

# Clustered Majority Judgement

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Abstract: In order to overcome the classical methods of judgement, in the literature there is a lot of material about different methodology and their intrinsic limitations. One of the most relevant modern model to deal with votation system dynamics is the Majority Judgement.

It was created with the aim of reducing polarization of the electorate in modern democracies and not to alienate minorities, thanks to its use of a highest median rule, producing more informative results than the existing alternatives. Nonetheless, as shown in the literature, in the case of multiwinner elections it can lead to scenarios in which minorities, albeit numerous, are not adequately represented.

For this reason our aim is to implement a clustered version of this algorithm, in order to mitigate these disadvantages: it creates clusters taking into account the similarity between the expressed judgements and then for each of these created groups, Majority Judgement rule is applied to return a ranking over the set of candidates. These traits make the algorithm available for applications in different areas of interest in which a decisional process is involved.

## 1 INTRODUCTION

Voting rules are different and behave differently according to their limitations or sometimes paradoxal traits. Asking for a more inclusive democracy also represents a modern citizens’ quest, but what does exactly it mean? First of all, we want to underline why a majority voting system embodies the best option between the classical judgement methods.

Consider three agents who express their binary judgement (“Yes” or “No”) for two statements  $A$ ,  $B$ ,  $A \wedge B$  and  $A \longleftrightarrow B$ . Premised-based rule take majority decisions on  $A$  and  $B$  and then infers conclusions on the other two propositions.

As shown in the table 1, results are quite different based on the used rule.

We now focus on Agent 2 case: he’s represented in just one of the single proposition ( $A$ ), and his judgement doesn’t agree with the outcome, in the other cases. So, a huge liability of this model could appear: Agent 2 could think about manipulating the outcome, pretending a disagreement for  $A$ . The premised model reacts by providing as final outcome on 3 agents’ votation a “No” for both  $A \wedge B$  and  $A \longleftrightarrow B$ , as originally expressed by Agent 2.

In such a way, a strategical approach on voting could lead to a deviation effect, providing as result the best tricker’s choice.

Table 1: Three agent case of voting.

	A	B	$A \wedge B$	$A \longleftrightarrow B$
Agent 1	Yes	Yes	Yes	Yes
Agent 2	Yes	No	No	No
Agent 3	No	Yes	No	No
<b>Premised rule</b>	Yes	Yes	Yes	Yes
<b>Majority</b>	Yes	Yes	No	No

Looking at the table, we can also highlight another paradoxal aspect: considering majority-based outcome, the latest two propositions are inconsistent with “Yes” value assigned to both  $A$  and  $B$ .

This is known as *discursive dilemma* and deals with inconsistency problem in judgement aggregation based on majority rule (G. Bellec, 2020).

Both premised and majority rule present drawbacks, but the latter has one important feature: it doesn’t suffer from deficiency shown by the first, so that, if an Agent care about the number of propositions agreeing with his own judgement, then it is always in his best interest to report his true preference. For this reason we focus our attention on majority rule as a transparent asset in decisional process, while trying to deal with its intrinsic problems related to judgement aggregation (Kleinberg, 2002).

Our attempt is not aimed to solve above-mentioned dilemma, rather joining a more refined majority rule (*Majority Judgement*) with cluster approach's advantages in aggregating similar patterns.

