

# Steiner Tree-based Collaborative Learning Group Formation in Trust Networks

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**Keywords:** Group Formation, Collaborative Learning, Trust Network, Steiner Tree.

**Abstract:** Group formation is one of the key problems for collaborative learning, i.e., how to allocate agents (learners) to appropriate groups in order to improve the learning utility of the system. Previous works often focus on investigating the potential factors that may influence the agent's learning utility from the perspective of intrinsic attributes of agents; however, the structural attributes of groups are rarely considered. Considering that trust is an important interactive and cognitive attribute in collaborative learning, which can influence not only the incentive of learners collaborating in a group but also the promotion of skills of agents in knowledge sharing, this paper studies the collaborative learning group formation problem in trust networks. We propose a Steiner tree-based group formation algorithm, which first allocates appropriate agents to groups as initiators by considering the skill mastery and the strength of trust in the groups to guarantee the opportunities for skill promotion and then select followers by searching locally in the trust network. Through experiments based on real-world network datasets, we validate the performance of the proposed algorithm by comparing to several benchmarks, e.g., the graph partitioning-based group formation algorithm and the simulated annealing-based group formation algorithm.

## 1 INTRODUCTION

Collaborative learning (CL), which is a paradigm organizing the learning process by dividing agents into multiple collaborative groups (Stahl et al., 2014; Matazi et al., 2014), is often used in employee training programs in enterprise management. Under this paradigm the employees' necessary working skills can be improved since employees can work under the guidance of senior professional practitioners, and can learn professional working skills from each other (Zhang et al., 2017); it is critical for the growth of enterprise (Zhang et al., 2017). Intuitively, the results of group formation could largely influence the learning utility of each agent and further determine the utility of the system. How to obtain an optimal group formation solution has been considered to be of great importance in CL area, which can be called the collaborative learning group formation (CLGF) problem.

There are many previous works trying to investigate potential factors that may influence the agent's learning utility in CL scenarios (Maqtary et al., 2019). Most of them focus on the *intrinsic attributes* of agents, e.g., the agent's gender (Curşeu et al., 2018),

skill level (Graf and Bekele, 2006), learning style (Abnar et al., 2012), etc. However, the *structural attributes* of groups are rarely considered, except for the communication constraint (Brauer and Schmidt, 2012) and the interaction cost (Zhang et al., 2017).

The structural attributes can be actually employed to indicate whether a group is efficient for collaboration from the perspective of interaction structure of a group. As an important interactive and cognitive attribute, trust reflects the belief between people (Granatyr et al., 2015) and plays an important role in knowledge sharing (Evans et al., 2019); it can influence not only the incentive of learners being involved into a collaboration group (Skinner, 2007), i.e., the reliability and efficiency of collaboration among agents, but also the promotion of skills of agents in the CL groups (Wickramasinghe and Widyaratne, 2012).

To this end, the trust network-aware collaborative learning group formation (T-CLGF) problem is studied in this paper, aiming at achieving the group formation solution that maximizes the learning utility of system considering the effect of trust on the collaboration of agents in groups. A trust network-aware collaborative learning model is first proposed which re-

fects the skill promotion of individuals (and groups) involved in CL scenarios considering the group cohesion and the skill absorption in terms of the trust network. The T-CLGF problem is then formally defined according to the coalition structure generation problem (Rahwan et al., 2015). Considering the group formation constraint of the trust network, a Steiner tree-based algorithm is proposed for the T-CLGF problem, which firstly select appropriate initiators for each group by considering the skill mastery and the strength of trust in the local trust network for potential promotion of members' skills, and then select followers for each group by searching locally in the trust network based on a greedy strategy. Through experiments on real-world datasets, the performance of the proposed approach are validated by comparing to several benchmark approaches, e.g., the graph partitioning-based algorithm, the simulated annealing-based algorithm and the trust-network constrained random algorithm.

The remainder of this paper is organized as follows. In Section II, we present the trust network-aware collaborative learning group formation (T-CLGF) problem. In Section III, we introduce the Steiner tree-based group formation algorithm for the T-CLGF. Then in Section IV, we present experimental results that validate our models. Finally, we conclude our paper and discuss the future work in Section V.

