LMI Stability Condition for NCS with Packet Delay and Event-triggered Control

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Abstract: This paper presents a controller design for networked control systems (NCS) with packet delay and event-triggered control. The total network delay is assumed to be an integer multiple of a fixed sampling period so that the overall system is time-varying with each model depending on the number of time delays. The design methodology is applicable to an arbitrary number of packet delays, regardless of whether the delays are random or deterministic. The methodology is applied to a simple example and Monte Carlo simulation results show that the controller stabilizes the NCS and is robust with respect to random variations in the sampling period and to changes in the probability of packet delays.

1 INTRODUCTION

Networked control systems (NCS) are control systems where the controller receives information from the plant and delivers control commands to the actuator through a communication network (Antaklis, et al., 2007; Li, et al., 2015). The shared network connection between different components of the control loop yields a flexible architecture and reduced installation and maintenance costs (Hespanha, et al, 2007)0. With limited network resources, in many applications it is beneficial to reduce the load on the network by using event-triggered control (Yang, 2006), (Ge et al., 2021), (Lemmon, 2010). Control actions are not updated unless this is warranted to maintain satisfactory operation of the control system and the need to relay information to the network from a remote controller during periods where the current control is satisfactory is eliminated.

With event-triggered control or with packet delay, the interval between updates of the control signal varies. This variation results in a system that switches between different plant models with each model corresponding to the interval between the last and current control update. Switching requires careful design to ensure that the switched system remains stable and perform satisfactorily.

Although there are multiple results in the literature for the stability analysis and design of linear NCS (Garcia et al., 2014), there is still a need for a simple design approach that yields a time-varying controller that can handle arbitrary packet delays. We exploit a well-known result for the stabilization of linear parameter-varying systems (Pandey et al., 2017) to design a time-varying controller for NCS with arbitrary packet delays. Although the result was intended for the design of gain scheduled control systems, a special case of the result allows us to exploit it for the design of NCS. The NCS model is adopted from (Montestruque and Antaklis, 2004). The resulting controller stabilizes the NCS regardless of the switching regime between the models corresponding to different packet delays. The controller is obtained by solving a set of linear matrix inequalities (LMIs). The number of inequalities solved for the controller depends on the maximum number of consecutive packet delays assumed for the design.

An example is provided to demonstrate the control system design. Simulation results show that the design stabilizes the NCS regardless of the switching regime associated with the packet delays. In addition, if the system is designed for switching at multiples of the sampling period, it is robust with respect to random variations in the sampling period. Thus, the sampling period need not be known exactly.

The next section reviews the NCS model of from (Montestruque and Antaklis, 2004) and some properties of switched systems. Section 3 presents our controller design methodology, which is the main

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result of this paper. Section 4 presents simulation results and Section 5 is the Conclusion.

2 NETWORKED CONTROL SYSTEM

Consider the linear plant

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) + D\boldsymbol{u}(t) \tag{2}$$

with networked control with constant matrices $A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^{n\times m}, C \in \mathcal{R}^{l\times n}, D \in \mathcal{R}^{l\times m}$. We adopt the NCS model of (Montestruque and Antaklis, 2004) and investigate the stability and controller design for the system.

The plant model is not exactly known, and the nominal model of the system with the matrices of the same order as their true counterparts in (1) and (2) is

$$\dot{\hat{\boldsymbol{x}}}(t) = \hat{A}\hat{\boldsymbol{x}}(t) + \hat{B}\boldsymbol{u}(t) \tag{3}$$

$$\boldsymbol{y}(\boldsymbol{t}) = \hat{C}\hat{\boldsymbol{x}}(t) + \hat{D}\boldsymbol{u}(t) \tag{4}$$

The discrepancy between the actual and nominal models results in the error

$$\boldsymbol{e} = \boldsymbol{x} - \hat{\boldsymbol{x}}, \qquad \boldsymbol{x} = \hat{\boldsymbol{x}} + \boldsymbol{e} \tag{5}$$

Subtracting the nominal from the actual dynamics gives the error dynamics

$$\dot{\boldsymbol{e}} = A\boldsymbol{e} + \tilde{A}\hat{x} + \tilde{B}\boldsymbol{u}(t) \tag{6}$$

where we use the perturbation matrices

$$\tilde{A} = A - \hat{A}, \qquad \tilde{B} = B - \hat{B} \tag{7}$$

For an observable system, we use the control

$$\boldsymbol{u}(t) = -K\hat{\boldsymbol{x}}(k), t \in [kh, kh + h] \tag{8}$$

where $\hat{\boldsymbol{x}}(t)$ is the state estimate. Here, we first assume that the state is measurable with a finite error in the measurement. Substituting the control in the system dynamics gives the closed-loop model