## 2 MAJORITY JUDGEMENT

### 2.1 A Brief Overview of Collective Decision Making

Many business magazines such as Harvard Business Review and Millionaire.it focus on business meetings, highlighting how inefficiently these meetings are conducted. Business meetings are often perceived as useless and unproductive for many reasons: scope meetings are quite vaguely defined, a very large number of people are involved and there is no such figure as a mediator, making decision processes increase in complexity and effort. (Streibel, 2003) provides a guide on how to plan and conduct a meeting in order to make it effective. In this work are defined different kinds of meeting and the main techniques which can be used to meet each kind of meeting's specific target. Talking about business meetings and management, leadership is a key variable. Tannenbaum and Schmidt (Tannenbaum and Schmidt, 2009) introduce model about leadership styles. The model's parameters take into account the leader's main features, his subordinates' features and the general context. Hence one can easily tell a good leader needs to know and adopt different leadership styles depending on different situations. The managerial grid proposed in (Blake et al., 1964) categorize leaders upon how focused they are across a production-oriented (completing tasks) and a people-oriented (supporting individuals) dimension. In 1973 Vroom and Yetton (Vroom and Yetton, 1973) presented some leadership styles that Vroom and Jabo revised later in 1988. Besides these descriptive decision theories, which provide theoretical models, many group decision making techniques have been studied to provide a means to choose between the alternatives proposed in a meeting. The most cited decision making techniques in (Verzuh et al., 2021) are based on brainstorming and mind mapping processes and focus on priority ranking techniques. The latter are based on voting techniques, such as Nominal Group Technique, Paired Comparison Analysis and Grid Analysis. During board of direction's meetings, voting is often used to converge into a final decision.

Social choice theory studies methods to consolidate the different views of many individuals into a single outcome. The main applications of social choice

theory are voting and jury decisions (Brandt et al., 2016). During voting, electors in a democracy choose one candidate among a list of many candidates, while in a jury decision the individual judges evaluate competitors in a competition (e.g sport competition, wine competitions, etc.), ranking them. Social choice theory's fundamental problem is to find a social decision function that elaborates the preference of judges or voters converging into a jury or electoral decisions while adhering to the main principles of fair voting procedures such as non-dictatorship, universality, independence of irrelevant alternative. Arrow's impossibility theorem shows that the fundamental problem has no acceptable solution in the traditional model (Arrow, 2012). In (Serafini, 2019) Condorcet and Borda methods and limits, Arrow's impossibility theorem and Majority Judgement are illustrated.

Majority Judgement (MJ) is a voting technique proposed by two mathematicians in 2007, Michel Balinski and Rida Laraki, aiming to overcome traditional voting methods' paradoxes and inconsistencies. In (Balinski and Laraki, 2007), published in 2007, Balinski and Laraki briefly describe MJ, moving from a social choice theory analysis which highlights traditional voting methods failures. Hence the need for a voting method where voters evaluate candidates in terms of a common language rather than simply ranking them. MJ makes it possible, since this method asks for electors/judges to express a judgment on all the candidates/competitors, using a known common language. Theorems and experiments confirm that, while there is no method which can completely overcome strategic voting, majority judgment strongly resists manipulation. Balinski and Laraki present MJ as a method both for evaluation and ranking of competitors, candidates or alternatives. In (Balinski and Laraki, 2014) authors explain how electors don't really make a personal ranking of candidates, as traditional methods input assume, and that this is the reason behind the inadequacy of traditional voting models. Forcing electors to rank candidates leads to incoherence, impossibility and incompatibility. Balinski and Laraki (Balinski and Laraki, 2011) present the case of the French presidential elections of 2002 and the results experiment related to the MJ conducted on the occasion of the French presidential elections of 2007. Analyzing data from the 2002 election, the authors describe the limits of the system First Pass the Post (FFP), which allows voters to express just one preference. Voters are induced to strategic voting: voting the candidate who is most likely to win against those deemed worst rather than voting the preferred candidate. In the 2002 French presidential election, Jospin, the left's leading candidate, was eliminated

in the first round, despite being one of the favorites according to polls. In the second round Chirac of the moderate right beat Le Pen, representative of the far right. The vast majority of Chirac’s votes were against Le Pen rather than him. They were, therefore, strategic votes. As a result of the 2002 experience, in the 2007 elections the number of registered voters increased sharply, from 41.2 million in 2002 to 44.5 million in 2007 and it was much discussed in the media whether there could have been strategic votes aimed at to avoid sensational defeats of the favorite candidates in the first round. According to a poll conducted on the election day, 30% of French voters voted strategically in 2007. The minor left candidates, in fact, obtained 27% in 2002 and 11% in 2007, the minor right 16%. in 2002 and 3% in 2002. This is a perfect example of Arrow’s paradox: the winner depends on the presence or absence of candidates, including those who have absolutely no chance of winning. Strategic nominations are also encouraged.