## 2 PROBLEM FORMULATION

We formalize the trust network-aware collaborative learning group formation problem in this section.

**Collaborative Learning Trust Network:** Let  $S = \{s_1, s_2, \dots, s_m\}$  be a set of skills. The collaborative learning trust network is defined as  $TN = (A, E, W)$ , where  $A = \{a_1, a_2, \dots, a_n\}$  is a finite set of learners in the collaborative learning system each element of which is associated with a vector of skill mastery  $P_i = \{p_i^1, p_i^2, \dots, p_i^m\}$  ( $\forall p_i^k \in P_i, p_i^k \in [0, 1]$ ), and  $E$  is two-element subsets of  $A$  called edges of  $TN$  representing the collection of trust between learners, in which  $e_{ij} \in E$  is associated with a weight  $w_{ij}$  ( $\forall w_{ij} \in W, w_{ij} \in (0, 1]$ ) representing the direct trust (obtained from direct experience of interaction (Granatyr et al., 2015)) between learners  $a_i$  and  $a_j$ .

The learners in the system will be divided into multiple groups in collaborative learning, i.e., the learner set  $A$  will be split into several subsets. The edges within a group is a subset of  $E$  representing the direct trust relationships among group members. It's worth noting that the trust from a learner to another in a group can be influenced by other group members in

collaborative learning process considering the effect of trust propagation and aggregation. The trust relationship, called *collaborative trust* actually can be built through the aggregation of the *direct trust* and *indirect trust* (Granatyr et al., 2015) assessed through paths along other group members.

**Collaborative Learning Group:**  $G_x = (A_{G_x}, T_{G_x})$ , where  $A_{G_x}$  represents the set of learners in the group  $G_x$ ,  $T_{G_x}$  represents the two-element subsets of  $A_{G_x}$  indicating the collaborative trust between group members within  $G_x$ .

Due to the effect of knowledge sharing and diffusion (Maleszka, 2019) (or group synergy) in the process of collaborative learning, group members usually have a positive trust tendency, that is, they tend to trust others through trust propagation. The collaborative trust from one learner to another in  $G_x$  can be assessed by trust propagation and aggregation with positive bias, which is defined as

$$trust_{G_x}(a_i, a_j) = \begin{cases} \max \prod_{e_{uv}} w_{uv}, & \forall \{e_{uv}\} \in \mathbb{P}(a_i, a_j) \\ & i \neq j \\ 0, & i = j \end{cases} \quad (1)$$

where  $\mathbb{P}(a_i, a_j)$  is the set of all the paths from learner  $a_i$  to  $a_j$  in group  $G_x$ ,  $e_{uv}$  is the set of edges representing direct trust along the path.

Completing a collaborative learning phase, the mastery level of a certain type of skill of each learner in a group has the opportunity to be improved because of the interactions among group members that will facilitate the knowledge and experience sharing. The improvement of skill mastery of learners can be mainly determined by the following three aspects: a) *Group cohesion*: collaborative trust of a group has a great influence on the cohesion of the group that reflects the communication efficiency and knowledge sharing opportunities within the group; group members are more likely to put in more effort in collaborative activities in a group with higher group cohesion. b) *Group skill mastery*: the improvement space of a learner is positively related to the gap between the skill mastery of the group and itself. The group skill mastery can be indicated by some experts holding highest skill mastery level in the group, since we take the CL scenario of hierarchical-structured skills in this paper (Zhang et al., 2017). c) *Learner trust on the group*: the extent of a learner's trust on the CL group determines the possibility of accepting specific knowledge concepts and improving the corresponding skill mastery. The last two factors are personalized for each group member, while the first factor is homogeneous for the group.

**Skill Mastery Improvement of Learners in CL:** the improvement of learner  $a_i$ 's mastery of skill  $s_k$  in

group  $G_x$  after a CL phase can be represented as

$$\text{imp}(a_i, G_x, s_k) = \text{Coh}_{G_x}(p_{G_x}^k - p_i^k) \text{trust}(a_i, G_x) \quad (2)$$

where  $\text{Coh}_{G_x} \in [0, 1]$  is the cohesion level of  $G_x$  and  $\text{trust}(a_i, G_x) \in [0, 1]$  is the trust of learner  $a_i$  on  $G_x$ .