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) - BK\big(\boldsymbol{x}(t) - \boldsymbol{e}(t)\big) \tag{9}$$

$$\boldsymbol{y}(\boldsymbol{t}) = C\boldsymbol{x}(t) - DK\hat{\boldsymbol{x}}(t) \tag{10}$$

Substituting in the error dynamics gives

$$\dot{\boldsymbol{e}} = \big(\hat{A} + \tilde{B}K\big)\boldsymbol{e}(t) + \big(\tilde{A} - \tilde{B}K\big)\,\boldsymbol{x}(t) \tag{11}$$

Combining error and nominal dynamics gives the augmented stated vector

$$\boldsymbol{z}(t) = \begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{12}$$

Combining (9) and (11), we have the augmented system dynamics

$$\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{e}}(t)\end{bmatrix} = A_n\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix} - B_nK(k)\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{13}$$

$$A_n = \begin{bmatrix}A & 0\\ \tilde{A} & \hat{A}\end{bmatrix}, \qquad B_n = \begin{bmatrix}B & -B\\ \tilde{B} & -\tilde{B}\end{bmatrix} \tag{14}$$

The system is digitally controlled with a control signal sent through a communication system to the actuator and the controller receiving a signal from the sensor. The following assumptions are used in the sequel:

(i) The total delay, including plant to controller T_{pc} and controller to actuator T_{ca}, plus the computational time is T_c satisfies

$$T_{pc} + T_{ca} + T_c \le lh, l\ integer \tag{15}$$

(ii) The delay T_{ca} can be predicted with sufficient accuracy to design the system using the sum

$$T_{pc} + T_{ca} + T_c = lh, l\ integer \tag{16}$$

(iii) The sampling period Nh provides sufficiently faster sampling than the Nyquist rate dictated by the dynamics of the closed-loop system.

(iv) The number of consecutive packet delays in the NCS does not exceed l_p.

(v) When event-driven control is used to reduce the required network bandwidth, the effective sampling period is in the range $[h, l_e h]$, where l_e is a variable but bounded integer, $l_e \le l_m$.

(vi) The number of sampling periods between two consecutive arriving packets does not exceed an integer bound N, that is

$$l_p + l_m \le N \tag{17}$$

Under the above assumptions, h is a suitable sampling period for the NCS and the NCS can function appropriately with the sampling period Nh

Discretizing the system with different sampling periods that are a multiple of the fixed sampling period h, we have a system that switches arbitrarily between the models

$$\left(e^{A_n lh}, \int_0^{lh} e^{A_n\tau} B_n d\tau\right), l = 1,2,\dots,N \qquad (18)$$

The state-space model of the system is of the form

$$\boldsymbol{x}(k+1) = A_i \boldsymbol{x}(k) + B_i \boldsymbol{u}(k), \quad i \in \{1,2,\dots,N\} \qquad (19)$$

$$\boldsymbol{y}(k) = \hat{C}\hat{\boldsymbol{x}}(t) + \hat{D}\boldsymbol{u}(t) \qquad (20)$$

The following result applies in this case.

Theorem 1. 0(Zhai et al., 2002) If all state matrices $A_i, i = 1,\dots,N$, are mutually commutative and Schur stable, then the switched system (19) is globally exponentially stable under arbitrary switching.

The result clearly applies in the case of Schur stable state matrices in the form

$$A_i = A^i, i = 1,\dots,N \qquad (21)$$

When applied to the NCS of (19), we have the corollary.

Corollary: If the discrete-time NCS model of (19) is Schur stable for a sampling period h, then the switched system (19) is globally exponentially stable under arbitrary switching between sampling periods $lh,\ l = 1,2,\dots,N$, that are integer multiples of h.

Remark 1

The stability condition is valid if the sampling period varies randomly because the state matrices remain mutually commutative based on the well-known properties of the matrix exponential.

Remark 2

The stability condition is valid in the case of a matrix perturbation $\Delta A \in \mathcal{R}^{n\times n}$ such that the state matrix $A + \Delta A$ is Schur stable.

Remark 3

A necessary condition for the system to remain stable under arbitrary switching is for all subsystem matrices to be Schur stable. Otherwise, switching to an unstable subsystem and subsequently remaining there would result in an unstable switched system.

