Maximum Correntropy Criterion-based UKF for Tightly Coupling INS and UWB with non-Gaussian Uncertainty Noise

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Abstract: In this paper, unscented Kalman filter (UKF) based on maximum correntropy criterion (MCC) instead of minimum mean square error (MMSE) criterion, and it is applied to tightly coupled integration of inertial navigation system (INS) and ultra wide-band (UWB). UWB can measure distance with an accuracy of less than 30cm in line-of-sight environment, but provides distance measurement with various types of non-Gaussian uncertainty noise in non-line-of-sight environment. In this case, if the INS/UWB system is configured with the existing MMSE-based filter, a large error occurs. To solve this problem, in this paper, UKF is designed based on MCC. Through simulation analysis, it is confirmed that the proposed filter has robust characteristics against UWB uncertainty and enables stable INS/UWB integration.

1 INTRODUCTION

Consider a system integrating inertial navigation system (INS) and ultra wide-band (UWB) for indoor navigation. Although INS can accurately calculate 6-degree of freedom motion, it has a problem of accumulation of errors over time. UWB enables accurate distance measurement and location estimation in a line-of-sight (LoS) environment, but in non-line-of-sight (NLoS) environment such as indoors, accurate becomes impossible as various types of uncertainty noise are included in distance measurements (Cho, 2019). To integrate these two systems, nonlinear filters such as extended Kalman filter (EKF) (Brown and Hwang, 2012) and unscented Kalman filter (UKF) (Julier et al., 2000) are generally used in consideration of their nonlinear characteristics. However, since these filters designed based on minimum mean square error (MMSE) do not properly respond to UWB uncertainty noise, a large error may occur. In this paper, considering this problem, maximum correntropy criterion (MCC)-based UKF (MCUKF) is introduced.

The MCC-based filter is a filter that maximizes the similarity between the measurements and the estiamtes, and is designed based on the kernel function reflecting the error characteristics of the measurement. The kernel function is to indicate the similarity of two random variables, and the total sum of kernel function values including innovation and residuals calculated in the measurement-update process is determined as a cost function. Then, the state variables are estimated so that this cost function is maximized. In this process, when uncertainty noise occurs in the UWB measurement, the MCC-based filter adjusts the P matrix and R matrix, thereby reducing the effect of the measurement uncertainity noise on the system (Chen et al., 2017).

Recent research on MCC-based filters has focused on kernel function design considering various error probability distributions (Li et al., 2022, Huang and Zhang, 2022). There are not many studies to apply MCC to nonlinear systems. Ther purpose of this study is to apply MCC to UKF so that is can be used in nonlinear systems. And the designed MCUKF is applied to tightly coupled INS/UWB integration system.

The performane of the MCUKF-based INS/UWB integrated navigation system is verified based on simulation. Through simulation analysis, if non-

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Gaussian uncertainty noise is included in the UWB measurement, it is confirmed that MCUKF provides a stable navigation solution by largely tuning the value of the R matrix corresponding to the erroneous measurement.

2 MCUKF-BASED INS/UWB INTEGRATION

Considering the following discrete-time nonlinear system model:

$$\begin{aligned} x_k &= f(x_{k-1}) + w_{k-1}, \quad w \sim N(0,Q) \\ y_k &= h(x_k) + v_k, \quad v \sim N(0,R) \end{aligned} \tag{1}$$

where subscript k is the discrete time, $x_k \in \mathbb{R}^N$ is the state, and $y_k \in \mathbb{R}^M$ is the output. w_k and v_k are uncorrelated zero-mean white Gaussian noise processes, and the covariance matrices of the processes are denoted by Q and R, respectively. The state variables considering INS are set as

$$x = \begin{bmatrix} Pos^L & Vel^L & Att & \nabla & \varepsilon \end{bmatrix}^T \tag{2}$$

where Pos^L and Vel^L are the position and velocity in the local level coordinate system, Att is the attitude expressed in Euler angles, and ∇ and ε are the accelerometer bias and gyro bias, respectively.