**Expert-led CL (ELCL):** there are some members acting as experts for teaching or training in the group. For each skill there is a corresponding expert, that is, the group member with the highest skill mastery level in the group (a member can be the expert of multiple types of skill in a group). Meanwhile, expert members also act as learners, developing other types of skill through collaborative learning. Let  $\text{Exp}_x = \{\text{Exp}_x^1, \text{Exp}_x^2, \dots, \text{Exp}_x^m\}$  be a set of experts of group  $G_x$  for each type of skill where  $\text{Exp}_x \in G_x$ , and  $\text{Exp}_x^k$  be the expert of  $G_x$  on skill  $s_k$ : a) The group cohesion distinguished by skill type can be determined by the average of collaborative trust of learners to the expert of each type of skill, which is represented as  $\text{Coh}_{G_x}^k = \sum_{a_i \in G_x} \text{trust}_{G_x}(a_i, \text{Exp}_x^k) / (|A_{G_x}| - 1)$ ; b) According to the hierarchical-structured of skills, the group skill mastery of  $s_k$  is represented by the skill mastery of the expert  $\text{Exp}_x^k$ ,  $p_{G_x}^k = \max_{a_i \in G_x} \{p_i^k\}$ ,  $A_{G_x} \subseteq A$ . c) The learner trust of  $a_i$  on the group  $G_x$  of skill type  $s_k$  is also represented by the collaborative trust of  $a_i$  on the expert  $\text{Exp}_x^k$ ,  $\text{trust}(a_i, G_x) = \text{trust}_{G_x}(a_i, \text{Exp}_x^k)$ .

**Skill Mastery Improvement of Learners in ELCL:** the improvement of learner  $a_i$ 's mastery of skill  $s_k$  in group  $G_x$  after a expert-led collaborative learning can be represented as

$$\text{imp}(a_i, G_x, s_k) = \frac{\sum_{a_i \in G_x} \text{trust}_{G_x}(a_i, \text{Exp}_x^k)}{|A_{G_x}| - 1} \cdot (p_{G_x}^k - p_i^k) \cdot \text{trust}_{G_x}(a_i, \text{Exp}_x^k) \quad (3)$$

**Learning Utility of CL Groups:** the learning utility of a CL group  $G_x$  is the sum of the learning utility of group members  $A_{G_x}$ , which is represented as:

$$U_{G_x} = \sum_{a_i \in A_{G_x}} \sum_{s_k \in S} \text{imp}(a_i, G_x, s_k) \quad (4)$$

**Trust Network-aware Collaborative Learning Group Formation (T-CLGF) Problem:** given a collaborative learning trust network  $TN$  with  $|A| = d \cdot q$  for some integer  $q$ , finding a collection of non-overlapping groups  $G^* = \{G_1, G_2, \dots, G_d\}$  over  $TN$  with the optimal learning utility:

$$G^* = \arg \max_G \left\{ \sum_{G_x \in G} U_{G_x} \right\} \quad (5)$$

where for any  $G_i, G_j \in G$ , with  $i \neq j$ ,  $A_{G_i} \cap A_{G_j} = \emptyset$  (i.e., no learner is in more than one group),  $\cup_{G_i \in G} A_{G_i} = A$  (i.e., a learner is at least in one group),

and  $\forall G_i \in G, |A_{G_i}| = |A|/d$  where  $|A|$  and  $|A_{G_i}|$  indicate the number of learners in  $TN$  and group  $G_i$  respectively.

### 3 COLLABORATIVE LEARNING GROUP FORMATION ALGORITHM

We present a Steiner tree-based group formation algorithm for T-CLGF in this section, which follows the initiator-follower group formation architecture, i.e., firstly select appropriate agents from  $TN$  to form the initiator set for each CL group, and then select followers (i.e., the other group members) locally for each group in a greedy manner.

