




# Electric Power System Operation: A Technique to Modelling, Monitoring and Control via Petri Nets

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**Keywords:** Petri Nets, Electric Power System, Modelling, Monitoring, Control, Switch Breaker, Disconnect Switch.

**Abstract:** Petri nets have been widely used as a tool to model, monitor and control several kind of systems. In this paper, Petri nets are used to model, monitor and control Electrical Power Systems (EPS). The electric power model will be expanded through a linear transformation. The restrictions imposed for that expansion specialize the new places with attributes that allow to monitor or control the dynamics of the original Petri net.

## 1 INTRODUCTION

Modern systems of production have been presenting high complexity degrees, modeling, analysis, planning, monitoring and controlling. These systems of production require appropriate modeling tools to ensure and validate the right procedures for their operations. The Electric Power Systems (EPS) can be classified as a class of such systems (Tekiner-Mogulkoc et al., 2012).


Generally, in the study of an EPS is considered that its operation should occur predominantly in a steady state. In this case, all of them the load changes, the opening or closing of disconnectors, the breakers, or any other occurrence that can generate transients in the power system are not considered. Thus, all variables are manipulated with dependence only on time in the strictly mathematical sense (Weiss and Schulz, 2015). However, when the dynamics of operation of an EPS is analyzed [i.e. the analysis through the conditional variables that although have a duration associated with their occurrence, they do not have any dependence on time to occur], its operation can be considered as a system whose dynamic is event-driven [i.e. it can be manipulated as a Discrete Event System (DES)] (Amini et al., 2019).


Originally, a dynamic system is considered as a Discrete Event System (DES) if its dynamic [i.e. its


states] is such that it can be considered as having no dependence on time, but it can be considered as having dependence on occurrences or events that can be, for instance, random. The elapsed time of the event is token usually as negligible. Afterward, some applications have presented dependence on timed occurrences or events. To deal with such requirements theoretical extensions taking into account timed events were developed. Several are the tools used to study DES for different kinds of systems. There exist also some systems with certain operational features such that they have dynamic called hybrid systems. Among the main tools used in the study of DES, one can point out Finite State Automata, Dioids, Queue Theory, and Petri Nets (Papadopoulos et al., 2019; Wu et al., 2019; Komenda et al., 2018; Lin et al., 2016).

Petri net is a well known DES tools. Its popularity comes mainly due to two factors: it has a compact representation and also has a graphic representation very easy. Over time, the Petri nets have been incorporating several resources to become them, more powerful as for instance Continuous Petri net and timed Petri net. These things have guaranteed for them, greater information richness in the applications and also allowed them can be applied to new classes of systems as for instance EPS (Murata, 1989; Casandras and Lafortune, 2009; David and Alla, 1994; Bin et al., 2015; Fendri and Chaabene, 2019).

The Modelling of a DES is building of model to represent its dynamic [i.e. the evolution of its states or events] using some suitable tool. The monitoring

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of a DES is the regular observation and recording of activities that have been occurring in such a system. It is a process of routinely collecting information about all states ([i.e. events] of the system. On the other hand, the basic idea behind controlling of a DES is to restrict the behavior of the system according to rules imposed by a controller. This restriction can be enforced by enabling or disabling controllable transitions [i.e. events] under some established conditions previously. For all of them, the Petri nets have been being shown as a suitable tool (Giua, 1992; Krogh and Holloway, 1991; Holloway et al., 1996; Dideban and Alla, 2009).

The EPS has some dynamic features that are dependent on the occurrences of events. The performance of maneuvering and protection devices are instances of elements whose dynamics can be completely modeled based on events. High levels of safety, quality, reliability and availability are some requirements in the EPS operation. An important factor in contributing to the fulfillment of these requirements is the existence of some modeling tool that allows an observation step-by-step of the evolution of an EPS in its varied stages. Traditionally, an EPS has been worked using contact diagrams and their simulation. This technique is inefficient when the systems become more complex [i.e. when more sophisticated interlocks are required].

This work is complemented with a section about Petri nets and supervisory control theory. In the following subsection is done an introduction about linear transformation to expand a Petri net after that a vision of monitoring and control. The third section is an introduction to EPS and in the fourth section, an application is used to validate the theory presented in the preview sections. The Conclusion are presented the mains results (Souza et al., 2016).