3 CONTROLLER DESIGN

This section presents a new approach for the design for NCS with arbitrary switching between models corresponding to different sampling rates. The varying sampling rates correspond to periods where no control signal is sent from the controller to the actuator. This results in a sampling period in the range, $lh, l = 1,2,\dots\dots N$, where h is the nominal sampling period of the system and N is an integer. The switching can be deterministic or random because the conditions are valid regardless of the switching mode.

The following theorem from (Pandey and Oliveira, 2017) provides stability conditions for a system that switches between different linear models.

Theorem 2. (Pandey and Oliveira, 2017)

Consider a time-varying discrete-time linear system of the form

$$A(k) = \sum_{i=1}^{N} \xi_i(k) A_i$$
$$B(k) = \sum_{i=1}^{N} \xi_i(k) B_i \qquad (22)$$

$$\sum_{i=1}^{N} \xi_i(k) = 1,\ \xi_i(k) > 0, i = 1,\dots,N \qquad (23)$$

The system is stable with the control

$$K(k) = \sum_{i=1}^{N} \xi_i(k) K_i\ K_i = L_i X_i^{-1} \qquad (24)$$

if there exist positive definite matrices $X_i, , Y_i, Z_i, Q_i, i = 1,\dots,N$ that the satisfy the LMI

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T & -L_i^T \\ A_i X_i & Q_j - R_{ij} & B_i Z_i - Y_i^T \\ -L_i & Z_i^T B_i^T - Y_i & Z_i + Z_i^T \end{bmatrix} > 0 \qquad (25)$$

$$R_{ij} = B_i Y_i + Y_i^T B_i^T, i,j = 1,\dots,N \qquad (26)$$

For an NCS, switching is between models that depend on the number of packet delays, or the period for event-triggered control. This effectively changes the sampling period from T to $lT, l = 1,2,\dots,N$, where $l-1$ is the number of packet delays. Applying Theorem 1 with $\xi_i(k) = 1$ for one i value at a time and $\xi_j(k) = 0, j \neq i$ gives the following theorem.

Theorem 3

The NCS with packet delay and event-driven control subject to assumptions (i-vi) such that the control input is changed every l sampling periods, $l \in \{1,2,\dots,N\}$ is stable with the control of (24) if the LMIs of (25) have positive definite solution matrices $X_i,, Y_i, Z_i, Q_i, i = 1,\dots,N$ for $i, j = 1,\dots,N$

Proof

Theorem 2 provides a controller for arbitrary switching subject to conditions (22-23). For NCS, switching is between matrices with different sampling periods corresponds to the case $\xi_i = 1, \xi_j = 0,\ j \neq i, j \in \{1,\dots,N\}$. This clearly satisfies condition (23). Hence, Theorem 3 follows directly from Theorem 2.

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Remark 4

Although Theorem 3 is stated for multiples of the sampling period, the result is clearly valid for any set of sampling periods. The results are even valid for arbitrary random switching between a set of sampling periods.

4 SIMULATION RESULTS

Consider the oscillatory behavior of the pair

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The pair is modelled as

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ -4.1 & 0.1 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The perturbation matrices are

$$\tilde{A} = A - \hat{A} = \begin{bmatrix} 0 & 0 \\ 0.1 & -0.1 \end{bmatrix}$$

$$\tilde{B} = B - \hat{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

We form the matrices of the augmented system

$$A_n = \begin{bmatrix} A & 0 \\ \tilde{A} & \hat{A} \end{bmatrix}, \qquad B_n = \begin{bmatrix} B & -B \\ \tilde{B} & -\tilde{B} \end{bmatrix}$$

For the purposes of controller design, assume that package delay and event-triggered control result in switching between two systems with sampling periods h and $2h, h = 0.04$ s. The switching is random with a probability p of the nominal sampling period h and probability $(1-p)$ of period $2h$ due to event triggered control or packet delay. The model corresponding to one sampling period $h = 0.04$ s with no delay in the arrival of a package is

$$A_1 = e^{A_n h} = \begin{bmatrix} 0.9968 & 0.04 & 0 & 0 \\ -0.1598 & 0.9968 & 0 & 0 \\ 0.0001 & -0.0001 & 0.9967 & 0.04 \\ 0.0043 & -0.0039 & -0.1641 & 1.0007 \end{bmatrix}$$

$$B_1 = \int_0^h e^{A_n \tau} B_n d\tau = \begin{bmatrix} -4 & -0.08 & 0 & 0 \\ 0.32 & -4 & 0 & 0 \\ 0 & 0 & -4 & -0.08 \\ 0.01 & 0.01 & 0.33 & -4 \end{bmatrix} \times 10^{-2}$$