#### 3.1 Steiner Tree-based Initiator Assignment

In order to increase the group skill mastery of the skills in  $S$  and guarantee the collaborative trust levels as well as the connectivity of trust among the group members, the initiator assignment algorithm first constructs an enhanced trust network  $H$  covering all the skills in the skill set  $S$  according to the approach presented in (Lappas et al., 2009), and then builds the skill Steiner tree for each group considering the skill mastery of agents and the trust among them (each vertices of the skill Steiner tree is an agent in the set of initiators of a group). The Steiner tree-based initiator assignment is presented in Algorithm 1.

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Algorithm 1: InitiatorAssignment algorithm for the T-CLGF problem.

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**Input:** Trust network  $TN(A, E, W)$ ; the individuals' skill vectors  $P = \{P_1, \dots, P_n\}$ ; the skills' vector  $S = \{s_1, \dots, s_m\}$ ; the group number  $d$ ; the specified group size  $q$ .

**Output:** A collection of non-overlapping groups  $G = \{G_1, \dots, G_d\}$

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1  $H \leftarrow \text{EnhanceGraph}(TN, P, S)$ 
2 for  $i \leftarrow 1$  to  $d$  do
3    $X_H \leftarrow \text{SteinerTree}(H, X_0, q)$ 
4    $G_i \leftarrow X_H \setminus X_0$ 

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To build the enhanced trust network  $H$ , first, for each skill in  $S$  a skill node needs to be added to the trust network  $TN$ ; Then the *leaders* of each skill (learners with the highest skill mastery) should be connected to the skill node. Intuitively, for each skill,

the learner whose skill mastery ranks in the top  $d$  can be one of the leaders; in this case, the set of leaders in  $TN$  can be represented as  $I_{TN}$ . However, when constructing the skill Steiner tree, learners along the paths between leaders need to be added to ensure the connectivity of trust network among group members. Hence, under the constraint of the limited group size, the selection conditions of leader need to be relaxed.

#### Leader Selection Redundancy:

$$r = \frac{|S| - R_{TN}}{|S| - 1} \quad (6)$$

where  $S$  is the set of skills that can be mastered by learners,  $R_{TN}$  is the average number of skills mastered by the original leaders in  $I_{TN}$ .  $R_{TN}$  is represented as

$$R_{TN} = \frac{d|S|}{|I_{TN}|} \quad (7)$$

where  $d$  is the number of groups to be formed. Note that  $R_{TN} \in [1, |S|]$ , and the redundancy  $r \in [0, 1]$ ; the closer  $R_{TN}$  is to 1, the greater the cost of building Steiner tree, and the less Steiner tree that can cover all skill nodes in  $H$ .

We don't present the detailed algorithm for the enhanced trust network construction (*EnhanceGraph*) in initiator assignment due to space limitation. The main idea is based on the algorithm introduced in (Lappas et al., 2009). It is important to note that by considering the leader selection redundancy  $r$ , the number of leaders should be  $(1+r)*d$ .

**Distance Between a Learner and a CL Group** in  $H$ : the distance between a learner (or a node)  $a_i$  to a CL group represented as  $X'$  here is defined as

$$Dist(a_i, X') = \min_{a_j \in X'} \{Dist(a_i, a_j)\} \quad (8)$$

where  $Dist(a_i, a_j)$  is the distance between two learners  $a_i$  and  $a_j$ .  $Dist(a_i, a_j)$  can be represented as

$$Dist(a_i, a_j) = \min_{\{e_{uv}\}} \sum [1 - \frac{1}{2}(w_{uv} + w_{vu})], \quad (9)$$

$$\forall \{e_{uv}\} \in \mathbb{P}_{TN \setminus \cup_{G_x \in G} G_x}(a_i, a_j)$$

where  $e_{uv}$  is the set of edges representing direct trust along a path between  $a_u$  to  $a_v$ ,  $\mathbb{P}_{TN \setminus \cup_{G_x \in G} G_x}(a_i, a_j)$  is the set of all the feasible paths from learner  $a_i$  to  $a_j$ . Here a feasible path from learner  $a_i$  to  $a_j$  means the path does not contain any node in  $G_x$  except  $a_i$  and  $a_j$ .