## 2.2 Formal Aspects

To introduce social choice theory formally, consider a simple decision problem: a collective choice between two alternatives. The first involves imposing some ‘procedural’ requirements on the relationship between individual votes and social decisions and showing that majority rule is the only aggregation rule satisfying them. May (1952) (May, 1952) (Caroprese and Zumpano, 2020a) introduced four such requirements for majority voting rule must satisfies:

- **Universal domain:** the domain of admissible inputs of the aggregation rule consists of all logically possible profiles of votes  $\langle v_1, v_2, \dots, v_n \rangle$ , where each  $v_i \in [-1, 1]$  (to cope with any level of ‘pluralism’ in its inputs);
- **Anonymity:** applying any kind of permutation on individual preferences does not affect the outcome (to treat all voters equally), i.e.,

$$f(v_1, v_2, \dots, v_n) = f(w_1, w_2, \dots, w_n) \quad (1)$$

- **Neutrality:** each alternative has the same weight and for any admissible profile  $\langle v_1, v_2, \dots, v_n \rangle$ , if the votes for the two alternatives are reversed, the social decision is reversed too (to treat all alternatives equally), i.e.

$$f(-v_1, -v_2, \dots, -v_n) = -f(v_1, v_2, \dots, v_n) \quad (2)$$

- **Positive responsiveness:** For any admissible profile  $\langle v_1, v_2, \dots, v_n \rangle$ , if some voters change their votes in favour of one alternative (say the first) and all other votes remain the same, the social

decision does not change in the opposite direction; if the social decision was a tie prior to the change, the tie is broken in the direction of the change, i.e., if  $[w_i > v_i$  for some  $i$  and  $w_j = v_j$  for all other  $j]$  and  $f(v_1, v_2, \dots, v_n) = 0$  or  $1$ , then  $f(w_1, w_2, \dots, w_n) = 1$ .

The May theorem (Theorem: “An aggregation rule satisfies universal domain, anonymity, neutrality, and positive responsiveness if and only if it is majority rule”) provides an argument for the majority rule based on four plausible procedural desires and the theorem helps us characterize other aggregation rules in terms of which desiderata they violate.

But that’s with regards to binary choice. Now, we consider a set  $N = [1, 2, \dots, n]$  of individuals ( $n \geq 2$ ). Let  $X = [x, y, z, \dots]$  be a set of social alternatives, for example possible policy platforms, election candidates, or other(List, 2022). Each individual  $i \in N$  has a preference ordering  $R_i$  over these alternatives that represents a complete and transitive binary relation on  $X$ . For any  $x, y \in X$ ,  $xR_iy$  means that individual  $i$  weakly prefers  $x$  to  $y$ . We write  $xP_iy$  if  $xR_iy$  and not  $yR_ix$  (‘individual  $i$  strictly prefers  $x$  to  $y$ ’), and  $xI_iy$  if  $xR_iy$  and  $yR_ix$  (‘individual  $i$  is indifferent between  $x$  and  $y$ ’). But we must specify that at the heart of social choice theory is the analysis of preference aggregation (Caroprese and Zumpano, 2020b), understood as the aggregation of several individuals’ preference rankings of two or more social alternatives into a single, collective preference ranking (or choice) over these alternatives (Garcia-Bermejo, 2011). In case of many successful alternatives, we need a more sophisticated model to deal with preferences’ aggregation (Klaus Nehring a, 2011). A multi-winner election  $(V, C, F, k)$  is defined by a set of voters  $V$  expressing preferences over a number of candidates  $C$ , and then a voting rule  $F$  returns a subset of size  $k$  winning candidates. A voting rule can perform its role on different types of ordered preferences, even though the most common refers to a pre-fixed linear order on the alternatives. In most of cases, these are chosen *a priori*. Formally we denote set of judgements performed by the  $i$ -th voter as profile preferences  $P_i$ . Each profile contains information about the grade of candidates by voters. The voting rule  $F$  associates with every profile  $P$  a non-empty subset of winning candidates. In multi-winner elections more precise traits are required, compared to the ones stated in May’s theory (Fabre, 2018). Indeed:

- **Representation:** for each partition of voters

$$V_i \in V \text{ (with } |V_i| \geq \lfloor \frac{n}{k} \rfloor \text{)} \quad (3)$$

at least one successful candidate is elected from that partition;

- **Proportionality:** for each partition of voters

$$V_i \in V \left( \text{with } |V_i| \geq \left\lfloor \frac{n}{k} \right\rfloor \right) \quad (4)$$

number of elected candidate is proportional to the partition’s size.

