

Sales Prediction Model based on Multifactorial Linear Regression

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Abstract: Motivated by the sales prediction of ABCtronics, this paper aims to provide integrated process for creating a proper multiple linear regression model and detailed analysis of according parameters. With multiple independent variables, the linear regression model exams the relationship between a collection of data-overall market demand, price per chip and economic condition to the single dependent variable sales volume of ABCtronics. The target is set to be future prediction of sales volume with three observable variables. All outputs are statistical results from Minitab which contain regression equation, model summary with R^2 value and analysis of variance stating the significance level of each variable. With detailed output, a refined multiple regression model is then developed to get rid of the overall market demand term. Pure regression equation brings only numerical understanding, i.e., visualization is the next focus. Based on comparison and contrast, it is verified the accurateness of the refined model. These results shed light on the proper use of multiple linear regression on sale prediction model.

1 INTRODUCTION

Sales prediction plays a key role in almost every successful operation of business. As one of the greatest inventions, it provides forecasting on future trend of target markets and an insight into proper allocation of resources e.g., labors and capitals. The way to maximize firm' sale target with limited resources is always considered the priority. Forecasting sales accurately for a new product is difficult and complex due to non-availability of past data. However, such forecast information is crucial for successful introduction of new products which, in turn, determines the survival of companies, in many cases (Meeran, et al, 2013). Hence, proper usage of sales prediction model can assist the firms with informed data on each input which may help refine future supply chain. Moreover, as the number of databases collected increases, corporations are able to gain less biased data which not only gives sales prediction but consumer preferences. Specifically, if linear regression model is formed, parameter of each variable would be given and that number tells the unit change of the regressors. By comparing the parameters, firms may attribute any boost of sales volume to some specific determinant factor, which can

be use to select suitable market target group and increase efficiency further.

Sales prediction models are now widely used in all fields. From prior literature, there are already intensive research on sales prediction in three major fields. First is the Microsoft Time Series algorithm. It provides us with optimized regression algorithm for forecasting continuous real-time values (Kohli, Shreya, et al, 2020). Time Series forecasting, which forecasts based on time-controlled variable, is an important tool under this scenario, where the research aim is to predict the behavior of complex systems solely by analyzing the past data (A Survey of Time Series Data Prediction on Shopping Mall, et al, 2013). For example, three researchers from Indian conduct a survey of time series data prediction on shopping mall to predict the next phase of the product price trends and sales volume. They propose a tree based data mining algorithm that treats market's behavior and interest as input & filter the desired output efficiently & a mining model of stream data time-series pattern in a dynamic shopping mall (A Survey of Time Series Data Prediction on Shopping Mall, et al, 2013). This is a comparatively complicated model because what involves are subjective investors who may cause significant error on the analyzing system, which leads to its complexity.

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Second is spatial data mining for retail sales forecasting. The study conducted by Maik Krause-Traudes, Simon Scheiderl and two other scientists aim to design a regression model to predict probable turnovers for potential outlet-sites of a big European food retailing company (Aina, Abidemi Ayodeji, et al, 2012). The forecast of potential sites is based on sales data on shop level for existing stores and a broad variety of spatially aggregated geographical, socio-demographical and economical features describing the trading area and competitor characteristics (Aina, Abidemi Ayodeji, et al, 2012). As a result, Support Vector Regression (SVR) is applied to provide the prediction of sales of existing outlets with attributes (e.g., floor space, number of parking lots, distance to the next competitor etc.) as well as the relationship between sales volume and these attributes. Finally, a novel trigger model for sales prediction with data mining techniques that focuses on how to forecast

sales with more accuracy and precision is proposed (Kohli, Shreya, et al, 2020). Then, researchers lay emphasis on online sales prediction instead of actual daily sales volume.

The approximate model they used is shown in Fig. 1. Raw data is first manipulated into available forms, and then a trigger model is proposed to do the classification. Next, the classification result shows the potentially best prediction model for each SKU. Finally, by use of the most appropriate model, the prediction is accomplished.

With the model, data is classified and grouped to facilitate analysis. The trigger model is a combination of several basic models with Mean Absolute Percentage Error (MAPE) to evaluate the performance of proposed method. Compared with the baseline model ARMA, the trigger model possesses more accuracy (Huang 2015).

