards the node order in favor of the edge order if the initial solution is not crossing minimal. Therefore, a special crossing minimization strategy is necessary.

A crossing minimization strategy, such as the barycenter heuristic (Sugiyama et al., 1981) or the median heuristic (Di Battista et al., 1999), can take the model order into account if it does layer sweeps and applies a strategy to order a bipartite graph—here the barycenter heuristic—such that it does not change an ordering if it is already optimal. The model order and the barycenter values form a partial and a total order on the nodes. Therefore, transitive orderings have to be taken into account and a sorting algorithm that is order preserving such as insertion sort has to be used.

As seen in Figure 6b, constraining the nodes creates additional crossings compared to Figure 6a but always maintains the node model order. Note that this approach might not find the optimal solution (edge 5 on top) in terms of edge crossings, since it is still limited by the used starting configurations.

Constraining only edges, as presented in this section for the nodes, constraints all parts of the graph except source nodes. Therefore, this strategy is discussed as part of the no crossing minimization approach.

5.3 No Crossing Minimization

For many models, crossing minimization might not be necessary if nodes and edges are already pre-ordered. Enforcing model order by pre-ordering nodes and edges with the `preferEdges`, `nodesAndEdges`, or `preferNodes` approach without doing crossing minimization at all is a viable solution.

If the edge order is enforced, it defines the order of the nodes and ports if a graph has only one source and no dangling source nodes are created. This is the case for SCCharts where each region must have exactly one initial state and all states must be reachable. If multiple sources are present, the node model order is used as a secondary criterion. This is enough to create any edge or node order inside a layer if the order of nodes and edges has no semantics. Since the pre-crossing minimization sorting strategy `preferEdges` fulfills these criteria, edge order can create a layout by sorting nodes and edges with this strategy and not doing crossing minimization at all, as seen in Figure 7a. This makes it compatible with the enforced cycle breaking and layer assignment breadth-first node order constraint.

If the node order should be enforced, the `preferNodes` pre-crossing minimization sorting strategy is used while also omitting crossing minimization. This results in a layout such as Figure 7c. Since this ordering depends on the long edge target nodes, it may be conflicting with a breadth-first node order. Since it is, therefore, not compatible with cycle breaking’s and layer assignment’s constructive breadth-first ordering constraint, it cannot be used to create any given layout, as seen in Figure 4a. If one would order the states breadth-first $\langle \text{init}, s1, s2, s5, s3, s6, s4 \rangle$ to create the layering $\langle \langle \text{init} \rangle, \langle s1, s2, s5 \rangle, \langle s3, s6, s4 \rangle \rangle$, the edge ordering would be different and create crossings if enforced, e. g., between the edge from $s5$ to $s6$ and the initial state to $s3$.

The `nodesAndEdges` strategy should only be used for a no crossing minimization approach if node and edge order are not conflicting. Otherwise, edge crossings are created as the one in Figure 7b.

For complicated models, we advise using crossing minimization since moving edges and nodes in the textual model—while an automatic solution is available—can control their placement but doing this manually on a regular basis is tiresome.

6 DISCUSSION

Before discussing what strategies can be used effectively in which scenario, we summarize the model order strategies for Sugiyama layouts with their respective slot in the algorithm:

Phase 1: Cycle breaking.

Greedy Model Order Cycle Breaking. Optimize for backward edges but use model order as a tie-breaker.

Model Order Cycle Breaking. Edges are reversed by model order such that edges always point to a node with a higher model order.

Phase 2 and intermediate slot l2: Layer assignment and post-processing.

Model Order Layerer. Promote nodes by model order after a simple long edge layering beginning from the sources.

Intermediate slot l2: Crossing minimization pre-processing.

PreferEdges. Order nodes and edges before crossing minimization primarily based on the edge order. (Domrös and von Hanxleden, 2022)

NodesAndEdges. Order nodes by node order and edges by edge order before crossing minimization. (Domrös and von Hanxleden, 2022)

PreferNodes. Orders nodes and edges before crossing minimization primarily based on the node order.

Phase 3: Crossing minimization.