An implicit assumption so far has been that preferences are ordinal and not interpersonally comparable: preference orderings contain no information about each individual’s strength or about how to compare different individuals’ preferences with one another. Statements such as ‘Individual 1 prefers alternative x more than Individual 2 prefers alternative y’ or ‘Individual 1 prefers a switch from x to y more than Individual 2 prefers a switch from x\* to y\*’ are considered meaningless. In voting contexts, this assumption may be plausible, but in welfare-evaluation contexts—when a social planner seeks to rank different social alternatives in an order of social welfare—the use of richer information may be justified.

### 2.3 Single-winner Majority Judgement

In order to describe the majority judgement, we need to use a table that refers to ranking for all the candidates C, by using tuples (Balinski, 2006). Suppose having six possible choices we may use the words: *excellent, very good, good, discrete, bad, very bad*. So each candidate is described by a bounded set of vote.

In general, letting  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$  be a candidate A’s set of n grades (written from highest to lowest,  $\alpha_i \geq \alpha_{i+1}$  for all i), there is a majority of (at least)  $n - k + 1$  for n A’s grade to beat most  $\alpha_k$  and at least  $\alpha_{n-k+1}$ , for all  $1 \leq k \leq \frac{(n+1)}{2}$ . We call this the (n-k+1) - majority for  $[\alpha_k, \alpha_{n-k+1}]$ .

As already mentioned any possible ranking tuple that we choose to describe must follow ordering relations. So the ranking should respect domination: namely, evaluate one candidate above another when that candidate’s grades dominate the other’s.

The described majority judgement is a single winner system, found comparing recursively median grade between candidates: first, grades are ordered in columns from the highest to the lowest according to the order relation, then the middle column (lower middle if number of grades are even) with the highest grade between candidates’row is selected. If there’s a tie, algorithm keeps on discarding grades equal in value to the shared median, until one of the tied candidate is found to have the highest median. Before describing how it’s possible to generalize this single winner system to a multi winner strategy, thanks to the use of clusters, we focus our attention on how these works, analyzing in particular *K-medoids*.



Figure 1: Example with 5 grades, between the dashed lines it’s reported the median grade. Highest occurrences in “Good” determines the winner.

## 3 CLUSTERING APPROACH

### 3.1 How Clusters Work

There’s no precise definition of clustering, mostly due to the huge variety in different clustering algorithms. We can state that they share the ability to divide data into groups with some common features. According to some general traits, we can distinguish types of clustering:

1. **Connectivity Models:** data points in a sample space exhibits similarity according to the distance between them. Two approaches are equally valid: *bottom-up* where each observation constitutes a group and then pairs of clusters are merged; *top-down*, where observation are included in one cluster and then it’s segregated; but in both approaches is not included the possibility of modifying a cluster once created;
2. **Distribution Models:** once created a cluster, model check probabilities on observations following a particular distribution. Good performances are not always guaranteed since these models are prone to overfit data if no constraint on complexity is made;
3. **Density Models:** areas of higher density are identified and local cluster are there created, while remaining data can be grouped into arbitrary shaped region, with no assumption about data distribution; for their flexibility, these models are fit to handle noise better than organizing data on fixed required body. Since we would like to model clusters that satisfy requirements expressed before, based on pretty fixed structure with no assumption about distribution followed by data, it seems more accurate considering a different class of clustering algorithm known as *centroid models*.

### 3.2 K-Medoids

Clustering is the process of grouping a set of objects in order to have each similar object to each other in one cluster, that are dissimilar to objects in other clusters.