## 2 PETRI NETS

Petri nets consist of a set of tools with graphic and mathematics resources that are fitted well to a set of applications in DES. They were used initially to create causal relationships among conditions and events in computer systems and currently present a powerful formalism for the DES. Petri nets can provide the dynamic behavior of a DES. It is possible to verify, for instance, whether there is parallelism or conflict between transitions, whether there is a possibility of occurrence of some event, as well as to follow the evolution of the system. The content of the section can be found especially in (Murata, 1989; David and Alla, 1994; Cassandras and Lafortune, 2009).

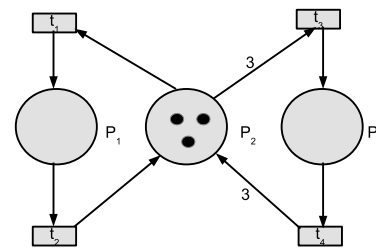


Figure 1: Petri Net Composition.

Graphically, a Petri net consists of a graph with three types of elements: places, transitions and directed arcs connecting places to transitions or transitions to places. Usually, places are represented by circles, while transitions are represented by narrow bars. In Figure 1 is shown an instance of Petri net containing three places ( $P_1, P_2$  e  $P_3$ ), four transitions ( $t_1, t_2, t_3$  and  $t_4$ ) and eight arcs connecting places to transitions or transitions to places. Mathematically, a Petri net can be defined as a quadruple  $R = \langle P, T, Pre, Post \rangle$  where  $P$  is a finite set of places,  $T$  is a finite set of transitions,  $Pre$  is a function that defines arcs linking places to transitions and  $Post$  is a function that defines arcs from transitions to places. The arcs can be weighted. If an arc  $Pre(P, t) = k$  or  $Post(P, t) = k$  then the weight of such arc is  $k$ . When  $k = 1$ , it does not need to be written. In Figure 1, the arcs  $Pre(P_2, t_3) = 3$  and  $Post(P_2, t_4) = 3$  are weighted by 3 and  $Pre(P_2, t_1) = 1, Pre(P_1, t_2) = 1, Pre(P_3, t_4) = 1, Post(P_1, t_1) = 1, Post(P_2, t_2) = 1$  and  $Post(P_3, t_3) = 1$  are weighted by 1.

On the other hand, the markup of a Petri net is a mapping  $M : P \rightarrow N$  that associates an integer number of marks or token to each place. Consequently,  $M(p)$  indicates the number of marks present in a place  $P$ . A marked Petri net is a double  $N = \langle R, M_0 \rangle$  where  $R = \langle P, T, Pre, Post \rangle$  and  $M_0$  is the initial marking.

The marks  $M$  are information assigned after the transition firing to represent the state evolution of the network at a given moment. Thus, to simulate the dynamic behavior of DES, the marking of a Petri net is modified after each action performed [i.e. each fired transition].

The evolution of marks can be made by enabling and firing transitions. A transition  $t$  is enabled if, and only if, in each of its input places  $P$  contains at least the number of marks equal to the weight of the arc that is connecting  $P$  to  $t$ . In Figure 1, all transition  $t$  are enabled.

The enablement is denoted by  $M[t]$ . The firing of a transition  $t$  removes from each place  $P$  the number of marks equal to the weight of the arc connecting  $P$  [i.e.  $Pre(P, t)$ ] to  $t$ . Besides, the firing of such transition will deposit at each outlet place  $P$  the number

of marks equal to the weight of the arc connecting  $t$  to  $P$  [i.e.  $\text{Post}(P, t)$ ]. Mathematically, the firing of the transition  $t$  in a  $M$ -marking leads to a new marking

$$M' = M - \text{Pre}(\cdot, T) + \text{Post}(\cdot, T).$$

Any  $M$ -marking achievable from the initial marking  $M_0$  by firing a sequence  $\sigma = t_1 t_2 \cdots t_n$  can be written as

$$M' = M_0 + B(P, t) * \sigma \quad (1)$$

where  $\sigma : T \rightarrow N$  is the count vector and  $B(P, t) = \text{Post}(P, t) - \text{Pre}(P, t)$  is called Incidence Matrix.

The Incidence Matrix provides the balance of the marks when the firing of transitions occurs. The set of all reachable marking from an  $M$ -marking in  $n$  is called  $A(n, M)$ . The behavior of a Petri net can be described by a graph of reachability  $GA(N, M_0)$ , whose nodes correspond to the achievable markings. In this graph, there is a tagged arc from node  $M[t \mapsto M']$  to node  $M'$ .

## 2.1 Supervisory Control using Petri Nets

In DES, supervisors or controllers are places added to a Petri net to avoid the occurrence of undesirable or prohibited states in the original model of a plant (Moody and Antsaklis, 2000). The states reached by the controller enable which states the plant can reach. The next subsection will present supervisory control using the place invariant property of Petri net models.