On the basis of the greedy heuristic for Steiner tree (Takahashi et al., 1980), we propose a Steiner tree algorithm for the initiator assignment of T-CLGF problem, which is presented in Algorithm 2. The algorithm adds the skill nodes and the nodes along the paths between the skill nodes to the set  $X'$  in turn. Specifically, it chooses the skill node  $v^*$  with the minimum distance from the group  $Dist(v^*, X')$  (line 3)

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Algorithm 2: SteinerTree algorithm for the T-CLGF problem.

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**Input:** Enhanced trust network  $H$ ; the skill nodes  $X_0$ ; the specified group size  $q$ .

**Output:** Group  $X' \subseteq A$

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1  $X' \leftarrow v$ , where  $v$  is a random node from  $X_0$ 
2 while  $X_0 \setminus X' \neq \emptyset$  do
3    $v^* \leftarrow \arg \min_{u \in X_0 \setminus X'} Dist(u, X')$ 
4   if  $Path(v^*, X') \neq \emptyset$  and
      $|\{X' \cup Path(v^*, X')\} \setminus X_0| \leq q$  then
5      $X' \leftarrow X' \cup Path(v^*, X')$ 
6   else
7     break
8 if  $|X'| = 1$  then
9    $X' \leftarrow \{v'\}$ , where  $v'$  is an available leader
     connected with  $v$ 

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and the nodes along the path  $Path(v^*, X')$  achieving the minimum distance  $Dist(v^*, X')$  (line 4-5) in each round. If  $v^*$  does not exist or the number of members in  $X'$  exceeds  $q$  after joining the new nodes, the algorithm stops building the Steiner tree (line 6-7). Moreover, if there is only one skill node and no other nodes in the Steiner tree, an available leader of any skill is returned (line 8-9).

### 3.2 Greedy Follower Selection

In order to guarantee the learning utility of each group while ensuring the connectivity of trust among group members, the follower selection algorithm selects members for each group in turn by searching the local environments of the initiators in  $TN$  in a greedy manner until all the agents in  $TN$  attend in the groups, which is presented in Algorithm 3.

**Neighbors of the CL Group:** the set of neighbors of a CL group  $G_x$  is represented as

$$N(G_x) = \{a_i | a_i \in A \setminus G_x, \exists a_j \in A_{G_x}, e_{ij} \in E\} \quad (10)$$

where  $a_i$  is a neighbor of  $G_x$  and  $a_j$  is a learner in  $G_x$ .

Because of the different size of Steiner trees formed in the initiator assignment step, the groups are first aligned through greedy selection (line 1-10). The algorithm sorts the groups in ascending order according to the number of neighbors, and the group with fewest neighbors choose follower first. After this operation, all the groups  $G_x \in G$  become the same size. Then, each group takes turns to select members through greedy selection (line 11-23). Each group can only greedily choose one follower from neighbors in each round; and the order of each round of selection is determined by the number of neighbors of each group,

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Algorithm 3: GreedySelection algorithm for the T-CLGF problem.

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**Input:** Trust network  $TN(A, E, W)$ ; the individuals' skill vectors  
 $P = \{P_1, \dots, P_n\}$ ; the skills' vector  
 $S = \{s_1, \dots, s_m\}$ ; the group number  $d$ ;  
the specified group size  $q$ .