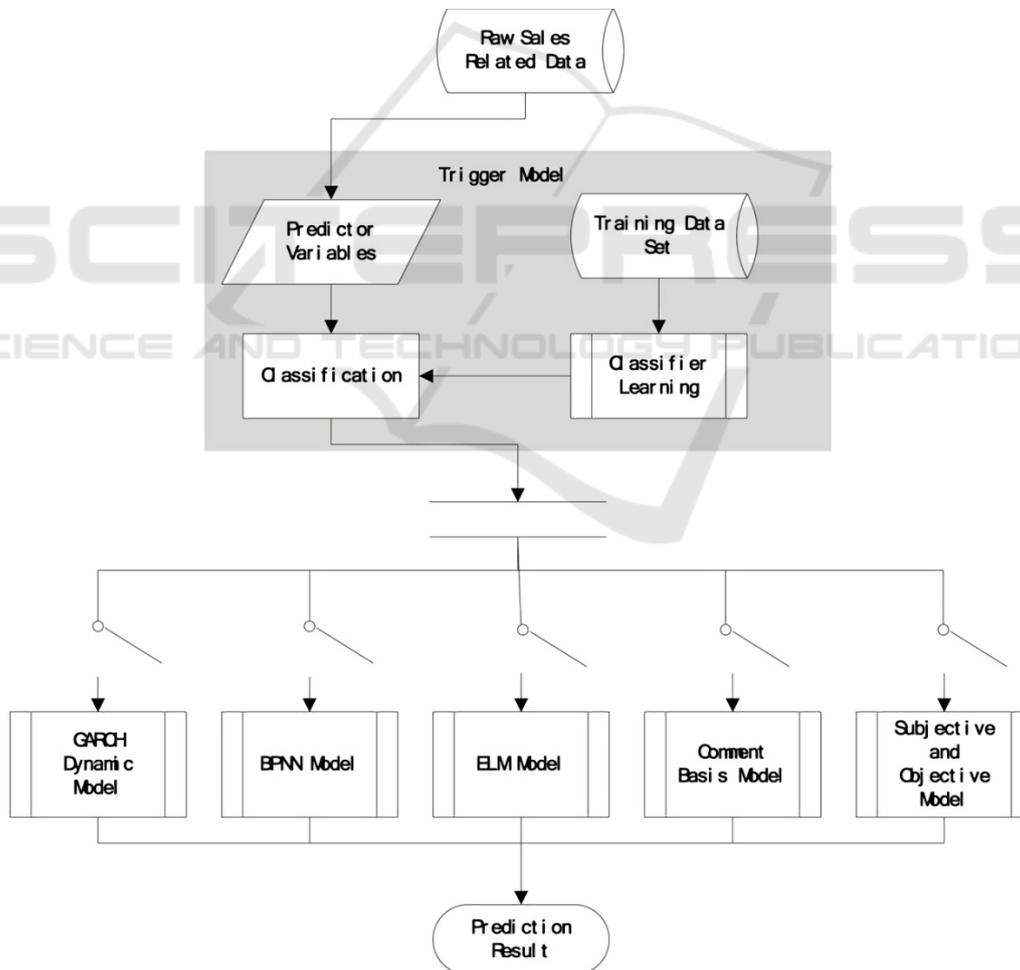


Figure 1: A sketch of the framework of Trigger Model System (Huang 2015).

The model considered to use to predict sales volume in this passage is multiple linear regression models. Regression analysis is a statistical technique for estimating the relationship among variables which have reason and result relation (Uyanık, Gülden Kaya, Neşe Güler, 2013). As a linear model, multiple regression provides most direct insight on how change in single unit of any independent variables affect the sales volume. Recent research shows extensive use of multiple linear regression in different fields. For example, the relationships between Chlorophyll-a and 16 chemical, physical and biological water quality variables in Camilleri reservoir (Ankara, Turkey) were studied by using principal component scores (PCS) in multiple linear regression analysis (MLR) to predict Chlorophyll-a levels (Çamdevýren, Handan, et al, 2005). In addition, the simulation of water table responses is discussed and the use of multiple linear regression as a modelling technique is considered. The model permits the consideration of changes in properties of recharge, discharge and aquifer parameters simultaneously (Hodgson, Frank D, 1978).