## 2.2 Development of Supervisor based on Place Invariants

Many researchers have been using Petri nets as a tool to model, analyze and synthesize control laws for DES (Murata, 1989).

Let be the Petri net model for a plant is  $(N, M_0)$ , where  $N = (P, T, B, W)$  is the graph with all its reachable markings. Let be also the control purpose is restricting the evolution of the state of the plant  $S$  to a subset  $S'$ , where this subset  $S'$  is described by the group of linear inequalities

$$\begin{pmatrix} L_1 \\ \vdots \\ L_q \end{pmatrix} * M(k) \leq \begin{pmatrix} b_1 \\ \vdots \\ b_q \end{pmatrix}, \quad (2)$$

or  $L * M(k) \leq b$ , Where  $L \in \mathbb{Z}^{q \times n}$ ,  $b \in \mathbb{Z}^n$ , and operator " $\leq$ " must be applied element to element.

The control goal is to prevent the firing of certain transitions. The controller implementation is done by

inserting new  $Pc_1, \dots, Pc_q$  and their respective tokens  $M_c(k) \in N_0^q$  which express the state of the controller. Both the initial marking and the way the controller is connected to the transitions of the plant model can be obtained by considering an extended Petri net with the marking  $(M, M_c)$ . If a transition  $t_j$  is fired, the state of the extended Petri net is changed according to the equation.

$$\begin{pmatrix} M(k+1) \\ \bar{M}_c(k+1) \end{pmatrix} = \begin{pmatrix} M(k) \\ \bar{M}_c(k) \end{pmatrix} + \begin{pmatrix} B \\ \bar{B}_c \end{pmatrix} * \sigma_j, \quad (3)$$

where  $\sigma_j$  is the  $n$ th firing sequence vector in  $\mathbb{Z}^m$  and  $B_c$  is unknown part of the incidence matrix.

A convention needs to be adopted such that for any pair  $Pc_i$  and  $t_j$ ,  $i = 1, \dots, n$ ,  $j = 1, \dots, m$ , be taken a arc of  $Pc_i$  to  $t_j$  or  $t_j$  for  $Pc_i$ . With this, a array  $B_c$  can be defined to specify the structure of connection between the controller and the plant transitions and thus to have the incidence matrix  $B_c^+$  as the positive values of  $B_c$  and  $B_c^-$  as negative values  $B_c$ .

The unknown parameters of the  $M_c(0)$  controller and  $B_c$  can be determined as following:

1. The Control specifications are given from Equation(2). So, one can do

$$L * M(k) + M_c(k) = b, \quad (4)$$

where  $k = 0, 1, \dots, m$  and  $M_c(k)$  is a positive vector of integers inserted to remove the inequality. The initial marking is gotten doing  $k = 0$  as

$$M_c(0) = b - L * M(0); \quad (5)$$

2. Multiplying both sides of Equation(3) by the matrix  $[L \quad I]$  and then applying invariance property from the Equation(4), it is determined that the controller incidence matrix is given by:

$$B_c = -L * B \quad (6)$$

The Equations(5) and (6) are used for solve supervisor control problems. The Equation(5) is used to compute initial marking to the controller. The Equation(6) show how places controller are connected with plant transitions (Zhou et al., 1992).

**Theorem 1.** *The control produced by Petri net  $(N, M_0)$  and constraints established in the Equation(2) by Equations(5) and (6) is at least restrictive or minimally permissive.*

**Proof 1.** *The Equation(4) remains constant  $\forall k = 0, 1, \dots, m$  of the closed loop system. Thus, assume that the closed-loop system is in the state  $(M'(k), M'_c(k))'$ , and that the transition  $t_j$  is disabled is to say that:*

$$\begin{pmatrix} M(k) \\ \bar{M}_c(k) \end{pmatrix} \geq \begin{pmatrix} B^- \\ \bar{B}_c^- \end{pmatrix} * \sigma_j$$

*which is a necessary condition for not availability of the transition from PN. This is possible if:*

- or  $M_i(k) < (B^- * q_j)_i$  for any  $i \in \{1, \dots, n\}$ , i.e. the transition is disabled in the PN uncontrolled  $(N, M_0)$
- or for some  $i \in \{1, \dots, m\}$   $M_{ci}(k) < (B_c^- * \sigma_j)_i = L_i * B * \sigma_j$ , e  $M_{ci}(k) = b_i - L_i * M(k)$  can be found that  $b_i < L_i * M_{ci}(k) + L_i * M(k)$

Meaning that the transition  $t_j$  could fire the state  $M(k)$  Petri net open loop  $(N, M_0)$ , the resulting state  $M(k + 1) = M(k) + B * \sigma_j$  would violate the specification given in Equation(2). Consequently, it showed that the transition  $t_j$  is disabled in the state  $(M'(k), M'_c(k))'$  of the closed-loop system if and only if it is disallowed in the state  $M(k)$  of Petri net uncontrolled  $(N, M_0)$  or if its fire violate the control specifications.