**Output:** A collection of non-overlapping groups  $G = \{G_1, \dots, G_d\}$

- 1  $maxSize \leftarrow \max_{G_i \in G} |A_{G_i}|$
- 2 sort  $G$  by the number of group's neighbors in ascending order
- 3 **for**  $i \leftarrow 1$  to  $d$  **do**
- 4      $diff \leftarrow maxSize - |A_{G_i}|$
- 5     **for**  $j \leftarrow 1$  to  $diff$  **do**
- 6          $V \leftarrow \{u | u \in N(G_i), \forall G_x \in G, G_x \cap u = \emptyset\}$
- 7         **if**  $V = \emptyset$  **then**
- 8             **break**
- 9          $v^* \leftarrow \arg \max_{u \in V} U(G_i \cup u)$
- 10          $G_i \leftarrow G_i \cup v^*$
- 11 **while** **true** **do**
- 12      $count \leftarrow 0$
- 13     sort  $G$  by the number of group's neighbors in ascending order
- 14     **for**  $i \leftarrow 1$  to  $d$  **do**
- 15         **if**  $|A_{G_i}| \geq q$  **then**
- 16             **continue**
- 17          $V \leftarrow \{u | u \in N(G_i), \forall G_x \in G, G_x \cap u = \emptyset\}$
- 18         **if**  $V \neq \emptyset$  **then**
- 19              $v^* \leftarrow \arg \max_{u \in V} U(G_i \cup u)$
- 20              $G_i \leftarrow G_i \cup v^*$
- 21              $count \leftarrow count + 1$
- 22     **if**  $count = 0$  **then**
- 23         **break**
- 24 **for**  $i \leftarrow 1$  to  $d$  **do**
- 25     **while**  $|A_{G_i}| < q$  **do**
- 26          $V \leftarrow \{u | u \in A, \forall G_x \in G, G_x \cap u = \emptyset\}$
- 27          $v^* \leftarrow \arg \max_{u \in V} U(G_i \cup u)$
- 28          $G_i \leftarrow G_i \cup v^*$

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the group with fewest neighbors choose follower first. Finally, due to the constraints of the trust network, some groups may not have available neighbors when the group size is smaller than the specified value  $q$ . For these groups, the algorithm selects the learners

who bring the highest learning utilities for each group from the remaining available learners in  $TN$  (line 24-28).

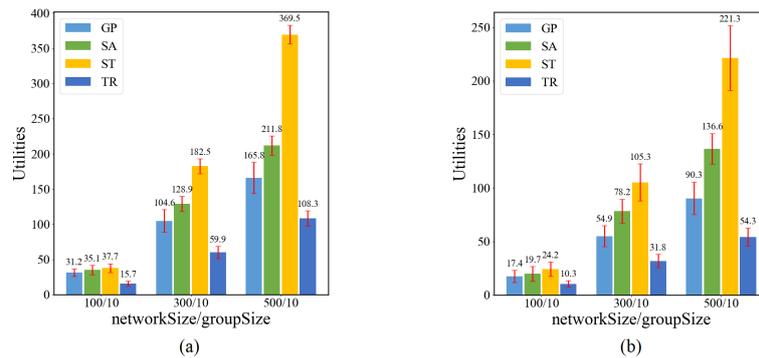
## 4 EXPERIMENTS AND ANALYSES

### 4.1 Experimental Settings

There are two real-world datasets used in the experiments: 1) *the research institution membership network* (Yin et al., 2017)(Leskovec et al., 2007) with 1005 nodes and 68430 edges (marked as  $N1$ ). The edges in the original dataset consist of the mail transfer relationships between members and the departmental colleague relations. We assume that there are interactions between nodes that have any of these relationships, i.e., the members have connections if and only if there is a mail transfer relationship between them or they belong to the same department. Since members of each department are interconnected with each other and the departments are connected by e-mail relations, this network has obvious community structure. 2) *the student friendship network* (Sapiezynski et al., 2019) with 851 nodes and 12834 edges. The edges are friendship relations among students collected from Facebook (marked as  $N2$ ). We also conduct experiments on the random network (Bollobás and Béla, 2001), and the small world network (Watts and Strogatz, 1998), investigating the effect of network density on the performance of the presented algorithm. In the experiments, the edge weights are generated randomly range from 0 to 1, and the number of skills is set to 5. The initial skill mastery of the learner is generated by the beta (4, 4) distribution, which is a continuous probability distribution defined in the interval (0, 1), and the expectation of beta (4, 4) is 0.5.

We evaluate the performance of the presented Steiner tree-based (ST) group formation algorithm compared with the following benchmark approaches.

- **Graph partitioning-based (GP) algorithm** (Karypis, 1997; METIS, 2021; Karypis and Kumar, 1998b): the GP algorithm is able to split a graph into several connected components. Let agents in  $TN$  be the vertices of a graph, and the direct trust relationships in  $TN$  be the edges, the GP algorithm can be used to solve the collaborative learning group formation problem. This algorithm can ensure the connectivity of each collaborative learning group, considering that the connectivity of a group has a great impact

Figure 1: Learning utilities of CL groups in the network  $N1$ (a) and  $N2$ (b).Table 1: Running time of algorithms in the network  $N1$  and  $N2$  (s).

