

Improved Predictive Fundamental Period Formula for Reinforced Concrete Structures through the Use of Machine Learning Algorithms

Nathan Carstens¹, George Markou¹ and Nikolaos Bakas²
¹*Department of Civil Engineering, University of Pretoria, South Africa*
²*Department of RnD, RDC Informatics, Athens, Greece*

Keywords: Machine Learning Algorithms, Fundamental Mode Formulae, Modal Analysis, Soil-structure Interaction, Finite Element Method, Reinforced Concrete, Hybrid Modelling.

Abstract: With the development of technology and building materials, the world is moving towards creating a better and safer environment. One of the main challenges for reinforced concrete structures is the capability to withstand the seismic loads produced by earthquake excitations, through using the fundamental period of the structure. However, it is well documented that the current design formulae fail to predict the natural frequency of the considered structures due to their inability to incorporate the soil-structure interaction and other features of the structures. This research work extends a dataset containing 475 modal analysis results developed through a previous research work. The extended dataset was then used to develop three predictive fundamental period formulae using a machine learning algorithm that utilizes a higher-order, nonlinear regression modelling framework. The predictive formulae were validated with 60 out-of-sample modal analysis results. The numerical findings concluded that the fundamental period formulae proposed in this study possess superior prediction ability, compared to all other international proposed formulae, for the under-studied types of buildings.

1 INTRODUCTION

The soil-structure interaction (SSI) phenomenon is a typical structural and geotechnical engineering issue, still open regarding its practical applications. Further investigation is required to develop simplified but reliable methods to account for such a phenomenon in routine structural analyses (Ceroni et al., 2012). In calculating the appropriate seismic loads, the fundamental period serves as one of the most critical dynamic characteristics. In the event of a seismic excitation, the interaction between the superstructure (building) and substructure (soil) becomes critical as it commences to alter the distribution of stresses and strains within the superstructure, which alters the expected results (Mourlas et al., 2019).

It is well known that computing the fundamental mode of fixed-base structures through design code formulae has its challenges (Mourlas et al., 2019). Furthermore, some shortcomings exist in the stiffness distribution of the structure due to a lack of adequate consideration of the effects of shear walls, especially in the Eurocode 8 design code (Gravett et al., 2019).

These considerations can cause a considerable amount of over or under designing of reinforced concrete (RC) structures, which can lead to inadequate designs liable to seismic conditions. Thus, it is crucial to establish a design tool that can successfully predict the dynamic properties of a variety of different RC structures.

It is usually not in favour of safety to analyse the response of a fixed-base structure by neglecting the SSI effect. In some cases, codes provide seismic design provisions by reducing the base shear of the fixed-base structures. In others, they suggest performing advanced analysis to investigate the overall effect (Mourlas et al., 2020). As a result, there is a need for more accurate design expressions for RC structures that can accurately predict their fundamental period while accounting for SSI effects.

When it comes to the SSI effect, the reaction of a building to a seismic event is evaluated in conjunction with the compressibility of its surrounding soil. The flexibility of the soil can impact its stress distribution and displacement profiles, which can be distinguished from standard fixed-base systems (Saadi, 2018, Markou et al., 2018).

A study conducted by Gravett et al. (2019) determined that the current design code formula assumes that all reinforced concrete structures have a fixed base, resulting in errors of up to 85% for international codes such as Eurocode 8. Upon further investigation, it was concluded that a RC structure's dynamic response could be significantly affected by SSI and stiffness redistribution when susceptible to seismic activity (Mourlas et al., 2019). See Table 1 below for a few design code formulae found in the international literature.

Table 1: International design codes in practice.

Relevant Code	Formulae
NEAK (New Greek Antiseismic Code)	$T_{NEAK} = 0.09 \frac{H}{\sqrt{L}} \sqrt{\frac{H}{H + \rho L}}$
Old Cyprus Code	$T_{Cyprius} = \frac{N}{10}$
Eurocode	$T_{EC} = C_T H^{0.75}$

In order to conduct this research report, the application of finite element modelling, using advanced modelling software, was utilised to construct models representing various RC structures. The finite element method (FEM) is frequently used in computing engineering, and mathematical models, the FEM allows for the numerical solution of differential equations.

For this research project, the constructed FEM models were analysed using modal analysis to get results that would help the researcher identify and understand the dynamic response of the various RC structures. Eigen-value problems are common in engineering. The parameter calculates the fundamental periods of a structural system (Felippa, 2004). The solution method used for this research project is called the subspace iteration algorithm (Bathe et al., 1980). The solution is ideally suited for large-scale structures.