### 2.3 Expansion of Petri Net

The expanding technique is as follows. Let be a Petri net  $N \in \mathbb{N}^{m \times n}$ . It is intended to expand this Petri net with the addition of a place  $P_c$  such that a new Petri net  $N_h \in \mathbb{N}^{(m+1) \times n}$  is built. This new Petri net  $N_h$  must have the following characteristics: (i) maintain the original Petri net dynamics; (ii) be able to identify possible changes in the dynamics of the original Petri net  $N$ .  $M^{m \times 1}$  is the vector of marks of  $N$ . The expanded Petri net can have two different behavior: (i) the inserted place  $P_c$  can behave passively in relation to the firing of the transitions of  $N$  such that the dynamics of  $N$  is not affected by the marking of the inserted place [characteristic typical of Monitor]; (ii) the inserted place  $P_c$  interferes in the dynamics of  $N$  [characteristic typical of controller].

The expanded Petri net  $N_h$  has a marking  $M_h^{(m+1) \times 1}$  such that

$$M_h = M_{h0} + B_h x, \tag{7}$$

which preserves its marking by a  $T$  transformation such that  $TM \implies M_h$ .

### 2.4 Monitoring Systems using Petri Net

An expanded Petri Net  $N_h$  can be generated such that it can ensure that the marking of the original Petri net  $N$  can be preserved [i.e. it works as a monitor]. In addition, it must be ensured that the sum of the markings of the new places is the sum of the markings of  $N$ . So, the marks lost in  $N$  must be absorbed by the monitors such that

$$\sum_{i=1}^d m_{ci} = \sum_{j=1}^m m_j, \tag{8}$$

where each  $m_{ci}$  can be given by

$$m_{ci} = c_1 m_1 + \dots + c_m m_m = l_1 m_1 + \dots + l_m m_m \tag{9}$$

or alternatively

$$m_{ci} = C_i M = LM, \tag{10}$$

where  $L$  is a line vector composed of zeros and ones and it enables those places to be monitored.

One can do still

$$LM - C_i M = 0$$

or

$$(L - C_i)M = 0.$$

The marking reached by the monitor is numerically equal to the sum of the marks of the monitored places. This is a property of a conservative Petri net [i.e. there is an invariant place between the monitors and the monitored places]. There are still transitions when fired, they can not be observed by an external element. These internal events, when occur, can not be monitored. When an unobserved event  $t_u$  occurs, the exchange of markings between the input and output places is not monitored.

Let  $N$  be a Petri net composed of a set of places  $P$ , where there is a place  $P_i \in P$  such that is connected to an unobserved transition  $t_u$ . A Monitor  $N_h$  for  $N$  can be determined by eliminating the influence of  $t_u$  on the dynamics of the monitor. The proposal to achieve this condition is to neutralize the action of  $t_u$  for  $N_h$ . For this, zeros are allocated to the column of the incidence matrix (Matrix B) that corresponds to  $t_u$ .

### 2.5 Control Systems using Petri Net

An expanded Petri Net  $N_h$  works as a control system when it forces the places invariants properties as following

$$\sum_{i=1}^d m_{ci} + \sum_{j=1}^m m_j = K \tag{11}$$

or

$$LM + C_i M = K. \tag{12}$$

This means that the sum of the markings obtained by the controller added to the sum of the markings obtained by the places in the portion of  $n$  to be controlled is always a constant.

## 3 ELECTRIC POWER SYSTEM

An EPS can be defined as a set of physical equipment connected as electrical circuit elements that work coordinately in order to generate, transmit or distribute electrical energy to consumers. The generation has the function of converting some other form of energy into electrical. The transmission carries the electricity from generation centers to consumption centers

or to some other electrical system connecting them. The distribution delivers the energy received from the transmission system to large, medium or small consumers (Vescio et al., 2015).

An electrical system should be carefully represented by a model using an appropriate modeling tool. The choice of the tool is related to the type of study to be performed. For instance, for protection studies, the values of short-circuit currents should be calculated. Therefore, each system component must be modeled and represented using the perspective of its behavior for short-circuit currents. This modeling is relatively easy due to the simplifications made in the equivalent circuits of the components. The suitability of the model for studies of short-circuit condition is made with the use of symmetrical components, which leads to the obtaining of three system models: positive sequence, negative sequence and zero sequence (Grainger and Stevenson Jr., 1994).