In UKF, these state variables are converted into sigma points. The number of sigma points is 2*N*+1, which is 31 because the system dimension *N* is 15 (Julier et al., 2000).

The system function can be expressed as an INS equations as follows (Farrell and Barth, 1999):

$$\begin{aligned} Pos_k^L &= Pos_{k-1}^L + Vel_{k-1}^L \cdot dt \\ Vel_k^L &= Vel_{k-1}^L + \Big\{ C_{b,k-1}^L (f_{k-1}^b - \hat{\nabla}_{k-1}) - \\ &\quad (2\omega_{ie,k-1}^L + \omega_{eL,k-1}^L) \times Vel_{k-1}^L + g_{k-1}^L \Big\} \cdot dt \\ Qtn_k &= Qtn_{k-1} + \frac{1}{2} \Big\{ Qtn_{k-1} * \Big(\omega_{ib,k-1}^b - \hat{\varepsilon}_{k-1} \\ &\quad - C_{L,k-1}^b (\omega_{ie,k-1}^L + \omega_{eL,k-1}^L) \Big) \Big\} \cdot dt \end{aligned} \tag{3}$$

where dt is the IMU output period, Qtn is the quaternion, f^b and ω_{ib}^b are the accelerometer output and gyro output, respectively, $\hat{\nabla}$ and $\hat{\varepsilon}$ are the estimated accelerometer bias and gyro bias, respectively, ω_{ie}^L is the Earth's angular velocity vector, and ω_{eL}^L is the rotational angular velocity vector of the local level coordinate system caused by the velocity.

In the time-propagation process, the sigma points are propagated in synchronization with the IMU output period based on (3).

When the distance measurement is obtained in UWB, the measurement-update is processed. The measurement function in (1) is as follows (Cho, 2019):

$$r_k(i) = \sqrt{(x(i) - Pos_k^x)^2 + (y(i) - Pos_k^y)^2} + w_k(i) \tag{4}$$

where $[x(i) \quad y(i)]^T$ is the location of the anchor node *i*, Pos^j is the *j*-axis location of the mobile node in the local level coordinate system, and *w*(*i*) is the noise contained in channel *i*. And $i \in \{1, 2, \cdots, M\}$.

In case of using calibrated UWB, *w* in (4) is modelled as zero-mean noise in the LoS environment. However, *w* may appear in the form of heavy-tailed impulsive noise in the probability distribution in the indoor environment. This error can be filtered out in an MCC-based filter. In MCUKF, the cost function is set as follows (Chen et al., 2017):

$$J(x) = \frac{1}{N+M} \sum_{i=1}^{N+M} G(e(i)) \tag{5}$$

where *G*() is a kernel function, and the Gaussian kernel function can be expressed as follows:

$$G(e) = e^{-e^2/2\sigma^2} \tag{6}$$

where σ is the kernel bandwidth and is an important design parameter in MCC.

The state variables are estimated to maximize the cost function.

$$\hat{x}_k = \arg\max_{x_k} J(x) \tag{7}$$

The convergence performance of the filter can be improved by iteratively using one measurement based on the fixed-point iteration algorithm in the measurement-update process (Chen et al., 2017). Using the innovation and residual calculated in this process, the *P* matrix and *R* matrix are adjusted as follows:

$$\begin{aligned} \bar{P}_{k(a)}^- &= B_P (C_{k(a)}^x)^{-1} B_P^T \\ \bar{R}_{k(a)} &= B_R (C_{k(a)}^y)^{-1} B_R^T \end{aligned} \tag{8}$$

where a is the iteration order, $B_P B_P^T = P_k^-$ and $B_R B_R^T = R$. P_k^- should be calculated before the measurement-update using the time-propagated sigma points as follows:

$$\hat{x}_k^- = \frac{1}{n}\sum_{i=1}^{n}\chi_k^-(i)$$
$$P_k^- = \frac{1}{n}\sum_{i=1}^{n}(\chi_k^-(i)-\hat{x}_k^-)(\chi_k^-(i)-\hat{x}_k^-)^T + Q \qquad (9)$$

where $n = 2N+1$, Q is the process noise covariance matrix, and $\chi_k^-(i)$ is the set of time-propagated i^{th} sigma points.