By utilising HYMOD (Markou et al., 2015), one can decrease the computational demands of the numerical model, allowing us to perform any type of analysis of various RC buildings at full-scale. With Reconan FEA (2020) software, analyses are performed to capture the complete nonlinear structural response, either with or without the SSI effect., while this was software used to perform the modal analyses. It must be noted here that the modal algorithm of Reconan FEA was validated through numerous experimental data.

Based on the procedure described in Taljaard et al., 2021 and Gravett et al., 2021, developing a dataset through the use of 3D detailed modeling and then using machine learning (ML) algorithms to develop closed form solutions can be a very powerful tool in developing new fundamental period formulae. Therefore, the objective of this research work is to extend the initial dataset developed by Gravett et al., 2021, and use the extended dataset in developing a more accurate fundamental period formula.

2 MACHINE LEARNING

This research work used the Julia ML framework. Similar to Python, this is an open-source, high-level language for dynamic programming. A mathematical model in ML is designed to develop generalised relationships between independent and dependent variables due to their nonlinear characteristic. As stated above, the focus of this research work is developing software generated data that is used to train ML algorithms to determine the fundamental period of RC structures.

Table 2, shows the high-order nonlinear regression algorithm that was used in this research work to develop the improved formulae. This algorithm was adopted from Gravett et al., 2021.

Table 2: Higher-Order Nonlinear Regression Algorithm (Gravett et al., 2021).

Input:	XX (matrix of Independent Variables), YY (Vector of Dependent Variable), nlf (number of nonlinear features to be kept in the model)
Output:	Prediction Formulae <ol style="list-style-type: none"> 1. Create all nonlinear features ($anlf$) 2. For i from 1 to nlf, do: 3. For j from 1 to $anlf$, do: 4. Add j^{th} feature to the model 5. Calculate Prediction Error, $MAPE_j$ 6. END 7. Keep in the model the j^{th} feature which yields the minimum prediction error 8. END
Return:	Prediction Formula

3 NUMERICAL CAMPAIGN

3.1 Database Development

In order to construct the extended database that would consist of various RC structures, various geometrical

parameters of the initial model were modified. These parameters include the height of the structures, base conditions, stiffness distribution throughout the structure, and the plan area of the structure. Figures 1 and 2 show different RC building models that were created for the needs of the dataset development.

With all the models created and the parameters for each model determined, the eigenfrequencies were determined, and all the data were stored in an Excel spreadsheet. Table 3 summarises the minimum and maximum geometrical properties for the models that were adjusted in this research work.

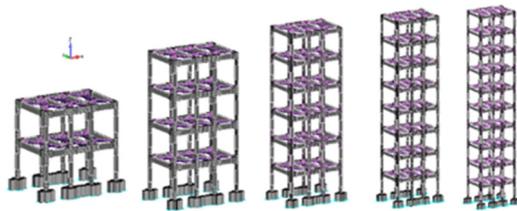


Figure 1: Different model geometries.

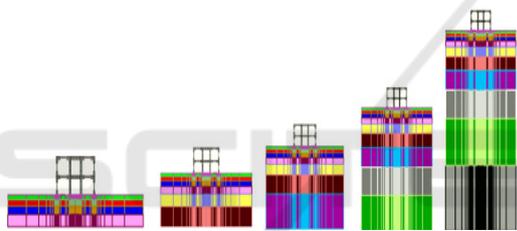


Figure 2: Different model base conditions.

Table 3: Minimum and maximum values of the newly obtained HYMOD meshes.

Variables	Minimum	Maximum
Soil Depth (m)	1	60
Soil E (kPa)	65,000	700000
H (m)	3	30
L (m)	3.4	34.4
B (m)	3.4	34.4
ρ (%)	0	85.29

3.2 Modal Analyses

For each of the numerical models, a modal analysis was performed to determine the eigenfrequencies of each model. For each model, only the two translational modes were used to construct the dataset. Translation oscillations along the global x- and y-axis directions.

Figure 3 shows the effect of shear walls on the computed fundamental period of the under-study RC structures. It is evident that when shear walls are

added to the RC buildings they significantly lower the fundamental period of the structure. Figure 4 shows the relationship of the period of a structure and how it is affected by the soil depth. It is easy to observe that the SSI effect reaches a plateau as the depth increases.

