

# Development of a New Fundamental Period Formula for Steel Structures Considering the Soil-structure Interaction with the Use of Machine Learning Algorithms

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**Abstract:** The fundamental period of buildings is an important parameter when designing seismic resistant structures. The current formulae proposed in design codes for determining the fundamental period of steel structures cannot accurately predict the fundamental period of real structures. In addition, most of the current formulae only consider the height of the structure in their formulation, while soil structure interaction (SSI) and the orientation of the I-columns that influence the fundamental period are usually neglected. This research focuses on the use of machine learning algorithms to obtain a new formula that accounts for different geometrical features of the superstructure, where the SSI effect is also considered. After training and testing a 40-feature formula, an additional 138 out-of-sample numerical results were used to further test the accuracy of the proposed formula's prediction abilities. The validation resulted in a correlation of 99.71%, which suggests that the proposed formula exhibits high predictive features for the steel structures considered in this study.

## 1 INTRODUCTION

An important structural feature related to the dynamic response of a structure is the fundamental period (Young, 2011). Current building codes use empirical equations to predict the fundamental period of structures (Jiang et al., 2020, Taljaard et al., 2021 and Gravett et al., 2021). For determining the fundamental period of buildings, current international codes have oversimplified formulae as they require only the height of the structure and do not account for the actual 3D geometry of the building nor account for the interaction between the superstructure and substructure.

The following design formulae are currently used in the estimation of the fundamental period of steel structures:

$$\text{EC8: } T_1 = C_t(H)^{0.75} \quad (1)$$

Where:

$C_t = 0.085$  for moment resistant space steel frames

$C_t = 0.075$  for eccentrically braced steel frames

$$\text{ASCE 7-05: } T_1 = 0.0724(H)^{0.8} \text{ for steel moment-resisting frames} \quad (2)$$

$$T_1 = 0.0731(H)^{0.75} \text{ for eccentrically braced steel frames} \quad (3)$$

Another formula proposed by Cinitha (2012) also takes into account the plan area of the building ( $L \times B$ ) and can be seen below:

$$T_1 = C_0(L \cdot B)^{0.3289-\alpha} \quad (4)$$

With

$$C_0 = 0.0247e^{0.1305 \cdot H} \quad (5)$$

$$\alpha = 0.4473e^{-0.0441 \cdot H} \quad (6)$$

Another work was also presented by Nassani, 2014, where a simple model for calculating the fundamental period of vibration in steel structures was presented. The proposed formulae in the aforementioned research works do not consider the SSI effect, therefore, the development of a formula that will be able to account this important feature is required. The phenomenon of SSI involves a multidisciplinary field of structural mechanics, soil mechanics and structural dynamics (Jayalekshmi and Chinmayi, 2013 and Gravett et al., 2021). It has been

found that the SSI can increase the fundamental period, thus is an important consideration when determining the fundamental period of a structure (Khalil et al., 2007 and Mourlas et al., 2020).

According to the research gap discussed in this section, the objective of this research work is to develop a new formula for predicting the fundamental period of steel structures that considers the SSI effect. Additionally, the proposed formula considers other parameters such as the base conditions and orientation of the I-columns. A total of 576 numerical models using Reconan FEA (2020) were created to obtain a dataset containing 1,152 numerical results. The dataset is used to train a machine learning algorithm to formulate a 40-feature formula, using a higher order NLR model, which was then validated through the use of out-of-sample data. It is important to note here that the ability of Reconan FEA to predict the fundamental period of structures was validated through the use of experimental data found in the international literature (Mourlas et al., 2019 and Mourlas et al., 2021).

## 2 MACHINE LEARNING

There are 18 independent variables used in this research work to train the machine learning algorithm. These include the initial parameters such as soil depth, Young's Modulus of soil, height, length and width of the superstructure, and the orientation of the I-columns. The modified parameters added during the training procedure to improve the predictability of the developed closed form solution included  $\ln(\text{parameter} + 1)$  and  $\frac{1}{\text{parameter}+1}$ .