### 3.1 Elements of Electric Power System

An EPS can be composed of some basic elements such that all of them together become able to generate, transmit, distribute or connect other electrical power systems (Grainger and Stevenson Jr., 1994). Some of these elements are:

**Generator** - Element that generates active power;

**Power Transformer** - Element that increases or decreases currents and voltages in an EPS;

**Transformer as Measure Instrument** - Element that is used to measure currents or voltages in order to monitor, control or protection the EPS;

**Bus** - Element that is used as link between the EPS and its components;

**Breakers** - Switching that is used to turn on or off an EPS under normal or abnormal conditions;

**Disconnecter** - Element that is used to isolate (sectioning) parts of EPS [i.e. subsystems, equipment etc]. It is installed aiming at breaking the grid to minimize the effects of outages, establish visible sectioning in equipment such as automatic reclosers, or establish bypass in equipment such as voltage regulators, etc;

**Protection Relays** - Element that using logic can distinguish the difference between short-circuit and load current. It is responsible by the decision to make a shutdown or not of breakers associated with it and quickly isolating the rest of the grid;

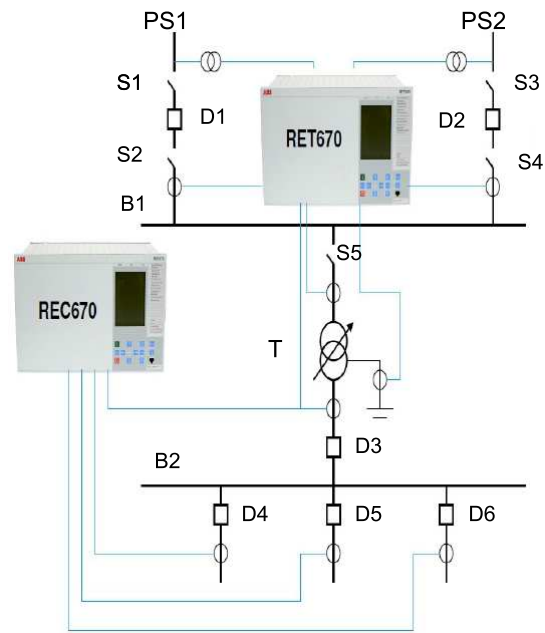


Figure 2: Bus Feeder System and Power Supply (Ltd, 2010).

## 4 APPLICATION TO A SUBSTATION

As application, it is made the modelling, monitoring and control of the EPS represented in Figure 2 by its single line diagram. It is a substation composed of two buses,  $[B_1$  and  $B_2]$ , six circuit breakers  $[D_1, D_2, \dots, D_6]$ , five disconnectors  $[S_1, \dots, S_5]$ , one transformer and three energy consumers.

First of all, considering the substation de-energized [i.e. the switch-disconnectors and circuit-breaker assemblies are all disconnected or open]. With this, the following procedure<sup>1</sup> can be proposed to energize the buses  $B_1$  and  $B_2$ :

1. To power on the Bus  $B_1$  via  $PS_1$ , must be given priority to close the disconnecting switches  $S_1$  and  $S_2$  and after these the circuit breaker  $D_1$  must be closed;
2. To power on the Bus  $B_1$  via  $PS_2$ , must given priority to close the disconnecting switches  $S_3$  and  $S_4$  and after these the circuit breaker  $D_2$  must be closed;
3. To power on the Transformer  $T$  after the Bus  $B_1$  is on, it must be given priority to close the disconnect switch  $S_5$ . So after that, the circuit breaker  $D_3$

<sup>1</sup>Any procedure proposed must be in agreement with local standards and aligned with the concessionary that holds the formal authorization to distribute electricity in the region in that the substation is installed.

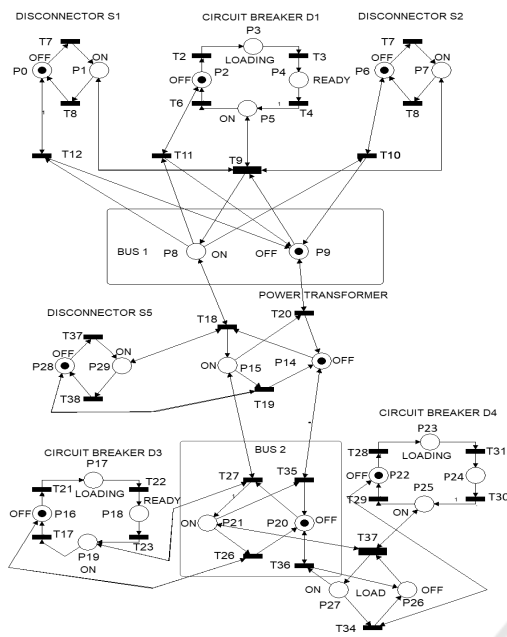


Figure 3: Substation Free Behaviour PN Modeling.