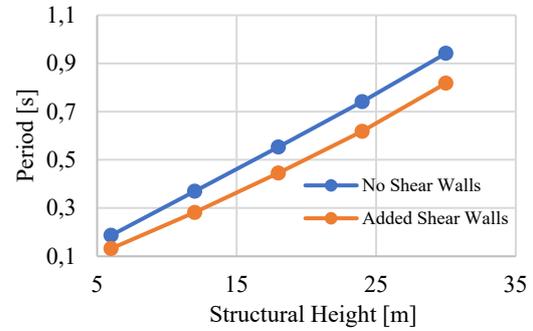


Figure 3: Shear wall effect on fundamental period.

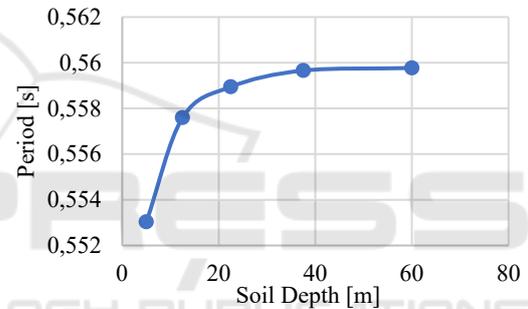


Figure 4: Soil depth effect on the fundamental period.

3.3 Proposed Fundamental Period Formulae

The ML algorithm is designed to determine the number of features used within the design formulae. For this research work, a design formula with 3, 5, and 20 features were developed and parametrically investigated. It must be noted here that the total number of fundamental period results used in the dataset to train and test was 790. Each fundamental period formula is constructed through the use of the following variables:

- H** the building's height (m)
- ρ** the percentage shear walls (%)
- E_s** the soils' modulus of elasticity (kPa)
- L** the length of building parallel to the oscillating direction (m)
- B** the width of the building perpendicular to the oscillating direction (m)
- D** the soil depth (m)

3.3.1 3-Feature Formula

It should be noted that the three feature formulae do not consider any SSI parameters. However, this formula still yielded an absolute mean error of 3.43%. The relationship can be seen in Eq. 1.

$$T = (0.0310197 \cdot H) - (0.00011254 \cdot \rho \cdot H) + (0.0000129093 \cdot H \cdot B^2) + 0.0110165 \quad (1)$$

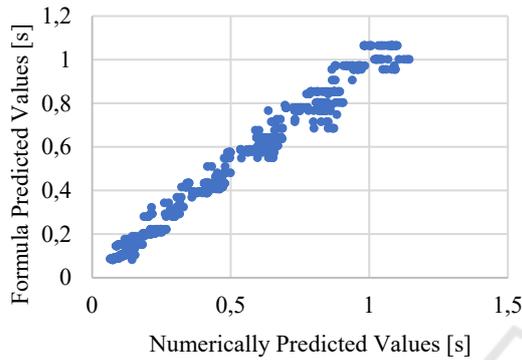


Figure 5: 3-Feature formula prediction vs numerically predicted results.

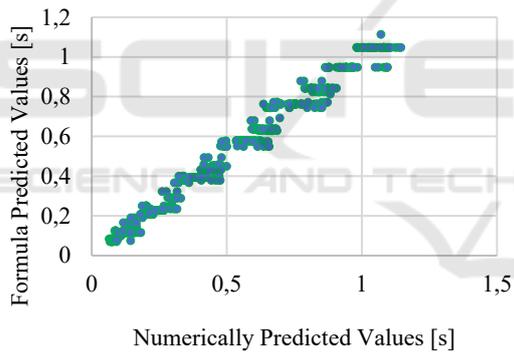


Figure 6: 5-Feature formula prediction vs numerically predicted results.

3.3.2 5-Feature Formula

Eq. 2 shows the 5-feature formula as it derived from the training and testing of the extended dataset. The absolute mean error for the 5-feature formula was calculated as 2.70%, which is more accurate compared to the 3-feature formula. This is attributed to the inclusion of additional parameters that affect the final predictions.

$$T = (0.0296602 \cdot H) - (0.000154717 \cdot \rho \cdot H) + (0.0000210854 \cdot L \cdot B \cdot H) + (0.0000042983 \cdot H^2 \cdot \rho) - (0.00000785228 \cdot \rho \cdot B \cdot H) + 0.0229 \quad (2)$$