Algorithm 1: Higher Order Regression.

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Input:  $XX$  (matrix of Independent Variables),  $YY$  (Vector of Dependent Variable),  
 $nlf$  (number of nonlinear features to be kept in the model)  
 Output: Prediction Formulae

1. Create all nonlinear features\* ( $anlf$ )
2. For  $i$  from 1 to  $nlf$  do
3.   For  $j$  from 1 to  $anlf$  do
4.     Add  $j^{th}$  feature to the model
5.     Calculate Prediction Error,  $MAPE_j^{**}$
6.   End
7.   Keep in the model the  $j^{th}$  feature which yields the minimum prediction error

5. End  
 Return: Prediction Formula

\*with all inter-items combinations up to the 3<sup>rd</sup> degree,  
 \*\*Mean Absolute Percentage Error (MAPE).

The features were created to contain a combination of the parameters up to the third degree (Dimopoulos and Bakas, 2019). The algorithm was set to use 85% of the data to train the algorithm and 15% of the data to test the proposed fundamental period formula. The algorithm shown below represents the applied procedure for developing the proposed formula (Gravett et al., 2021).

## 3 DEVELOPMENT OF NUMERICAL MODELS AND DATASET

The main challenge for proposing a new design formula is in the development of a sufficient number of models that have varying soil depths, number of stories, plan area and orientation of I-columns. In addition to the test dataset, a validation dataset is developed that contains the numerically obtained fundamental period results of models that foresee out-of-sample parameters as discussed below.

The finite element software Femap is used to graphically create the models and Reconan FEA (2020) is used to analyse and obtain the fundamental periods numerically. The models were created using a varying number of stories, bays and base conditions. The development of the models started with an initial model, which is a single storey, single bay structure with a height of 3.5 m and a raft foundation assuming a fixed base (see Figure 1). The geometry of the single bay has a length of 5 m (in the x-direction) and a width of 3 m (in the y-direction). The initial model was used to develop additional building geometries by altering the number of stories, number of spans, depth of soil and orientation of I-columns.

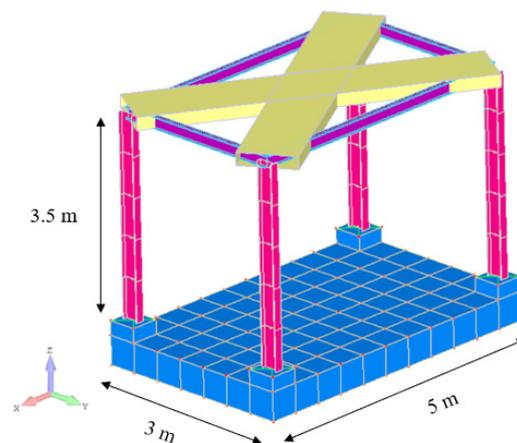


Figure 1: Initial model.

The initial model was modified to develop new models that foresaw 2, 4, 6, 8 and 10 stories, each with a 3.5 m height. Each of these models were then modified to contain single, double, and triple spans along the x-axis and, single and double spans along the y-axis. The largest total plan area used to develop the dataset foresaw a 15x6 m plan view, where the smallest was 5x3 m.

The models were further modified to include the SSI effect as seen in Figure 2. The discretization of the soil domain foresaw depths of 1, 5, 12.5, 22.5 and 37.5 m. It is important to note that the superstructure was discretized through the use of Natural Beam-Column Flexibility-Based (NBCFB) finite elements, where the raft slab and the soil domain were discretized through 8-noded isoparametric hexahedral elements. Three soil types were considered in this research investigation, namely: soft soil with a Young’s modulus of 65 MPa, soft to medium soil with a Young’s modulus of 350 MPa and medium soil with a Young’s modulus of 700 MPa.