Weighting. Include weights for node and edge order additionally to edge crossings that rate the crossing minimization runs. (Domrös and von Hanxleden, 2022)

Enforce Node Order. Instead of pure barycenter-based crossing minimization, the node order primarily determines the ordering.

No Crossing Minimization. Pre-ordering nodes and edges is sufficient; the crossing minimization step does nothing.

In the following we discuss these strategies depending on their usage for SCCharts and Lingua Franca. A study on cycle breaking for SCCharts, quantitative analysis, and feedback of expert SCCharts and Lingua Franca developers serves as a basis for this. The feedback was collected in multiple iterations during weekly developer meetings, while verifying the extracted information on existing models. We recommend a similar process when considering model order for other languages to identify desired order and ordering conventions in the textual and graphical model.

6.1 Model Order Cycle Breaking in SCCharts

We analyzed 265 SCChart models using the greedy (see Algorithm 1), the breadth-first, the depth-first, and the model order cycle breaker (see Algorithm 2) and the nodesAndEdges crossing-minimization pre-ordering. The textual models were created without model order influencing the drawing. Instead, the greedy cycle breaker optimized backward edges, the layering optimized for minimum edge length, and the crossing minimization optimized the number of edge crossings.

Only 47 of these SCChart graphs resulted in unique solutions for all algorithms. Eight selected graphs were rated by 27 participants based on their first impression, crowdedness, ability to follow edges, state grouping based on perceived semantics deduced from state names, and final impression on a Likert scale. On average the model order cycle breaking returned the best results before the depth-first, the breadth-first, and the greedy cycle breaker. Participants rated the clear edge routes and state grouping as their primary incentive for their decisions. Other factors mentioned are that the initial state should be in the first layer and the final state be in the last, symmetry, edge crossings, no dangling nodes, and understandable and readable node labels, which is partly based on the compactness of the graph.

The greedy algorithm performed worst, mainly because state grouping is only randomly created with this approach and because dangling nodes seem to disturb the flow of the diagram (see n3 in Figure 1a).

The breadth-first approach created compact drawings with fewer layers but more edge crossings. Sometimes, however, it is rated above average if the breadth-first order creates thematic state groups as seen in Figure 8. Here, the second layer holds all stationary states (stretched, stopped, and contracted) and the third all intermediate states (stretching and contracting). Although many edge crossings are created, half of the participants preferred this solution. The second half heavily disliked the drawing because of the introduced edge crossings. This effect occurred for all participants regardless of their prior knowledge in computer science or graph drawing.

To summarize: The node model order between connected nodes should be utilized during cycle breaking since the analyzed models show that the developer desires the same ordering in the diagram. Using this, control-flow loops introduce the backward edge just at the end of the loop, where it semantically fits best. Further analysis revealed that the few exceptions are copy-pasted models and models

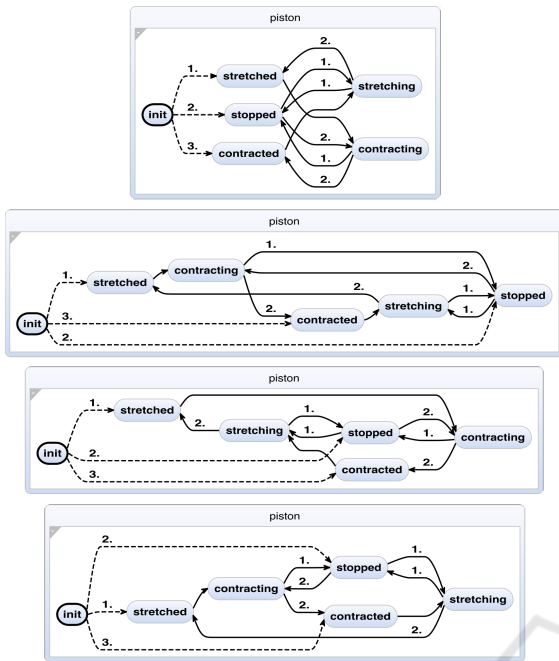


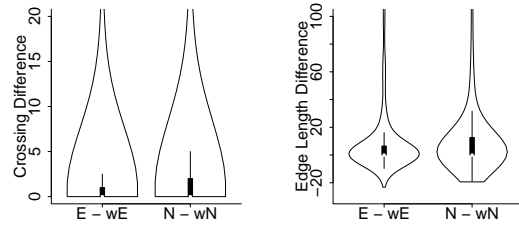
Figure 8: Different cycle breaking strategies compared. From top to bottom: breadth-first cycle breaker, greedy cycle breaker, model order cycle breaker, depth-first cycle breaker.