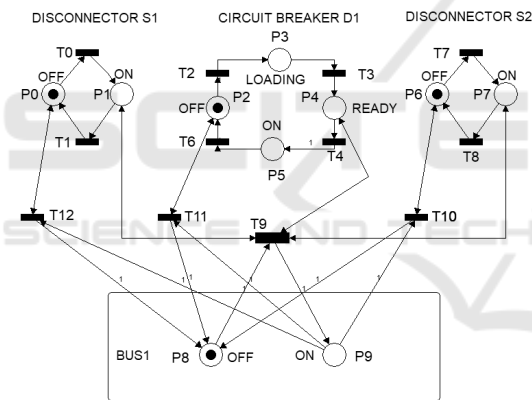


Figure 4:  $PS_1$  Free Behaviour PN Modeling.

can be closed and consequently the bus  $B_2$  will be on;

4. To energize some electric loads after bus  $B_2$  is on, one circuit breaker must switch on  $D_4$  to  $D_6$  without setting an order of priority;
5. The procedure for shutdown the substation must obey an inverse prioritization order those done for startup it [i.e. first of all, it must disconnect the loads by switching off  $D_4$  to  $D_6$ , then isolating the bus  $B_2$  through  $D_3$ , following with the de-energizing of the transformer through  $S_5$  and so on];

### 4.1 Free Behavior Modeling for Substation

Some specifications are adopted for modeling of the substation as following. The disconnect switches have two possible states: (i) the connected state; and (ii) the disconnected state. The circuit breakers have 4 states: (i) the Off state; (ii) Carrying the Spring State; (iii) ready to go Into Operation State; and the On State. For this modeling, abnormal conditions of the equipment are not considered [for instance, the broken state]. The buses also are represented by two states: (i) de-energized; and (ii) energized.

The presence of the token depicts the current state of the equipment. For instance, considering a disconnect switch where  $P_1$  is the state that symbolizes when it is disconnected and  $P_2$  the state representing when it is connected, then if a token is in  $P_2$  it represents that the disconnect switch is closed, otherwise [token in  $P_1$ ] it is open.

In Figure 3, a summarized version of the free behavior for the substation presented in Figure 2 is presented. The substation Petri net model consists of an input  $PS_1$  composed of two disconnect switches  $S_1$  and  $S_2$  and a circuit breaker  $D_1$ , followed by the representation of the input bus  $B_1$ . After this, there is the model of the disconnect switch ( $S_5$ ), responsible for energizing the primary winding of the transformer. To the right side of  $S_5$  model is the representation of the possible states of the power transformer. In the bottom of the Petri net, all of the other models are presented: the model to the circuit breaker  $D_3$ , the output bus  $B_2$  and the energizing circuit breaker of a load  $D_4$ .

### 4.2 Controller Design for Sequence Operation

In Figure 4 is shown the Petri net model of the free behavior for the input  $PS_1$  energizing the bus  $B_1$ . This representation does not contemplate the restrictions for the operation of the disconnect switches and circuit breakers that affect the safe energization or shutdown of the bus  $B_1$ . These constraints are intimately related to the constructive aspects of these operational control elements. For example, the manipulation of the disconnect switches must not be carried out with the circuit-breaker switched on. Thus, the correct way to energize the bus  $B_1$  is closing the disconnect switches  $S_1$  and  $S_2$  and after that close the circuit breaker  $D_1$ .

For the energizing and de-energizing operation of the  $PS_1$ , it is necessary to compute a controller that can restrict some actions of the free behavior model.

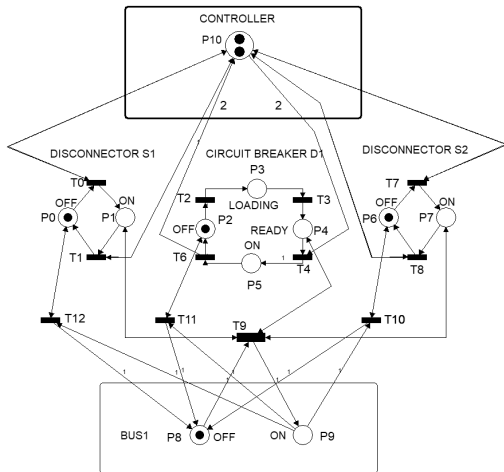


Figure 5:  $PS_1$  System Control Modeling Using PN.