In (8), $C_{k(a)}^x$ and $C_{k(a)}^y$ are calculated as follows:

$$C_{k(a)}^x = \begin{bmatrix} G\left(e_{k(a)}^x(1)\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G\left(e_{k(a)}^x(N)\right) \end{bmatrix}$$
$$C_{k(a)}^y = \begin{bmatrix} G\left(e_{k(a)}^y(1)\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G\left(e_{k(a)}^y(M)\right) \end{bmatrix} \qquad (10)$$

where

$$e_{k(a)}^x = B_P^{-1}(\hat{x}_k^- - \hat{x}_{k(a-1)}^-)$$
$$e_{k(a)}^y = B_R^{-1}(\tilde{y}_k - \hat{y}_{k(a)}^-) \qquad (11)$$

Through this, the P matrix and R matrix are adjusted based on the values calculated by inputting the normalized innovation and the normalized residual as input to the kernel function, respectively.

When the fixed-point iteration algorithm is finished, the state variables and error covariance matrix are updated as follows:

$$\hat{x}_k = \hat{x}_k^- + P_{xy,k(a)}P_{yy,k(a)}^{-1}(\tilde{y}_k - \hat{y}_{k(a)}^-)$$
$$P_k = \bar{P}_{k(a)}^- - P_{xy,k(a)}P_{yy,k(a)}^{-T}P_{xy,k(a)}^T \qquad (12)$$

where

$$P_{xy,k(a)} = \frac{1}{n}\sum_{i=1}^{n}(\hat{\chi}_{k(a)}^- - \hat{x}_{k(a-1)}^-)(\tilde{y}_k - \hat{y}_{k(a)}^-)^T$$
$$P_{yy,k(a)} = \frac{1}{n}\sum_{i=1}^{n}(\tilde{y}_k - \hat{y}_{k(a)}^-)(\tilde{y}_k - \hat{y}_{k(a)}^-)^T + \bar{R}_{k(a)} \qquad (13)$$

In (12), $\hat{y}_{k(a)}^-$ is constructed by calculating for each channel based on (4).

3 SIMULATION RESULTS

To verify the performance of the proposed MCUKF-based INS/UWB integrated navigation, a simulation was carried out to apply this filter to navigation for robots and pedestrians in an indoor environment. The IMU used in the simulation is OEM-IMU-EG-320N, and the specifications are as follows:

- gyro bias repeatability is 0.5 deg/sec
- angular random walk is 0.1 deg/√hr
- accelerometer bias repeatability is 15mg, and
- velocity random walk is 0.05 m/s/√hr.

The output frequency of the IMU was set to 50Hz, and that of the UWB was set to 1Hz. Four anchor nodes are located as shown in Figure 1, and the mobile node moves along the trajectory shown in this figure for 60 seconds.