with changed ordering with the hope to influence the layout. This can be generalized to control-flow languages that model connections explicitly. Therefore, we propose to use the model order cycle breaker for such languages. Furthermore, we see that, as already explained in Section 4, a breadth-first node model order is uncommon in SCCharts models and hence probably also in other languages that model control-flow. However, it can be used to create any desired layering if necessary.

6.2 Model Order Crossing Minimization in SCCharts

As mentioned earlier, edges in SCCharts are numbered by their priorities. The placement inside the diagram has no semantics, therefore, their drawing order can be freely chosen. Expert developer feedback revealed that although a correct ordering is helpful, it should be overridden if the drawing can be simplified by reducing the edge crossings. The order of nodes itself is less relevant and should be disregarded in favor of edge order.

The layered algorithm should, therefore, apply the preferEdges crossing minimization pre-processing and weight node and edge order violation as, e. g., one tenth as important as edge crossings (Domrös and von Hanxleden, 2022). For complete control over the lay-



(a) Edge crossing difference (b) Edge length difference

Figure 9: The preferEdge/preferNodes (E/N) approach without crossing minimization compared to the crossing minimization approach (wE/wN) with node and edge order violation weights during crossing minimization. The edge crossing difference is cut off at 20 crossings to increase the readability.

out the model order layer assignment and preferEdges without crossing minimization should be used, since it can recreate any layout given a breadth-first node model order.

The same conclusions can be drawn for other control-flow languages for which the drawing order of edges or ports and nodes has no semantics.

6.3 Model Order Without Crossing Minimization

We evaluated 357 SCCharts with at least 3 states and more than 3 edges between them created by students during lectures and projects between 2014 and 2022. The results can be seen in Figure 9.

When using preferNodes without crossing minimization, 151 models are different compared to the solution that uses pre-ordering and crossing minimization with a node and port order violation weight of 0.1. When using preferEdges, only 127 are different. We see very few models that create many additional edge crossings. Edge crossings tend to get induced primarily when models are particularly complicated, relied on copy and paste during creation, or model order had no effect on the layout during their creation. For the preferEdges approach, 75% of all models create at most one additional crossing. For preferNodes, 75% of all models create up to two additional crossings. For both approaches the median is 0, which emphasizes that most models are already layouted optimally by just following the model order. Since model order did not affect the creation on these models, we expect that this number will only increase in the future, which will be evaluated in future work.

The edge length only gets significantly worse for the same big, not carefully designed models mentioned earlier. For other graphs it increases on average only slightly.

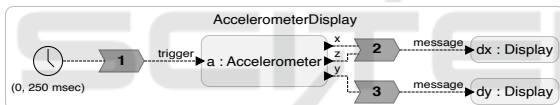
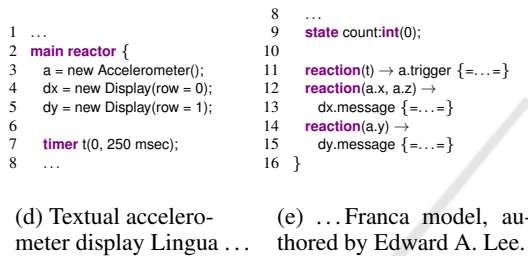
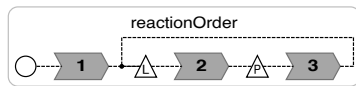
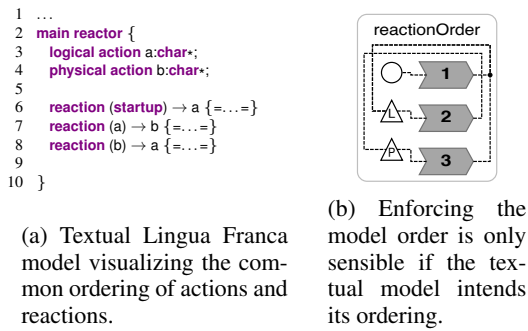


Figure 10: Textual and graphical Lingua Franca models. The startup node is drawn as a circle, a timer as a clock, an action as a triangle, reactions as an arrow shape with a number, and reactors as rounded rectangles.