For our goal, namely selecting winners from a group of candidates, *K-medoids* clustering are used, because

medoids are the representative objects that are considered, in order to have a result that belongs to the group of candidates: it is based on the most centrally located object in a cluster, so it is less sensitive to outliers in comparison with the K-means clustering, which is not the best model in our case since it could result in something that is not present in the candidate list due to the fact that is an average-based method rather than median. In fact, the medoid is a data point (unlike the centroid) which has the least total distance to the other members of its cluster.

Another advantage for this choice is that the mean of the data points is a measure that gets highly affected by the extreme points; so, in K-Means algorithm, the centroid may get shifted to a wrong position and hence result in incorrect clustering if the data has outliers because then other points will move away from. On the contrary, the K-Medoids algorithm is the most central element of the cluster, such that its distance from other points is minimum. Thus, K-Medoids algorithm is more robust to outliers and noise than K-Means algorithm.

The K-medoid we use is part of the python `sklearn` library (Pedregosa et al., 2011), which is oriented to machine learning. This library supports *partitioning around medoids* (PAM) (Leonard Kaufman, 2015) proposed by Kaufman and Rousseeuw (1990), that is known to be most powerful. The workflow of PAM is described below (Hae-Sang Park, 2008).

The PAM procedure consists of two phases: *BUILD* and *SWAP*:

- In the BUILD phase, primary clustering is performed, during which  $k$  objects are successively selected as medoids.
- The SWAP phase is an iterative process in which the algorithm makes attempts to improve some of the medoids. At each iteration of the algorithm, a pair is selected (medoid and non-medoid) such that replacing the medoid with a non-medoid object gives the best value of the objective function (the sum of the distances from each object to the nearest medoid). The procedure for changing the set of medoids is repeated as long as there is a possibility of improving the value of the objective function.

Suppose that  $n$  objects having  $p$  variables each should be grouped into  $k$  ( $k < n$ ) clusters, where  $k$  is known. Let us define  $j$ -th variable of object  $i$  as  $X_{ij}$  ( $i = 1, \dots, n$ ;  $j = 1, \dots, p$ ). As a dissimilarity measure is used the Euclidean distance, that is defined, between object  $i$  and object  $j$ , by:

$$d_{ij} = \sqrt{\sum_{a=1}^p (X_{ia} - X_{ja})^2} \quad (5)$$

where  $i$  and  $j$  range from 1 to  $n$ . The medoids is selected in this way:

- calculate the Euclidean distance between every pair of all objects;
- calculate  $v_j = \sum_{i=1}^n \frac{d_{ij}}{\sum_{i=1}^n d_{il}}$ ;
- sort all  $v_j$  for  $j = 1, \dots, n$  in ascending order and select the first  $k$  object that have smallest initial medoids value;
- from each object to the nearest medoid we can obtain the initial cluster result;
- calculate the sum of distances from all objects to their medoids;
- update the current medoid in each cluster by replacing with the new medoid, selected minimizing the total distance from a certain object to other objects in its cluster;
- assign each object to the nearest medoid and obtain the cluster result;
- calculate the sum of distance from all objects to their medoids, so if the sum is equal to the previous one, then stop the algorithm; otherwise, go back to the update step.

In our case, prior knowledge about the number of winners is required, and identified clusters are restricted in minimum size that is number of voters on the number of candidates ( $\frac{n}{k}$ ).

### 3.3 Clustered Majority Judgement

Multi winner majority judgement exploits clustering approach to apply to each group majority judgement (Andrea Loreggia, 2020). Given  $k$  the number of candidates to be elected, algorithm seeks the optimal number of cluster to create.

This ranges from 1 to  $k$  and has to satisfy an important additional requirement: once selected a number of clusters, if a tie occurs and so  $k'$  vacant seats are left, algorithm is repeated  $k'$  times until tie's broken. In case there's no broken tie, fixed number of cluster is changed.