The model corresponding to a sampling period $h = 0.08$ s with delay in the arrival of a package due to package delay or event triggered control is

$$A_2 = e^{A_n 2h} = \begin{bmatrix} 0.9993 & 0.0191 & 0 & 0 \\ -0.0764 & 0.9993 & 0 & 0 \\ 0 & 0 & 0.9993 & 0.0191 \\ -0.002 & -0.0019 & -0.0784 & 1.0012 \end{bmatrix}$$

$$B_2 = \int_0^{2h} e^{A_n \tau} B_n d\tau = \begin{bmatrix} -7.97 & -0.32 & 0 & 0 \\ 1.28 & -7.97 & 0 & 0 \\ 0 & 0 & -7.96 & -0.32 \\ -0.04 & 0.03 & 1.31 & -8 \end{bmatrix} \times 10^{-2}$$

We solve the LMIs of Theorem 3 to obtain the controller parameters using the MATLAB LMI solver. The solutions can be improved by imposing constraints on the norms of the matrices. Solving the LMIs gives the gain matrices

$$K_1 = \begin{bmatrix} -1.3001 & 0.3355 & 0.0264 & -0.0189 \\ 0.4269 & -0.5425 & -0.0092 & 0.012 \\ 0.0263 & -0.008 & -1.35 & 0.3613 \\ -0.0226 & 0.0128 & 0.4567 & -0.5667 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -1.3180 & 0.2617 & 0.0217 & -0.0148 \\ 0.3617 & -0.466 & -0.0062 & 0.0092 \\ -0.0213 & -0.0059 & -1.1788 & 0.2618 \\ -0.0175 & 0.01084 & 0.3845 & -04855 \end{bmatrix}$$

The design obtained by solving the LMIs results in a slightly faster response for the pair (A_2, B_2) corresponding to a longer delay but both subsystems are asymptotically stable.

Using 100 Monte Carlo simulations for the system under different conditions, we compare the simulation results for the zero-input response. To assess the robustness of the systems to random changes in the sampling period, the system is simulated (a) with switching between sampling period h and sampling period $2h$, the (b) with the sampling period randomly switching between h and $h+\Delta h, \Delta h \sim U[-0.1h, 0.1h]$. Plots of the average evolution of the state variables are shown in Figures 1 and 2. Figure 3 shows the random switching between sampling period h and sampling period $2h$, with an initial sampling period equal to h. Figure 4 shows the random variation of the sampling period $\Delta h \sim U[-0.1h, 0.1h]$. The simulation results show that the state variables of the system converge to zero with the controller resulting in a stable well-behaved system. The random variation of the sampling period results in a larger first peak and a more oscillatory response but does not destabilize the system.

The system also performs well for different probabilities p of a sampling period $h = 0.04\,s$. Figures 5 and 6 show the state evolution for $p = 0.6$ and $p = 0.8$, with probabilities of sampling period $2h = 0.08\,s$ equal to 0.4 and 0.2, respectively. Because the LMI for the delay that results in doubling the sampling period gives a faster response, contrary to intuition, the response is faster for the lower probability $p = 0.6$. However, the system performs well for both probabilities, as do others not included in the paper.

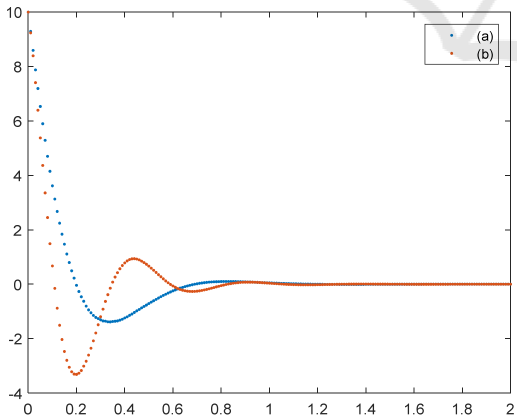

Figure 1: SEQ Figure * ARABIC 1,. Plot of x_1 versus time (a) switch between sampling period h and sampling period 2h (b) random variation Δh around h, $\Delta h \sim U[-0.1h, 0.1h]$.