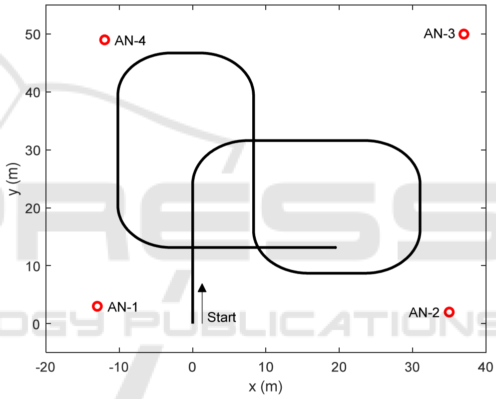
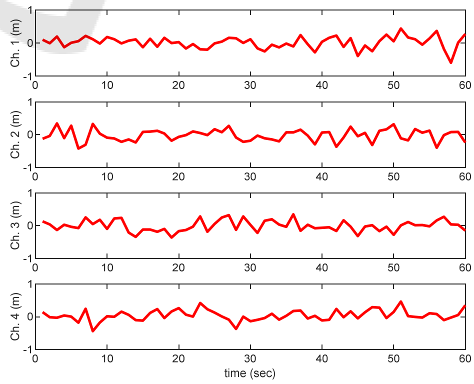
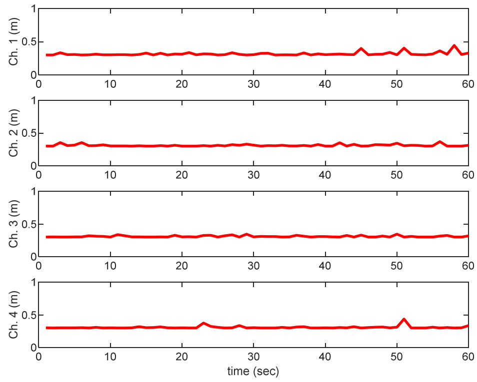


Figure 1: Simulation trajectory.

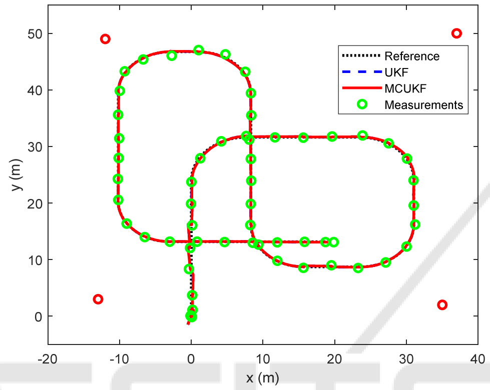


(a) measurement error

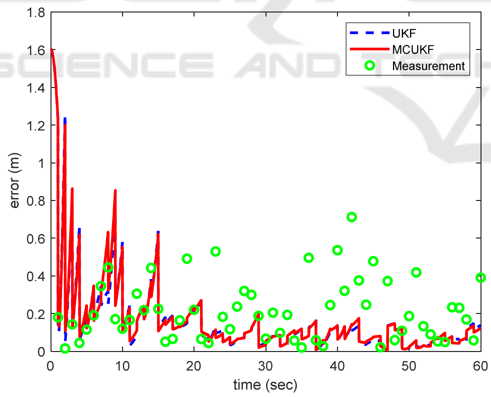
Figure 2: Simulation result in the first case.



(b) square root of adjusted R matrix



(c) position estimates

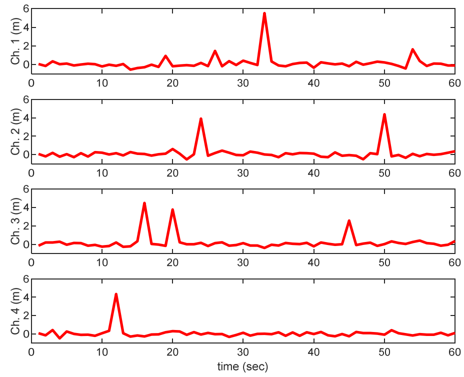


(d) positioning error

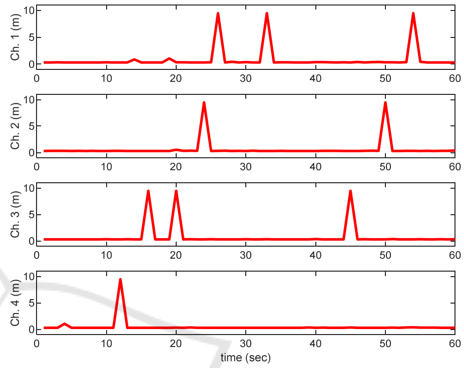
Figure 2: Simulation result in the first case (cont.).