The rest part of the paper is organized as follows: In the first following part, case used for multiple regression model in this passage will be introduced with data and model presented. For visualization, output from Minitab and graphs would be presented, along with assumptions for proper utilization of the model, and how the model is evaluated. Moreover, results output of software, not only the output function, given R² value and VIF but also the Four in one graph for the observational data versus the predicted data with residuals would be explained and presented one by one with clarification. One can understand what the data represents and what the graph indicates without any background knowledge. Based on analysis provided and output given by graph, final conclusion would be given with discussions on limitation of the model. Finally, conclude upon all the paragraph including future expectations on prediction of the sales model.

2 DATA AND METHOD

The whole analysis of sales volume and model prediction is based on the ABCtronics case. In this case, reader serves as an intern of the ABCtronics firm which specializes in producing IC chips. Recent consumer feedback and data shows a dramatic increase in the rejection rate from XYZfirm, which causes doubt from the Audit committee (Adhikari, Arnab, et al, 2016). As the main focus of this passage the use of multiple linear regression model, analysis

on consumer feedback and rejection rate are ignored, the main focus would be prediction of future sales volume by multiple linear regression. Table I provided historical sales.

Table 1: Historical Sales Figure of Abctronics (Adhikari, Arnab, Et Al 2016).

Year	ABCtronics' sales Year volume (in millions)	Overallmarket demand (in millions)	Priceper chip (in \$)	Economic condition *
2004	2.39	297	0.832	0
2005	3.82	332	0.844	1
2006	3.33	195	0.854	0
2007	2.49	182	1.155	1
2008	1.56	93	1.303	0
2009	0.97	98	1.265	0
2010	1.32	198	1.368	1
2011	1.42	188	1.208	0
2012	1.48	285	1.234	1
2013	1.85	264	1.282	1

Note: *Economic condition : 1 signifies favorable market condition and 0 signifies otherwise.

For multiple linear regression model to be applicable, at least two independent variables are needed. For multiple linear regression, the proposed model is shown below:

$$Y = \beta^0 + \beta^1 X^1 + \beta^2 X^2 + \beta^3 X^3 + \varepsilon \quad (1)$$

where $\varepsilon \sim \text{Normal}(0, \sigma^2)$. Here, an error term of normal distribution with mean 0 and standard deviation σ^2 is included accounting for instability of this model.

From the table, statistics show three different independent variables and one dependent variable-ABCtronics' sales volume. Since statistically significance of any independent variable cannot be determined only by looking at the raw data. The first try includes all three independent variables in the model and the corresponding results are shown in Eq. (1) Table II Table III and Table IV.

Table 2: Coefficients for all independent variables including the constant term.

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	8.86	1.35	6.57	0.001	
Overall market demand	-0.00524	0.00258	-2.03	0.089	3.27
Price Per Chip	-5.505	0.881	-6.25	0.001	2.54
Economic Condition	1.130	0.342	3.30	0.016	2.44

Table 3: Overall Model summary for three independent variables.

S	R-sq	R-sq(adj)	R-sq(pred)
0.346274	90.75%	86.13%	72.75%

Table 4: Additional Information Including The F-Value.

Source	D F	Adj SS	Adj MS	F- Value	P- Value
Regression	3	7.0606	2.3535	19.63	0.002
Overall market demand	1	0.4930	0.4930	4.11	0.089
Price Per Chip	1	4.6786	4.6786	39.02	0.001
Economic Condition	1	1.3089	1.3089	10.92	0.016
Error	6	0.7194	0.1199		
Total	9	7.7800			

$$\text{sale} = 8.86 - 0.00524 \text{ market demand} - 5.505 \text{ Price Per Chip}$$

$$+ 1.130 \text{ Economic Condition} \quad (2)$$

Adjusted R² from model summary shows that only 86.13% of the variation in the sales volume is explained by this regression model. Adjusted R² is preferred here since it takes into account the real variation by adding more variables, which still shows high percentage of prediction coverage. To justify the accurateness of this model, p-values for each different variable are required, which is shown as the last column of Analysis of Variance part. Taking a 95% confidence level, p-value of Overall market term equals 0.089 which is greater than 5%, the passing line for statistical significance. For any p-value greater than 5%, independent variable with that specific p-value is considered statistically insignificant. Getting rid of that term would provide more accurate result since insignificant variable attributes little to predict future sales volume. A refined model is provided taking only Price per chip and economic condition as independent variables. With the same regression model, data output from Minitab is shown in Table V, Table VI and Eq. (2).

$$\text{sales} = 6.452 - 4.136 \text{ Price Per Chip} + 0.606 \text{ Economic Condition} \quad (3)$$

Table 5: Overall Model summary or two variables.