3.3.3 20-Feature Formula

Finally, the most accurate formula is presented in Eq. 3. The absolute mean error of the 20-feature formula was calculated as 1.49%. It is evident that the use of SSI related parameters in this relationship, makes this formula the most accurate when used on the training and testing datasets. Fig. 7 shows the comparison between the predictions derived from the proposed formula and the numerical results.

$$T = (0.0292939 \cdot H) - (0.000150825 \cdot \rho \cdot H) + (0.00000582242 \cdot H \cdot B^2) + (0.00000330369 \cdot \rho \cdot H^2) + (0.000215881 \cdot H \cdot L) - (1.89375 \times 10^{-15} \cdot E_s^2 \cdot D) + (0.00000323855 \cdot L \cdot H \cdot D) - (0.00000646154 \cdot \rho \cdot B \cdot H) - (0.000000000925478 \cdot H \cdot E_s \cdot D) - (0.000000000406192 \cdot \rho \cdot E_s \cdot D) + (0.000000194394 \cdot D \cdot \rho^2) + (0.0037148 \cdot B) + (0.000000358861 \cdot \rho \cdot H \cdot D) + (0.000000000662381 \cdot E_s \cdot D^2) - (0.000000278639 \cdot D^3) - (0.000000000113737 \cdot L \cdot E_s \cdot D) - (0.0000016727 \cdot B^3) + (0.0000309934 \cdot L \cdot D) - (0.00178654 \cdot L) + (0.000000645744 \cdot L^3) + 0.00239996 \quad (3)$$

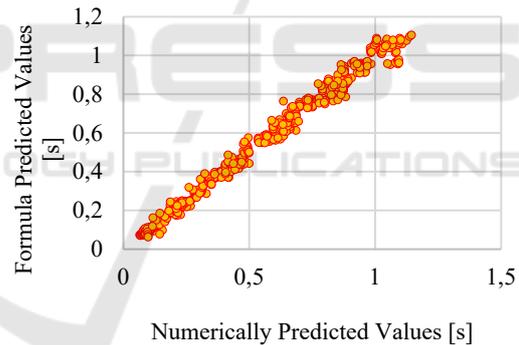


Figure 7: 20-Feature formula prediction vs numerically predicted results.

4 VALIDATIONS OF RESULTS

To further test the ability of the proposed formulae to predict the fundamental period of RC structures, a dataset was developed for validation purposes. For this reason, 60 out-of-sample building models were constructed and used to further validate the ability of the proposed formulae in predicting the fundamental period of RC buildings with and without SSI effects.

From Figures 8 - 10 it is evident that comparing the proposed formulae manage to predict the out-of-sample data with high accuracy. The most significant improvement was seen with the 3-feature proposed

formula that improved with 4.19% from the results obtained by Gravett et al., (2021). The 20-feature formula also had a significant improvement of 0.22%.

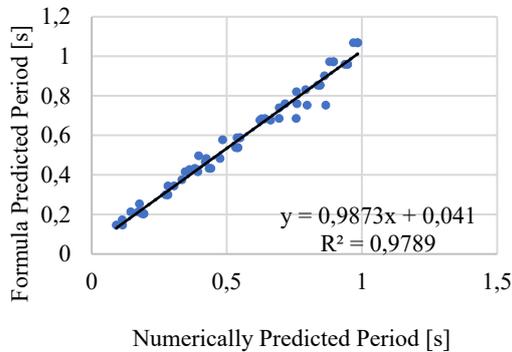


Figure 8: Correlation of 3-feature formula on validation dataset.

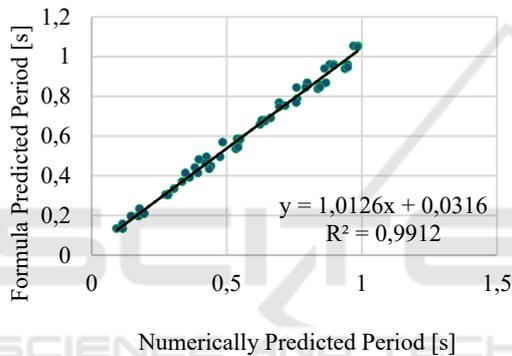


Figure 9: Correlation of 5-feature formula on validation dataset.

5 CONCLUSIONS AND FUTURE WORK

790 fundamental period results were used to train an ML algorithm and develop accurate design formulae to calculate the fundamental period of RC structures. The three proposed formulae were then tested with out-of-sample data comprising 60 new RC models constructed in a manner that foresaw the use of new parameters compared to the models used to train and test the formulae. This served as the validation phase in which the design formulae showcased a high degree of correlation, effectively proving their accuracy and extendibility.