These results emphasize that node and edge model order is in most cases intended by the SCCharts developer and should, therefore, be respected in the drawing, as also reported by (Domrös and von Hanxleden, 2022).

6.4 Model Order in Lingua Franca

Lingua Franca models are data-flow based, as seen in Figure 10. Even though the textual order of reactors, reactions, actions, and timers, is interchangeable, they are usually grouped by category, e. g., first all action then all reactions, as seen in Figure 10a, 10d and 10e.

Not every node has a model order. In Figure 10a, the round startup node is not explicitly modeled. It is created when the textual model is synthesized into a view model, which creates the graph. Here, the diagram synthesis takes care of giving the startup node

the correct order inside the list of view model nodes. For the startup node this question is easy, since it is the entry point of execution and should, therefore, be the first node. For other nodes this is not trivial and has to be done with care, especially if model order is used as a constraint. Therefore, it is a valid option to not give these nodes a model order. If this is the case, cycle breaking may ignore them, which works as long as no cycles are created by them, and crossing minimization sorts them by their incoming connections.

6.4.1 Cycle Breaking in Lingua Franca

Since nodes are ordered by convention only inside their respective category, the model order cycle breaker performs badly, as seen in Figure 10b. Declaring the actions (line 3 and 4) before the reactions (line 6-8) results in backward edges from the reactions to the actions. The greedy model order cycle breaker should be used instead to get a drawing with minimal backward edges that creates a deterministic solution, as seen in Figure 10c. Minimizing the backward edges is especially important for Lingua Franca, since they have to be routed around the nodes since outgoing edges are always on the right of a node and incoming edges always on the left side. As seen in Figure 10b, backward edges create, therefore, dummy nodes routing the output backward and the input forward, which increases the clutter, makes edges harder to follow, and may increase the size of the drawing.

The model order cycle breaker can still be used if the textual convention is disregarded to be able to use model order to constrain the drawing. This is also the only use case for model order layer assignment for Lingua Franca. Additionally, the diagram can be configured assigning model order only to reactions, to reactions and reactors, or to everything, since each might be the intended behavior.

6.4.2 Crossing Minimization in Lingua Franca

In contrast to other data-flow languages, Lingua Franca deems the order of connected edges as unimportant, as reported by expert developers, since the edges can be identified by port labels and not relate to real hardware. Hence, changing the port order from x y z to x z y in Figure 10f is viable to reduce the number of crossings.

Enforcing the node order inside a layer improves orientation in complex models such as Figure 11. Here, the order of the Ghosts pinky, blinky, inky, and clyde is maintained.

Doing no crossing minimization often proved to be ineffective, since edges are not explicit and many dummy nodes are created to route backward edges.