For this, it is necessary to specify the controller actions to guarantee the correct sequence of events firing. A control system is presented in Figure 5. The control specifications are represented by a set of linear equations where their variables represent the marking of the controlled places. The constant on the other side of the equation will mean that the controlled places will form a place invariant with the controllers [see Equation(12)].

To specify the controllers, it is necessary to find the incidence matrix  $B_c$  of these places and also their initial marking  $M_c(0)$ . The incidence matrix  $B_c$  is obtained from both the control specifications and the incidence matrix of the original Petri net.

From Figure 5, the constraints on this feeder can be found by making the following considerations with respect to feeder free behavior model  $PS_1$ :

1. If the current state of the circuit-breaker is closed [i.e.,  $P_5$  with marks], it forces that the current states of the disconnect switches are also on states [ $P_1$  and  $P_7$ ];
2. Generating equation  $P_5 + P_1 + P_7 = 3$ ;
3. If the current state of one of the connect switches is opened [i.e.,  $P_0$  or  $P_6$  with marks] the breaker can not to close [ $P_5 = 0$ ];
4. Generating equation  $P_5 + P_0 + P_6 = 1$ ;

Adding the two equations above one can get

$$2P_5 + P_0 + P_1 + P_6 + P_7 = 4$$

and consequently

$$L = [1, 1, 0, 0, 2, 1, 1, 0, 0].$$

With  $L$  and the incidence matrix, the weights of the arc that interconnects the controller with the transitions of the original Petri net are obtained from

$$CB = -LB = [\pm 1, \pm 1, 0, 0, -2, 2, \pm 1, \pm 1, 0, 0].$$

The determination of the initial marking of the controller is obtained through Equation(12). With the value of the initial marking of the controlled places [i.e.,  $P_0, P_1, P_5, P_6$  and  $P_7$ ], the vector  $L$  and the constant  $K, K = 4$ . Found:  $M_{c0} = 2$ . With this information the Petri net of Figure 5 is drawn.

The terms represented by  $\pm$  correspond to the nonzero position in the vector  $L$  that resulted in zero in the calculation of Equation(12). In the formation of the controlled Petri net, these positions will be represented by autoloop. see Figure 5. The autoloop present between the transitions of the connect switches models and the controller allows these transitions to fire without changing the controller marking. This condition releases the models of the connect switches to move from the off state ( $P_0$  and  $P_7$  with tokens) to on state ( $P_1$  and  $P_8$  with tokens) freely. When the model representing the circuit breaker goes to the on state ( $P_5$  with mark), the  $T_4$  transition removes two marks from controller ( $P_{10}$ ). The controller place,  $P_{10}$ , without marking inhibits firing of the transitions belonging to the switch models ( $T_0, T_1, T_7$ , and  $T_8$ ). This condition ( $P_{10}$  without tokens) keeps the model of the disconnect switches in the state that were before the  $T_4$  transition firing.

In agreement with the constraints imposed to determine the controller, the following sequence of events are possible:  $T_0T_7T_2T_3T_4$  representing the on states of the switches, breaker and bus ( $P_1, P_5, P_7$  and  $P_9$  with marks) and another sequences are  $T_5T_1T_8$  or  $T_5T_8T_1$  representing the states off the disconnect switches, circuit breaker and bus ( $M_0$ ). In relation to the bus, it already loses the on status as soon as  $T_5$  fires since the model contemplates the way the switches and circuit breakers are physically interconnected(series association).

### 4.3 Development of the Monitor

In Figure 3 is shown the Petri net model for the input power supply PS 1 of the sub-station presented in Figure 6. A new transition  $T_5$  has been inserted to represent a failure event. This event connects both the place that represents the Ready Circuit Breaker Status and the place that represents off State Circuit Breaker Status. Thus,  $T_5$  emulates possible problems that may appear on the circuit breaker, such as loss of elastic characteristics of the spring assembly, jammed or worn contacts, etc. Consider  $T_5$  be uncontrollable and observable means that the controller can not intervene in its firing but the monitor alarms if it comes to fire.