In MCUKF, the kernel bandwidth was set to 1.3.

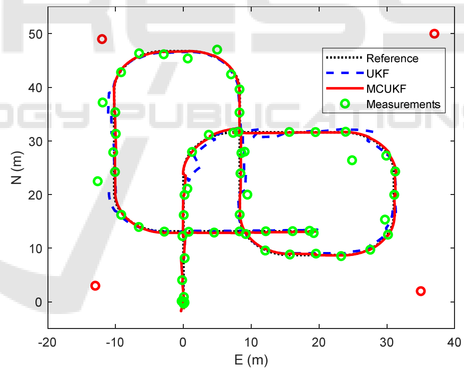
First, it is the case where w in (4) is white Gaussian noise with zero-mean. The result is shown in Figure 2. As can be seen in this figure, the measurements have only Gaussian noise, and the R matrix adjusted in MCUKF is similar to that set as the initial value. And it can be confirmed that the positioning performance of MCUKF is almost similar to that of UKF in general case.



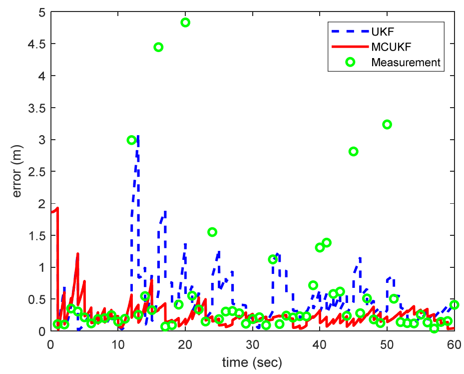
(a) measurement error



(b) square root of adjusted R matrix



(c) position estimates



(d) positioning error

Figure 3: Simulation result in the second case.

Table 1: Summary of the simulation results.

positioning errors (first case)		
filter	mean	standard deviation
UKF	0.16127	0.21896
MCUKF	0.16602	0.22280
positioning errors (second case)		
filter	mean	standard deviation
UKF	0.51414	0.44778
MCUKF	0.26019	0.25141

In the second case, a total 10 impulse errors were randomly generated in *w*, and the result of MCUKF in this case is shown in Figure 3. It can be seen that large and small impulse errors are randomly entered into each channel of the distance measurements. And it can be seen that the adjusted R in MCUKF increases largely according to the impulse error generated for each channel. Due to this, the channel with the impulse error momentarily loses its function, and the INS error is corrected using the measurements obtained from the remaining channels. Therefore, MUCKF is hardly affected by the impulse error.

It can be seen that the UKF positioning result is greatly affected by the impulse error and the error increases. The reason is that UKF is a filter designed based on MMSE and cannot cope with non-Gaussian noise. On the other hand, it is confirmed that the positioning result of MCUKF is not affected by impulse error. Therefore, MCUKF is evaluated to be able to provide stable navigation information regardless of positioning error.

The number of the estimated position information for 60 seconds is 3000, and the mean and standard deviation of the positioning errors are calculated for each filter. And the result is summarized in Table 1. Based on this table, the excellent performance of the proposed MCUKF can be confirmed.

4 CONCLUSIONS

In this paper, MCUKF-based INS/UWB integrated navigation system was introduced. To use MCC in nonlinear system, MCUKF was designed by combining MCC with UKF. And this filter was used to integrated INS and UWB. UWB has non-Gaussian uncertainty noise in an indoor environment. While this causes a large estimation error in the existing UKF, it is proven based on simulation that MCUKF provides a stable navigation solution by tuning the *R* matrix for each channel in which this error occurs. Based on this paper, it is expected that stable variables can be reliably estimated in a nonlinear system including heavy-tailed non-Gaussian impulse noise.

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