### 3.4 Algorithm

In order to explain how the algorithm deals with polarization problem, most relevant steps are described in pseudocode and in annotated strides:

1. set the number of winners as maximum number of clusters;

2. cluster are created decreasing the previous maximum number of clusters until the optimal number is not achieved. This number is bound by the size of cluster, that satisfies the following proportion:  $\text{number of voters} : \text{number of winners} = \text{number of voters in one cluster} : \text{one winner}$  (line 8 in pseudocode);
3. the function *winners* calculates the median for every created cluster (line 15 of pseudocode);
4. check that winners from cluster are different between each other (line 29 in pseudocode); in case it's not true (condition="ko" on pseudocode) algorithm goes back to step 2 with a maximum number of cluster equal to number of vacant seats and the proceedings are held until all seats have been filled.

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Algorithm 1

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**Require:**  $k \geq 0$

**Ensure:**  $n\_winners = (n_1, \dots, n_k), k > 1$

```

k ← number_winners
max_cluster ← k
condition ← "ko"
while condition = "ko" do
    cluster_list ← cluster(vote_list)
    for all list_cluster do
        winners_per_cluster ← compute_winners(cluster)
        all_winners ← list_of_all_winners(winners_per_cluster)
    end for
    list_winner_distinct ← list_of_all_distinct_winners(all_winners)
    option_remaining ← number_winners - len(list_winner_distinct)
    if option_remaining = 0 then
        condition = 'ok'
    else
        k ← option_remaining
        condition ← 'ko'
    end if
end while

```

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### 3.5 Case Studies

In this section, we describe two interesting comparisons of majority judgement (MJ) and clustered majority judgement (CMJ).

The first one is a simulation of the use of MJ for deciding the name of a new security software to be release for the bank employees (we point out here that is has been a real life case study but the bank management asked to hide the references for privacy reasons). The

second scenario we tested MJ is the case of a local bank BCC Naples that used MJ in order to test if their credit policy could be encoded in an automatic system.

#### 3.5.1 Case Study 1: President of the Republic Election

In order to test our algorithm, we asked an heterogeneous group of voters to express judgements on a pre-defined list of possible candidates as President of the Republic before the elections took place. This list has been created according to the rumours circulated on that period, creating a bias effect on our results, as it was excluded a possible rielection of Sergio Mattarella.

In spite of it, we focus on how the algorithm has worked in order to balance polarization, returning a subset of winners with size chosen *a priori*, that we may interpret as best solutions for majority of people who took part into the venture.

Input parameters of Clustered Majority Judgement test are *Excellent*, *Very Good*, *Good*, *Acceptable*, *Poor*, *To Reject*, *No Opinion* and the number of winners is set a priori equal to 3. 125 voters took part into this election and the algorithm form three clusters, exactly like the number of winners.

Testing our algorithm on the described election has shown how difference preference has a leverage on judgement aggregation: for example, voters in Cluster 1 are more bound to express "Good" judgement for candidates considered neutral in terms of political ideas, than the cluster 3 in which voters have a tendency in judging neutral ones as "Fair" or "Poor". Cluster 2 has intermediate traits and no particular tendency is emphasized. We can compare CMJ results

Table 2: CMJ results.

Cluster	Cluster size	Winner
Cluster 1	65	Mario Draghi
Cluster 2	35	Paolo Gentiloni
Cluster 3	25	Anna Finocchiaro

Table 3: Top 3 of single-winner Majority Judgement applied to voters.

Ranking MJ	Candidate
1	Mario Draghi
2	Paolo Gentiloni
3	Emma Bonino

with single-winner MJ ranking, comparing the tables tab3 and tab2. The comparison shows different results for the third candidate, highlighting how clus-

tering influences outcome, giving more weight to minorities' judgement.

### 3.5.2 Case Study 2: Deciding the Name of Security Software

Another interesting comparison case is our second case study, which concerns the application of MJ and CMJ to a business case where a well-known international credit institution shared the need to have an custom branded proprietary cyber security system. The same judgments paradigma of the case study 1 were used: *Excellent, Very Good, Good, Acceptable, Poor, To Reject, No Opinion*. Every voter will have to respond with these judgments to these alternatives proposed by managers: *3PM, ARTOOL, CARLO/CARLOW, KY3P, MEDUSA, PANORAMA, TRIMTOOL*.