Network $N1$				
Network/Group Size	GP	SA	ST	TR
100/10	$0.054 \pm 6.960e-4$	$34.679 \pm 13.925$	$0.738 \pm 5.0069e-2$	$0.0578 \pm 2.7423e-3$
300/10	$0.1759 \pm 1.700e-3$	$48.6867 \pm 16.6237$	$13.319 \pm 0.5750$	$0.24055 \pm 6.3046e-3$
500/10	$0.30996 \pm 1.6323e-3$	$53.9094 \pm 12.5364$	$57.401 \pm 2.2445$	$0.50464 \pm 1.0701e-2$
Network $N2$				
Network/Group Size	GP	SA	ST	TR
100/10	$0.05322 \pm 6.09593e-4$	$81.2871 \pm 36.2385$	$0.45856 \pm 7.1254e-2$	$0.05229 \pm 2.2329e-3$
300/10	$0.16927 \pm 5.6813e-3$	$150.389 \pm 39.3495$	$6.77763 \pm 0.9683$	$0.1988 \pm 6.3734e-3$
500/10	$0.29062 \pm 1.4750e-3$	$187.978 \pm 50.2311$	$28.4208 \pm 3.6787$	$0.38074 \pm 5.8080e-3$

on the learning utility of the group. The tool used in this article is METIS (Karypis, 1997; METIS, 2021). METIS realizes both the multi-level k-way division (Karypis and Kumar, 1998b) and the multi-level recursive division (Karypis and Kumar, 1998a). Since METIS with multi-level k-way division can guarantee the connectivity of subgraphs, we adopt this tool for the T-CLGF problem.

- **Simulated Annealing-based (SA) algorithm** (Keinänen, 2009): the SA group formation algorithm is a representative heuristic algorithm which can be applied to large-scale group formation problems in multi-agent systems. The SA algorithm starts by randomly selects a feasible group structure  $G$ , and then generates a neighbour group structure  $G'$  of  $G$  at each iteration. If the utility of the newly generated group structure  $G'$  is better than  $G$ ,  $G'$  will be adopted; otherwise, there is still a certain probability that  $G'$  will be adopted, which is controlled by a temperature parameter that will decrease after each iteration. Through a series of experiments of SA, we use 1 as the initial temperature and 0.99 as the annealing rate in the following experiments
- **Trust-network constrained random (TR) algorithm**: the random group formation algorithm can naturally achieve the heterogeneity of groups to some extent, i.e., it can guarantee the group learn-

ing utilities to a certain extent, with a low computation complexity. The random group formation algorithm is extended to match the scenario of the collaborative learning trust network considered in this paper. Firstly, it randomly selects the initial agents for all the groups; then it continues to randomly select agents to join the groups from the neighbor agents who have direct trust relationships with the group members.

## 4.2 Results and Analyses

The experiments are conducted on the research institution membership network  $N1$  (Yin et al., 2017)(Leskovec et al., 2007) and the student friendship network  $N2$  (Sapiezynski et al., 2019). For each simulation with different network sizes (100, 300 and 500), the agents and the network among them are generated from the original network randomly, and the group size is set to 10, i.e., each group has 10 members. For the research institution membership network  $N1$ , the mastery of each skill an agent mastered is generated by the beta (4, 4) distribution and is independent of each other. For the student friendship network  $N2$ , it is assumed that there is a correlation between the mastery of skills for a student in such a learning environment in a university. For example, for a student with a high cognitive level, he/she may have a high mastery of the skills to be