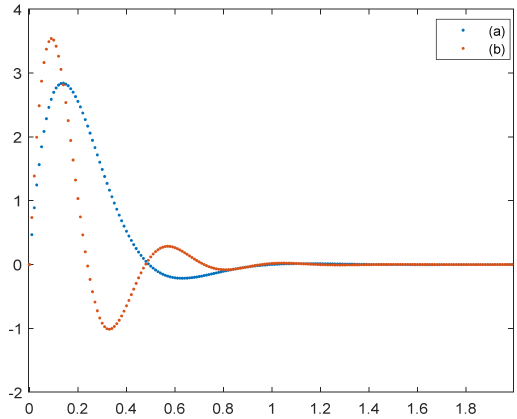

Figure 2: Plot of x_2 versus time (a) switch between sampling period h and sampling period 2h (b) random variation Δh around h, $\Delta h \sim U[-0.1h, 0.1h]$.

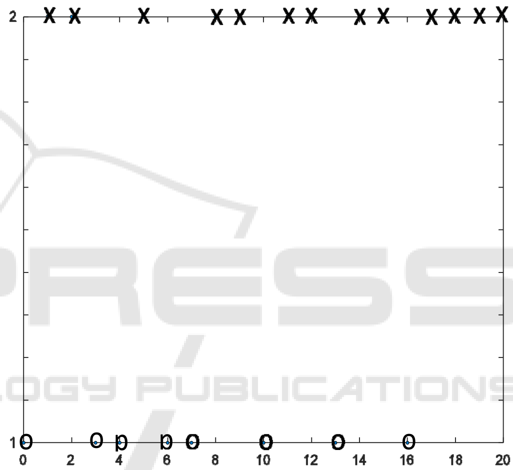

Figure 3: Switching between the sampling periods h and 2h for the NCS.

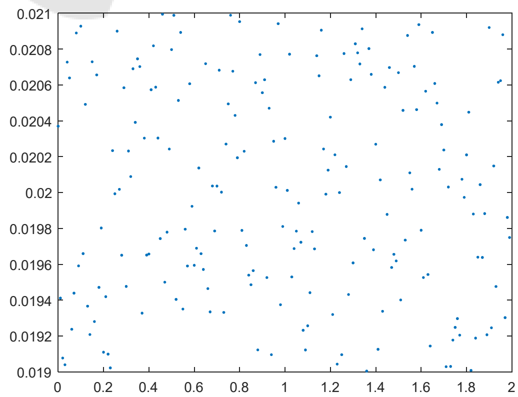

Figure 4: Plot of randomly varying sampling periods.

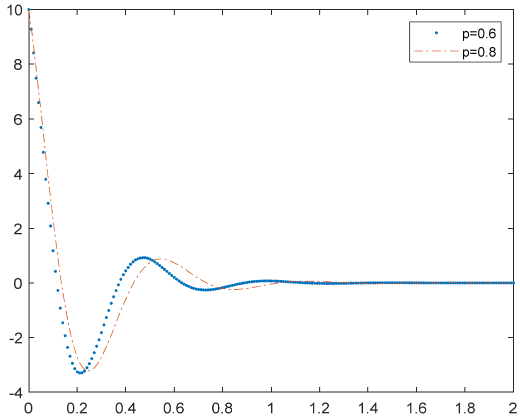

Figure 5: Plot of x_1 versus time for probability of sampling period h (a) p = 0.6 (b) p = 0.8 s.

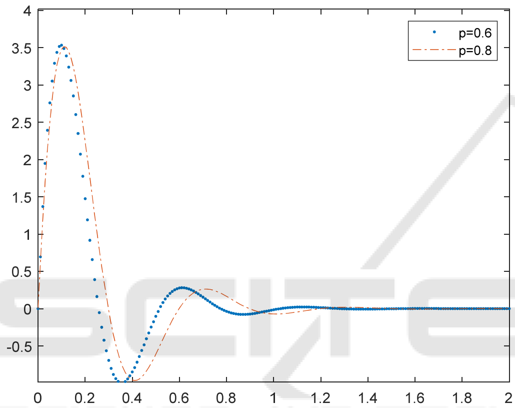

Figure 6: Plot of x_2 versus time for probability of sampling period h (a) p = 0.6 (b) p = 0.8 s.

4 CONCLUSIONS

This paper presents a new controller design for linear NCS with packet delays, event triggered control that is robust with respect to random variations in the sampling period. The approach is applicable to an NCS with known upper bound on the number of sampling periods between consecutive received packages. The approach is valid for arbitrary random switching between different models. Simulation results show that the system is stabilized with random switching between models and remains stable for different probabilities of switching and random variations in the sampling period. Future work will provide an analysis of the robustness of the design with respect to modelling errors and changes in the sampling period and probability of switching.

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