S	R-sq	R-sq(adj)	R-sq(pred)
0.416172	84.42%	79.96%	64.42%

Table 6: Coefficients summary for two variables and the constant term.

Term	Coef	SE Coef	T- Value	P- Value	VIF
Constant	6.452	0.766	8.42	0.000	
Price Per Chip	-4.136	0.680	-6.08	0.001	1.05
Economic Condition	0.606	0.269	2.25	0.059	1.05

The p-value here are comparatively small, which proves the two independent variables to be statistically significant. Furthermore, for it not to be a biased model, multicollinearity still needs to be tested. Multicollinearity defines the correlation between two independent variables so if too high, the prediction will not be accurate as variation in one variable affects the other as well. Variance inflation factor (VIF) is then chosen to exam the correlation. Here, coefficient component of above graph gives result of VIF to be 1.05, which is much smaller than the standard value 5. To sum up, there is no multicollinearity between these two variables and the refined model seems to be suitable for predicting the sales figure of ABCtronics.

Table 7: Coefficient for single variable 'overall market' term.

Term	Coef	SE Coef	T-Value	P- Value	VIF
Constant	0.753	0.779	0.97	0.362	
Overall market demand	0.00614	0.00344	1.79	0.112	1.00

3 RESULTS AND DISCUSSION

For the modified model:

$$\text{Sales Volume} = 6.452 - 4.136 \text{ price per chip} + 0.6063 \text{ economic condition} \quad (4)$$

Adjusted R² in this model shows that around 79.96% of the variation in the sales volume is explained by this regression model, which is comparatively high for a sales prediction model. Further clarification for choosing confidence level at 95% is needed. According to the analysis, a simple linear regression model with overall market demand was ran before any multiple linear regression.