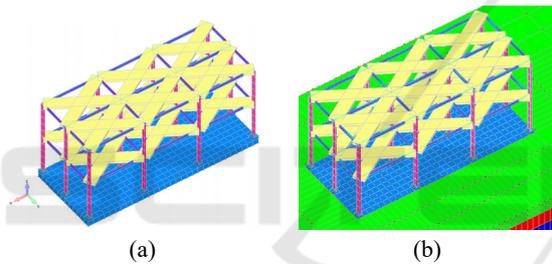


Figure 2: 2-storey steel building. Triple span in long direction, double span in short direction (a) fixed base with raft foundation (b) flexible base with soil hexahedral mesh.

Table 1 contains the minimum and maximum values that each parameter had according to the design of the geometrical features of the buildings and the soil domains.

Table 1: Minimum and maximum parameter values for model development.

Parameter	Minimum	Maximum
Soil Depth [m]	1	37.5
Soil E [kPa]	65 000	700 000
Height [m]	3.5	35
Length (along x-axis) [m]	5	15
Width (along y-axis) [m]	3	6

After the construction of the initial numerical models that foresaw the use of the positioning of the steel IPE columns’ section along a specific direction, the number of models was increased by changing the

orientation of the columns’ section by 90°. The columns’ strong axis orientation was modified from being parallel to the global x-axis to being parallel to the global y-axis direction of the structure, thus allowing the investigation of this feature on the fundamental period. It is important to note here that the IPE200 section was used for all beams and the IPE300 for constructing all columns.

Additionally, the slabs of the buildings were assumed to be reinforced concrete (RC) slabs and were modeled as diaphragms with a mass equal to the mass of a 150 mm thick slab that foresees a live load of 2 kN/m<sup>2</sup>.

Figure 3 shows the first two modal shapes of a 4-storey, 1-bay steel building with 1 m soft soil.

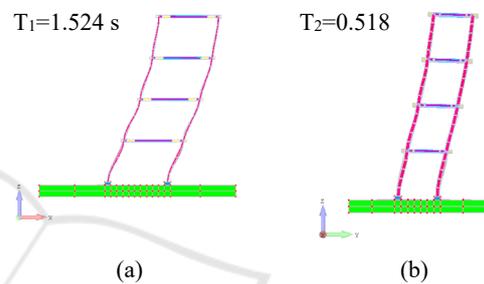


Figure 3: Modal shape (a) 1 and (b) 2 of a 4-storey, 1-bay steel building founded on 1 m deep soft soil.

#### 4 PROPOSED FUNDAMENTAL PERIOD FORMULA

The proposed formula for determining the fundamental period of steel structures was determined from the numerical results of 1,152 data points. The formula contains 40-features, which are a combination of the following parameters:

- $T$  is the fundamental period (s)
- $D_s$  is the depth of soil (m)
- $E$  is the soils Young’s Modulus (kPa)
- $H$  is the building height (m)
- $L$  is the length of the building parallel to the oscillating direction (m)
- $B$  is the width of the building perpendicular to the oscillating direction (m)
- $CO$  is the orientation of the columns (either a 1 or 2)
- $lParameter$  is  $\ln(Parameter + 1)$  i.e.,  $lD_s = \ln(D_s + 1)$
- $InvParameter$  is  $\frac{1}{Parameter+1}$  i.e.,  $InvD_s = \frac{1}{D_s+1}$

The developed formula is given in Equation 7. It must be noted here that numerous formulae have been

investigated that foresaw 5, 10 and 20 features, where it was found that the 40-feature formula was able to provide with the highest accuracy in terms of fitting into the training and test data, but most importantly in terms of predicting accurately the out-of-sample data compared to other machine learning generated formulae.