The results of MJ for this election appear to lead to a tie; let's consider the judgments attributed to the alternative *MEDUSA* and *ARTOOL*:

- *MEDUSA*: {*Excellent, Very Good, Acceptable, Acceptable, Poor, To Reject*}
- *ARTOOL*: {*Excellent, Very Good, Acceptable, Acceptable, Poor, No Opinion*}

In this case, that is if two alternatives have the same majority grade, a ballot eliminates the common majority grade. If the latter is also common, the procedure is repeated until these bonds are broken, applying this algorithm, we get the following results:

- *MEDUSA*: {*To Reject*}
- *ARTOOL*: {*No Opinion*}

*MEDUSA* has the highest rating, so it wins. This iterative proceedings is capable to break the tie, but it leads to different results, in the case of an even number of voters, if the highest median rating is considered as majority grade. This discrepancy is due to the definition of median and therefore of majority grade: in case of an even number of voters, there are two median judgments and therefore it is difficult to determine exactly which one to consider as majority grade. Furthermore, considering the lowest median rating, there is an absolute majority of ratings at least equal to the majority grade.

Also in this case we want to see if there are substantial differences in the result using the CMJ. Unlike the first case study, we point out number of voters here is very small.

By setting 2 as the number of winners, clustered results is: *MEDUSA* as the first winner and *ARTOOL* as the second. As we can see, the results of the CMJ coincide with that of the traditional MJ. A naive explanation could be linked to the small number of voters, who show a marked polarization for this vote.

### 3.6 Case Study 3: Working Hours per Week

The last case study is a good paradigm for deciding how to manage working hours in the office, given a fixed number of working hours to be done (18 hours). In this case, we asked 160 students of University Federico II of Naples to choose the best combination of working hours, in presence (P) or with online lectures (O). We used again the grades *Excellent, Very Good, Good, Acceptable, Poor, To Reject, No Opinion* and the five options are:

1. 6 hours (P) - 6 hours (O) - 6 hours (P or O)
2. 10 hours (P) - 4 hours (O) - 4 hours (P or O)
3. 8 hours (P) - 6 hours (O) - 4 hours (P or O)
4. 7 hours (P) - 9 hours (O) - 2 hours (P or O)
5. 5 hours (P) - 5 hours (O) - 8 hours (P or O)

The results of MJ method, with the traditional compute of medians takes back as winner the option 4 (7 hours (P) - 9 hours (O) - 2 hours (P or O)) that has the highest number of "Good" votes.

Instead the compute of winner with CMJ method takes back a different situation: we fixed 2 as number of winners (and number of clusters) and the first one is the option 3 (8 hours (P) - 6 hours (O) - 4 hours (P or O)) and the second one is the option 4, the same winner of MJ method.

As we can see, probably because the number of voters is quite high, the results are not the same like in case study 2. With CMJ, we take into account the wide spectrum of preferences, with special regards for the most polarising ones, which are the most influent in creating different clusters.

Especially for this reason, we may prefer CMJ to MJ for this case-study's lookalike situations, where a shared solution should be taken, considering the different impact it can have on the heterogeneous groups (*clusters*) the judgement is made by.

## 4 CONCLUSIONS

In section 1, we dealt with logical issues involved in voting rules and judgement aggregation, highlighting majority rule's resistance to strategical vote.

In section 2, a more fined model of majority rule, Majority Judgement, has been presented as an option to better estimate the most shared candidate.

In section 3, the related works have been shown and in section 4, all possible categories of clustering approach has been reported in order to choose the fittest one for our generalization of Majority Judgement as a

multi-winner strategy. After that, three different case studies are reported, with a particular attention to the comparison between MJ and CMJ results.

In spite of non-deterministic nature of K-Medoids, Clustered Majority Judgement is thought to be used in high populated disputes. For these reasons, we feel confident about clustering's role of taking into account all different perspectives could be shown in such situation.

Moreover, our implementation is not strictly linked to political field, as is clearly shown in the case studies (except the first one), mostly because it requires only some fixed parameters: number of winners, number of grades and grades themselves.

An important future challenge could be speeding up the algorithm or making a more flexible structure, even though all the constraints already explained in previous sections need to be satisfied.

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