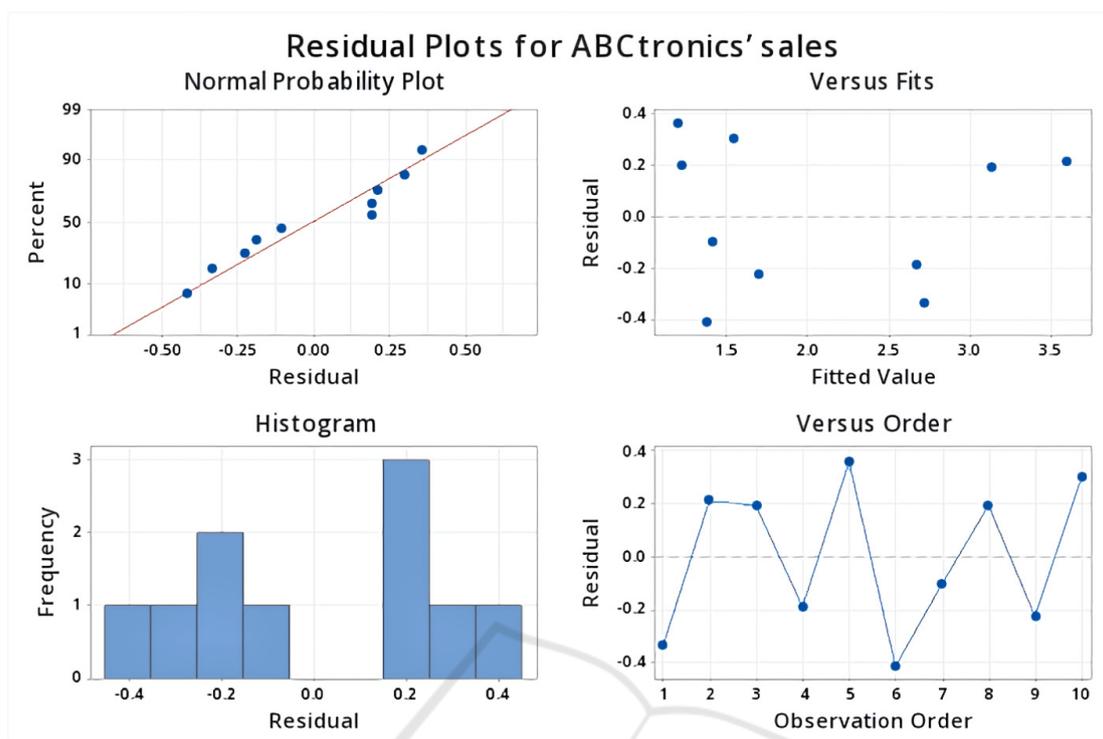


Figure 2: Residual plots for ABCtronics' sale for all three independent variables.

P-value of this single independent variable is 0.112, which is higher than 10%, corresponding value of 90% confidence level. Here, even with a 10% level of significance, overall market demand is still defined as statistically insignificant, not to mention the 5% level. It might work for lower levels of significance but overall precision would decrease choosing too low a significant level for any model. Therefore, getting rid of the demand term may help people to better estimate sales volume using a multiple regression model.

From the model, unit increases in price per chip would lead to approximately 4.136 units decrease in the sales volume. For the binary economic condition value, signifies favorable market condition would lead to 0.6 unit increase in sales volume. For instance, ABCtronics should operate in favor of market condition and control their chip price comparatively low to maximize sales volume.

To visualize and better compare two different regression modes with and without overall market demand variable, Four-in-one graphs with normal probability plot, histogram, versus fit and versus order are shown in Fig. 2. With all three independent variables, top left plot shows how fit the real data to the fitted line; top right is the verification that residuals are randomly distributed; bottom left is the

histogram with frequency and bottom right is data of residual in order.

The top left normal probability curve is a visualization of R^2 value: predicted value forms the best fitted line shown as the red straight line. For each point, it represents the real sales volume from 2004 to 2013. The more variation that is explained by the model, the closer the data point fall to the fitted regression line. The distance between real value point and the predict line is defined as residuals. The lower the residuals, better fit is the model of prediction. From above top left plot, real points are all shown to be close to the predicted line, which signals high R^2 corresponding to our result-90,75%.

The top right residual fits graph plots residual on y axis and fitted value on x-axis. The residuals versus fits plot is used to verify the assumption that the residuals are randomly distributed and have constant variance. Therefore, ideally, the points should fall randomly on both sides of 0. To sum up, the versus fit plot is at ideal status.

The bottom left histogram graph is easy to understand -it shows the frequency of each residual. Larger frequency with small residuals is favorable since it represents more accurate prediction. But here, data with 0 residual does not exist due to flaws on design of the model.

The versus order graph is similar to the versus fits graph but plotted in order that of the date collected. Independent residuals show no trends or patterns when displayed in time order. From the graph,

residuals near year 2005 and 2006 may be correlated since the difference is comparatively low, but the overall trend of residuals fall randomly around the center line.