According to the numerical investigation, the most accurate proposed formula on train, test and validation data was the 20-feature, which was found to be improved compared to the one proposed by Gravett et al., 2021.

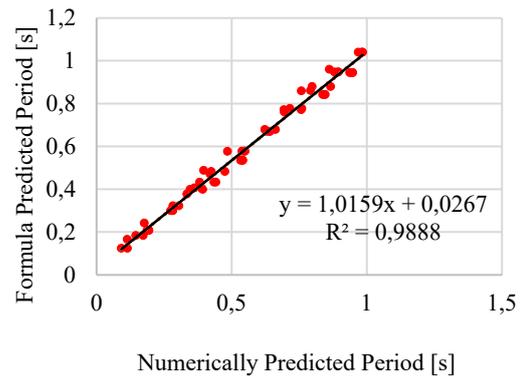


Figure 10: Correlation of 20-feature formula on validation dataset

Finally, this research work will foresee a further dataset extension and also take into account the infill walls of RC buildings. The asymmetry of buildings should also be investigated in the near future and how that affects the fundamental period of RC structures when the SSI effect is accounted for.

REFERENCES

- Bathe, K.J. and Ramaswamy, S., 1980. An accelerated subspace iteration method. *Computer Methods in Applied Mechanics and Engineering*, Vol 23, No 3, pp. 313-331.
- Ceroni, F., Sica, S., Pecce, M. and Garofano, A., 2012. Effect of Soil-Structure Interaction on the dynamic behaviour of masonry and RC buildings. *Proceedings of the 15th World Conference on Earthquake Engineering (WCEE)*, p. 10.
- Felippa, C., 2004. *Introduction to finite element methods*, University of Colorado., p. 885.
- Gravett, D.Z., Mourlas, C., Markou, G. and Papadrakakis, M., 2019. Numerical performance of a new algorithm for performing modal analysis of full-scale reinforced concrete structures that are discretised with the HYMOD approach, *COMPADYN 2019, 7th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 24-26 June 2019, Crete, Greece..
- Gravett, Z.D., Mourlas, C., Taljaard V.L., Bakas P.N., Markou, G. and Papadrakakis, M. (2021), *New Fundamental Period Formulae for Soil-Reinforced Concrete Structures Interaction Using Machine Learning Algorithms and ANNs*, *Soil Dynamics and Earthquake Engineering*, 144 (2021), 106656.
- Markou, G., Sabouni, R., Suleiman, F. and El-Chouli, R., 2015. Full-Scale Modeling of the Soil-Structure Interaction Problem Through the Use of Hybrid Models (HYMOD), *International Journal of Current Engineering and Technology*, Vol 5, No2, p. 15.
- Markou, G., AlHamaydeh, M. and Saadi, D., 2018. Effects of the soil-structure-interaction phenomenon on RC

- structures with pile foundations. Proceedings of the 9th GRACM International Congress on Computational Mechanics.
- Mourlas, C., Gravett, D.Z., Markou, G. and Papadrakakis, M., 2019. Investigation of the soil-structure interaction effect on the dynamic behaviour of multi-storey RC buildings. VIII international conference on computational methods for coupled problems in science and engineering.
- Mourlas, C., Khabele, N., Bark, H.A., Karamitros, D., Taddei, F., Markou, G. and Papadrakakis, M., 2020. The Effect of Soil-Structure Interaction on the Nonlinear Dynamic Response of Reinforced Concrete Structures. International Journal of Structural Stability and Dynamics, Vol 20, No 13), p. 39.
- Reconan FEA v2.00, User's Manual. 2020.
https://www.researchgate.net/publication/342361609_Reconan_v200_Finite_Element_Analysis_Software_User's_Manual
- Saadi, D., 2018. Nonlinear FEA of Soil-Structure-Interaction Effects on RC Shear-Wall Structures. PhD thesis. American University of Sharjah.
- Taljaard, V.-L., Gravett, D.Z., Mourlas, C., Bakas, N., Markou, G. and Papadrakakis, M., "Development of a New Fundamental Period Formula by Considering Soil-Structure Interaction with the Use of Machine Learning Algorithms", COMPDYN 2021, 27-30 June 2021, Streamed from Athens, Greece.