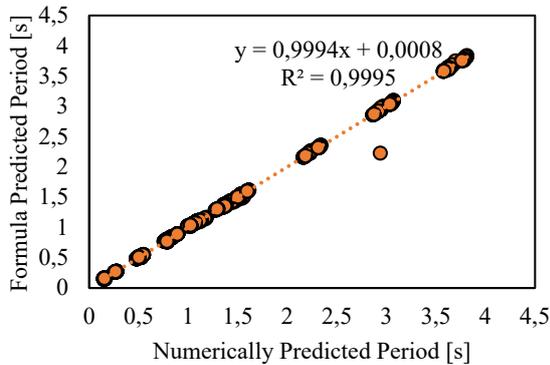


Figure 4: Relationship between numerically predicted and formula predicted fundamental periods on test and train dataset.

$$\begin{aligned}
 T = & 0.194630 \cdot LH^2 + 0.0580556 \cdot CO^2 \cdot B \\
 & -9.39027 \cdot InvCO \cdot InvB \cdot IB \\
 & -8.49213 \cdot InvL \cdot CO \cdot H \\
 & -41.8498 \cdot InvCO \cdot LL \cdot H \\
 & -8.14564 \cdot InvE \cdot E \cdot H - 0.800465 \cdot CO \cdot B \cdot H \\
 & +114.808 \cdot InvCO \cdot InvB \cdot H \\
 & +46.6778 \cdot InvCO \cdot InvB^2 + 0.0631499 \cdot B^2 \cdot H \\
 & +4.20803 \cdot IB \cdot CO \cdot H - 0.144945 \cdot LL \cdot H \cdot L \\
 & +0.847694 \cdot B \cdot H \cdot InvL + 9.37930 \cdot InvL^2 \cdot H \\
 & -1.08930 \cdot InvCO^2 \cdot L + 4.04342 \cdot InvL \\
 & -0.251627 \cdot InvL \cdot CO \cdot B \\
 & -0.00783561 \cdot InvB \cdot lCO \cdot lE \\
 & +0.523388 \cdot LL^2 \cdot InvCO \\
 & +0.0947335 \cdot InvH \cdot lH \cdot L \\
 & +46.8309 \cdot InvE \cdot H \cdot lDs + 0.00764850 \cdot lH \cdot B \\
 & +0.000161108 \cdot LL \cdot L \cdot lE \\
 & -20.5554 \cdot InvE \cdot CO \cdot Ds \\
 & -0.00474725 \cdot InvL^2 \cdot InvDs \\
 & +2.73101 \cdot InvL \cdot InvH \cdot CO \\
 & +0.403996 \cdot InvCO \cdot lB \cdot L \\
 & -0.0105914 \cdot LL \cdot L \cdot B \\
 & -0.228100 \cdot lB^2 \cdot CO \\
 & +0.00265642 \cdot InvL \cdot H^2 \\
 & -2.58386 \cdot InvB \cdot InvH \cdot CO \\
 & +5.84142 \cdot InvCO \cdot H \cdot L \\
 & +29.5168 \cdot InvCO \cdot H \\
 & +0.849560 \cdot InvL \cdot lH \cdot CO \\
 & -2.14776 \cdot InvB \cdot lH \cdot lCO \\
 & +1.34222 \cdot lB \cdot lH \cdot InvH \\
 & -0.00333495 \cdot lE \cdot L \cdot InvH \\
 & -2.64111 \cdot InvB^2 \cdot InvH \\
 & +71.1358 \cdot InvH \cdot Ds \cdot InvE \\
 & -17.9194 \cdot InvE \cdot lE \cdot LL - 1.16636
 \end{aligned}
 \tag{7}$$

Figure 4 shows the similarity ratio of the proposed formula compared to the numerically predicted data used to train and test the developed relationship. The correlation ratio was found to be 99.95% as it derives from the training and testing procedure.