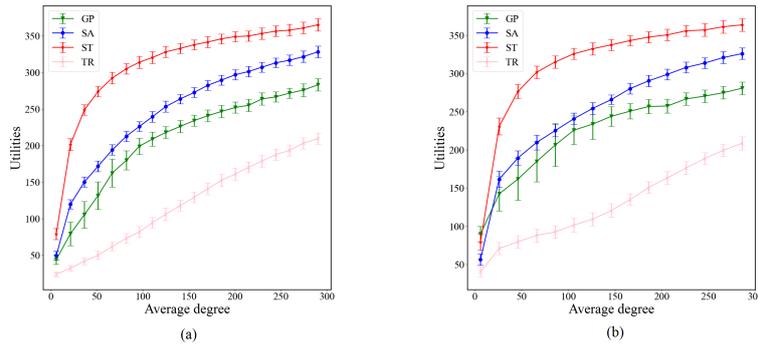


Figure 2: Learning utilities of CL groups in the ER network and the small-world network.

mastered, and vice versa. Hence, for  $N2$ , this paper generates a mastery baseline  $mb_i$  for each student through the distribution of  $\text{beta}(4, 4)$ , and the mastery of each skill is randomly generated from  $[\max\{0, mb_i - 0.1\}, \min\{mb_i + 0.1, 1\}]$ .

Fig.1 and Table 2 show the results of learning utilities and average running time of the algorithms in networks  $N1$  and  $N2$ , respectively. From the results, it can be observed that the learning utility of the ST algorithm is significantly better than the benchmarks. The main reason is that the ST algorithm can divide the agent set in the network hierarchically and allocate the agents into groups more properly, and it can also guarantee the network connectivity among members in the groups. In addition, it can be observed that the performance advantage of the ST algorithm increases with the increase of network scale (the number of agents increases from 100 to 300 and then to 500). The learning utility of the SA algorithm is inferior to the ST algorithm but better than the GP and TR algorithms. Note that, the ST algorithm is much faster than the SA algorithm in most cases, but in some special situations such as the cases where the network size is 500, the SA algorithm may take less running time, the potential reason is that the neighbor partition can be found quickly in each iteration and the number of iterations is limited in large-scale dense networks like  $N1$  and  $N2$ . The learning utilities obtained by the GP and TR algorithms are significantly worse than that of the ST and SA algorithms but with lower running time; the main reason is that the GP algorithm ignores the member attributes in group formation, while the TR algorithm only performs local search of the trust networks. Moreover, it can be found that the ST algorithm performs well in both the situation where the mastery of each skill an agent mastered is independent of each other ( $N1$ ) and the situation where there is a correlation between the mastery of skills for an agent ( $N2$ ). Therefore, it can be concluded that the ST algorithm has significant advantages in the collaborative learning utility compared with the benchmarks

in real world networks  $N1$  and  $N2$ , and the advantage is increasing with the increase of the network scale.

Moreover, in order to investigate the influence of network density on the algorithm performance, experiments on the ER random network (Bollobás and Béla, 2001) and the WS small-world network (Watts and Strogatz, 1998) are conducted and the results are shown in Fig. 2. The trust networks generated consist of 300 agents, and the group size is set to 10; the mastery of each skill an agent mastered is generated by the  $\text{beta}(4, 4)$  distribution and is independent of each other. From the results, the ST algorithm can obtain the highest learning utility of the system for different settings of network average degree; the performance of the SA algorithm is superior to the GP and TR algorithms in most cases; and the TR algorithm get the lowest cooperative learning utility of the system. When the average degree of network is low, the performance gap between these algorithms is small; but as the network size increases, the performance gap between the algorithms increases significantly.

## 5 CONCLUSIONS

This paper first formalizes the collaborative learning group formation problem in trust networks (T-CLGF), considering the influence of trust on not only the incentive of learners collaborating in a group but also the promotion of skills of agents in knowledge sharing; and this paper then proposes a Steiner tree-based group formation algorithm to solve the T-CLGF problem, which first allocates appropriate agents to groups as initiators by considering the skill mastery and the strength of trust in the groups to guarantee the opportunities for skill promotion and then select followers by searching locally in the trust network.

We validate the performance of the proposed algorithm by comparing with the graph partitioning-based algorithm, the simulated annealing-based algorithm and the trust-network constrained random algorithm

through experiments. From the results, it can be concluded that our algorithm has significant advantages in the learning utility of the system compared with the benchmark algorithms and has a practical running time. Moreover, considering some other collaborative learning scenarios, e.g., the non-expert led scenarios, we plan to extend the group formation problem and our algorithm to these scenarios in our future work.

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