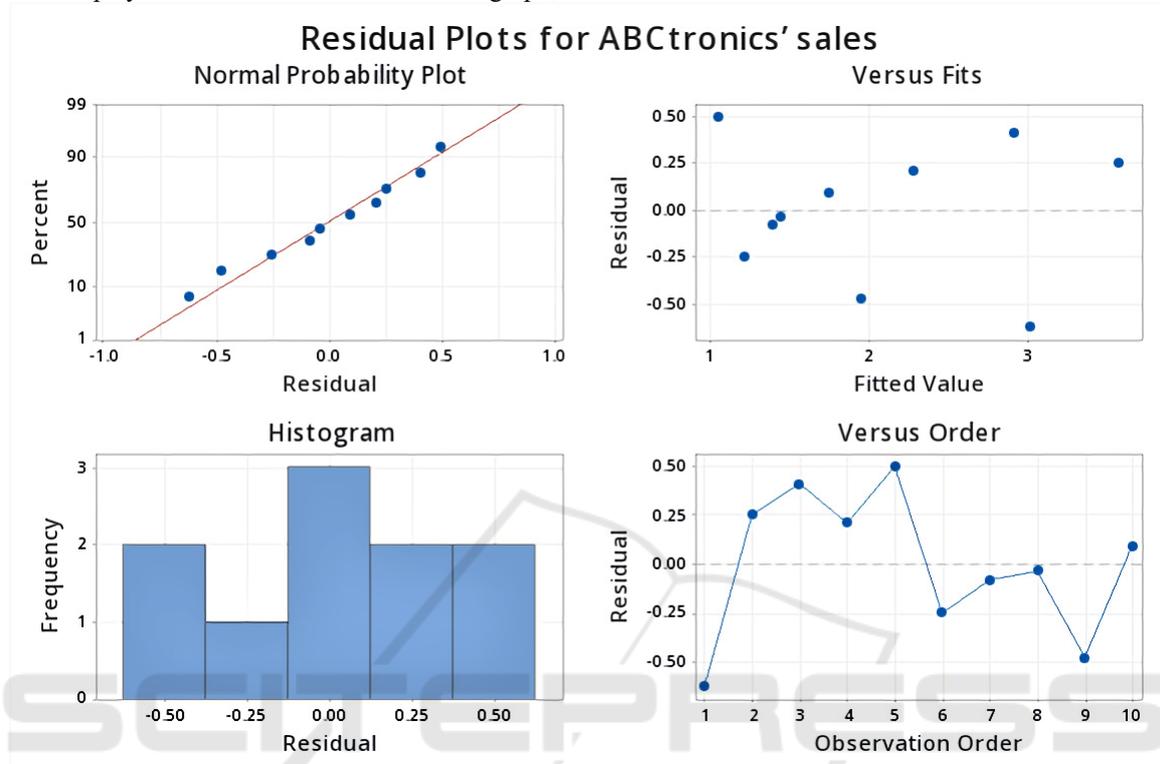


Figure 3: Residual plots for ABCtronics' sales for two variables.

Getting rid of the overall market term, with two independent variables, the top left plot shows a better fit of the real data to the fitted line; top right is still the verification that residuals are randomly distributed; bottom left is the histogram with residual frequency more in the middle and bottom right is data of residual in order.

people from ABCtronics is flawless and contains all factors influential to sales figures. First of all, whether there is an omitted variable that is not mentioned in the model is unclear. If there are omitted variables which also determine changes in the sales volume, the model is then incomplete and biased since not all factors are included. Moreover, without telling whether each independent variable correlates with existing error terms, people are not sure that this is a good model for prediction. If correlation exists, IV regression needs to be done to find other better independent variables. Additionally, real world economy is more complicated with nonlinear relationships; only using multiple linear relationships may not provide desirable results. There are many clinical problems which do not allow the option of corroboration by more invasive and disruptive approaches. For such problems, a more penetrating approach to data analysis may be the only way to do justice to the data set (BLACK, A.M.S., P. FOÉx, 1982).

Now, one is able to reproduce the four-in-one graph with only two independent variables. The residual plot seems to be closer to the fitted line which shows an increase in R^2 value. Versus fit and order graphs still show a random distribution of the residual points around the center line. The histogram is with more differences—frequency of predicted data with small residuals increase a lot comparatively, which shows a pattern closer to the normal distribution. Hence, visualization for a model of prediction shows the same result as a pure linear equation that getting rid of the overall market variable helps refine the multiple regression model.

A multiple linear regression model seems to provide a proper prediction for future sales volume. However, nothing can be guaranteed that the table provided to

4 CONCLUSION

In summary, sales prediction models based on multiple linear regression is investigated based on multiple independent variables related to sales volume. The article starts with examples from historical successful sale model prediction to the use of multiple linear regression models among different fields. Next comes the description of ABCtronics case which would be used for building own sales prediction model. Though as a linear regression, multiple regression models vary a lot depends on how many independent variables to include. Keep adding meaningless variables affects nothing about the model but create more bias. As a result, final model created in the passage excludes one independent variable from data provided to promote accuracy. Though the best fitted model is found using multiple linear regression here, in real world, linear model is comparatively incapable of producing accurate sale figure prediction. Furthermore, if people keeping conducting more investigations, researchers may find other correlated independent variables which become determinant factors to predict sales volume. Including those factors can further refine the multiple regression model and give accurate prediction. These results offer a guideline for more complicated models developed in sale volume prediction model and give a chance for people to create their own prediction model even if one does not specialize in it.

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