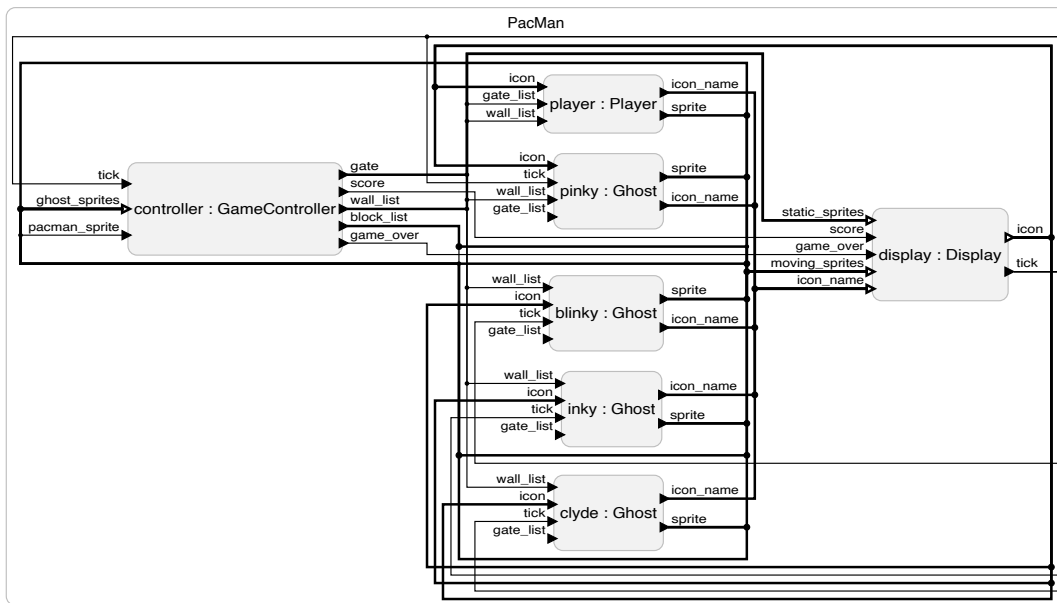


Figure 11: The PacMan Lingua Franca model, which contains the ghosts pinky, blinky, inky, clyde in that order.

Model order could be inferred for dummy nodes such that an implicit ordering could be calculated with other means than the connection to a previous layer, which is evaluated as part of future work on model order. Currently, it is solved by greedily switching edges and nodes in a one-pass algorithm instead of doing crossing minimization.

7 FURTHER RELATED WORK

(Gansner et al., 1993) state that even cyclic graphs have an edge direction based on their graph input that represents their natural direction. They do, however, only conclude that depth-first cycle breaking is preferable to other approaches and do not use the model order—which they implicitly already do for traversing nodes and edges—for cycle breaking.

(Nikolov and Tarassov, 2006) propose two node promotion algorithms that are applied after a longest-path-layering, which is also the starting point for the model order layerer. Instead of considering the ordering, the algorithms minimize the layer width while also considering dummy nodes. Their algorithms optimize the edge length and edge density but utilize node promotion similar to our approach.

Several solutions exist to constrain node and edge order via absolute or relative constraints (Böhringer and Paulisch, 1990; Waddle, 2001; Mennens et al., 2019). Even though enforced model order can replicate any layout by breadth-first order, it is not an alternative to using constraints but rather one additional

instrument to constrain nodes and edges. Since layout constraints need a reference layout and, therefore, two layout runs, the first layout run can use model order and the second one can evaluate the constraints and move nodes. This tends to create the desired layout with fewer constraints if the model order is sensible, since the model order produces a stable initial layout, which is most likely already desired. Since model order layer assignment requires an often not existing breadth-first model order, constraints can be used instead to force nodes in the correct layers.

8 CONCLUSION AND OUTLOOK

We presented a full overview on model order strategies for Sugiyama layouts. Moreover, we suggest strategies that are applicable to control-flow graphs and data-flow graphs based on developer feedback, studies, and quantitative analysis.

Future work on this topic goes in several directions.

Model order currently assumes that the given linear textual ordering can be applied to all elements. Since this is not the case for Lingua Franca, the given model order strategies should also be able to handle different model order groups that represent different kinds of semantic elements.

During cycle breaking and layer assignment, all nodes are expected to have a model order or to be dummy nodes. In practice this might not always be the case, as shown on the example of Lingua Franca.

As part of future work, the algorithm should infer the model order of these nodes.

Lingua Franca proposes another additional use case in form of hyperedges, which are currently treated as normal edges. Future work should investigate what special treatment might be required to apply model order to more general graphs.

Lingua Franca does not explicitly model edges but rather ports. Future work should investigate whether the port and edge order, which are handled equivalently in SCCharts, need to be distinguished for Lingua-Franca-like languages by having an explicit order for input ports.

(Purchase, 1997) found that user performance increased from drawings from medium symmetric drawings to fully symmetric drawings. Since the nature of model order and the grouping it creates relates to the way symmetry allows us to quickly access a model, part of future work should focus on whether model order is also more effective if everything or nearly everything is ordered.

Since the model order tends to produce more stable layouts than randomized solutions, its relation to the mental map should be investigated, as it is deemed as one of the most important aspects of graph drawing (Purchase et al., 2006). In particular, it should be investigated whether the mental map, which is represented by the textual and graphical model, is emphasized using model order in an interactive scenario.

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