## 5 FURTHER VALIDATION OF THE PROPOSED FORMULA

A set of 138 fundamental periods were generated through the use of additional models that were not used during the training and testing procedure so as to investigate the performance of the proposed 40-feature formula when out-of-sample data are used. The validation dataset was created using random parameter values not included in the train and test datasets. 3, 5, 7 and 9-storey models and models with Young's modulus of 10 MPa and 100 MPa were used. Figure 5 shows two of these models that were developed for the validation stage that foresaw 5 and 9 storeys. The out-of-sample parameters were assumed to validate whether the new proposed formula would be able to accurately predict the fundamental period of steel structures that had parameter values that differ from those that were used to train and test the predictive model.

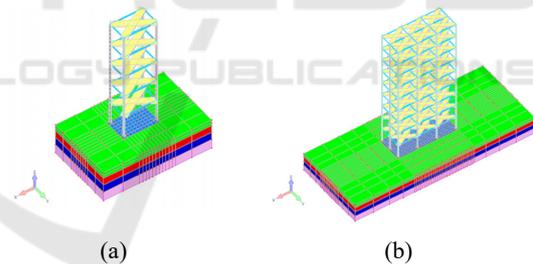


Figure 5: (a) 5-storey, 1-bay, (b) 9-storey, 3-bay 5m soil models developed for validation stage.

The numerically predicted periods were plotted against those obtained using the proposed formula as seen in Figure 6. By evaluating the correlation between the numerically predicted periods and those obtained using the proposed formula, it is observed that a high correlation ( $R^2 = 99.71\%$ ) was achieved. This shows that the formula yields a high accuracy prediction and can be used to predict the fundamental period of framed steel structures that have geometrical features within the limits presented in Table 1.

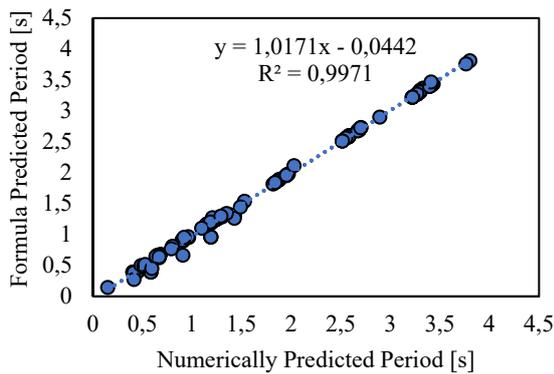


Figure 6: Relationship between numerically predicted and formula predicted fundamental periods on the out-of-sample validation dataset.

Table 2: Comparison of fundamental period error predictions on the validation dataset.

Description	Formula	Mean absolute error
40-feature formula	Equation 7	2.8%
EC8	Equation 1	76%
ASCE	Equation 2	76%
Cinitha (2012)	Equation 4	92%

Table 2 shows the comparison between the numerically obtained fundamental periods and those obtained using the proposed formula as well as the formulae currently found in design codes and the international literature. It is evident that the current design codes estimate the fundamental period with a high mean absolute error as compared to the new proposed formula.

## 6 CONCLUSIONS AND RECOMMENDATIONS

A newly proposed formula for predicting the fundamental period of steel structures with the use of machine-learning algorithms was presented. The proposed formula considers the depth of soil, Young's modulus of soil, height and plan area of the structure, as well as the orientation of the I-columns. The 40-feature formula proposed was developed using an algorithm combining the parameters using a higher order NLR.

The proposed fundamental period formula was tested on out-of-sample steel structures, where a correlation of 99.71% was achieved. This shows that the proposed formula produces accurate results and can be further used to predict the fundamental period of out-of-sample results. Design code formulae for

the calculation of the fundamental period of steel structures were compared to the proposed formula, where it was found that the proposed predictive model derived a 27 times smaller mean absolute error. In addition to that, the proposed fundamental period formula was found to be superior to other existing proposed equations found in the international literature when used on the under-study datasets.

The study focuses on steel structures with regular plans. To expand the dataset and further investigate the dynamic response of steel framed structures, irregular in plan buildings will be investigated, where braced and infill frames will be modeled in future research work. Finally, for each type of steel framing system, larger models will be created to develop formulae that will be applicable to a broader spectrum of frame geometries.

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