So, given the Incidence matrix  $B$  of the Petri net of Figure 6 and knowing that when specifying a monitor

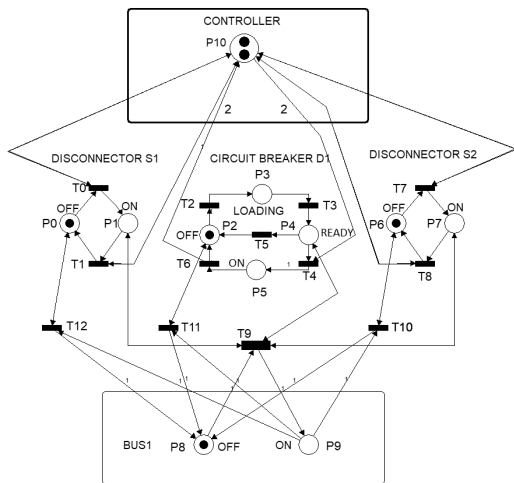


Figure 6: PN Model for PS1 With Uncontrolled and Observable Transition.

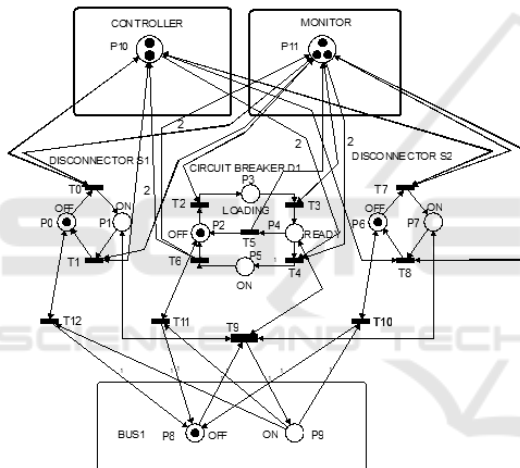


Figure 7: Structure of a Monitor for PS1 With Failure Transition.

for the main places those represents  $S_1, S_2$  and  $D_1$  the vector  $L$  will be.

$$L = [1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1]$$

And applying the condition for monitoring in the Equation(10). It is determined that the weight of the arcs between the monitor place and the Petri net will be:

$$B_m = (\pm 1 \pm 1 \pm 1 \pm 1 - 2 \pm 1 \pm 1 \pm 1 \pm 1 0 0 \pm 1)$$

Transition  $T_5$  is occupying the last column of the incidence matrix  $B$ . It is a non-controllable transition and therefore the monitor will not be able to intervene in its fire. This condition eliminates in the computation of vector  $B_m$  the arc that will link the place monitor to the transition  $T_5$ . That way, the monitor will know if the transition  $T_5$  has fired but it does not interfere

in your fire. The last field of the  $B_m$  vector loses the value -1 and retains the +1 value. See  $B_m$  below.

$$B_m = (\pm 1 \pm 1 \pm 1 \pm 1 - 2 \pm 1 \pm 1 \pm 1 \pm 1 0 0 1)$$

The initial marking for the monitor will be obtained through Equation(10). Thus, the new Petri net is shown in Figure 7

By analyzing the Petri net of Figure 7 are extracted the following informations:

1. The firing of all controllable transitions remains invariant between the monitored and monitor places.
2. The fire of the uncontrolled transition causes the breaking of the invariant place, allowing the identification of the failure.
3. The number of monitor marks expresses the number of times the failure occurred.
4. The occurrence of the fault causes loss of control by the controller  $P_{10}$ .
5. The Monitor does not interfere in the actions of the controller.

## 5 CONCLUSION

The approach adopted in this work proved to be a tool capable of expanding a Petri net to an enlarged Petri net without, however, interfering with the marking of the original one. The Invariant Place property of the Petri net, when forced, such a place acquires characteristics that allow monitoring, controlling and diagnosing plants just like an EPS. By marking the place added to the original Petri net, it is possible to associate it with a system operating state and identify an abnormal operating condition (monitoring feature) or allow it to intervene in the system with a control action (control feature). Thus, the use of Place-Transition Petri nets proved to be very useful in modeling, monitoring and controlling the operation of an EPS, making such tasks simpler, elegant and effective.

It is found that an inserted place gains the property of monitoring the original Petri net when the place monitor has the same total number of marks of the monitored model. The divergence between the monitor marking and the monitored model marking is a flag that occurs something wrong.

When it is imposed, the sum of the markings of the place inserted with the markings of the original Petri net is equal to a predefined constant  $k$ . This place gains ownership of controlling Petri net actions from constraints imposed. This new property is acquired



by the added place, via the geometric approach created using the free model for a circuit breaker and a disconnecter. Thus, it was verified that the inserted place was able to control the sequence of those models that represent this equipment validating the model.

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