

Graph Convolutional Networks for Turn-Based Strategy Games

Wanxiang Li, Houkuan He, Chu-Hsuan Hsueh and Kokolo Ikeda

School of Information Science, Japan Advanced Institute of Science and Technology, Nomi, Ishikawa, Japan

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Abstract: In this paper, we research Turn-Based Strategy (TBS) games that allow players to move multiple pieces in one turn and have multiple initial states. Compared to a game like Chess, which allows only one piece to move per turn and has a single initial state, it is difficult to create a strong computer player for such a group of TBS games. Deep learning methods such as AlphaZero and DQN are often used to create strong computer players. Convolutional neural networks (CNNs) are used to output policies and/or values, and input states are represented as “image”-like data. For TBS games, we consider that the relationships among units are more important than their absolute positions, and we attempt to represent the input states as “graphs”. In addition, we adopt graph convolutional neural networks (GCNs) as the suitable networks when inputs are graphs. In this research, we use a TBS game platform TUBSTAP as our test game and propose to (1) represent TUBSTAP game states as graphs, (2) employ GCNs as value network to predict the game result (win/loss/tie) by supervised learning, (3) compare the prediction accuracy of GCNs and CNNs, and (4) compare the playing strength of GCNs and CNNs when the learned value network is incorporated into a tree search. Experimental results show that the combination of graph input and GCN improves the accuracy of predicting game results and the strength of playing TUBSTAP.

1 INTRODUCTION

Turn-Based Strategy (TBS) games are a popular game genre where players take turns to perform actions. Classical board games such as Chess and Go are a special case of TBS games that players usually operate one unit in each turn (for example, moving a piece in Chess or putting a stone in Go). Under a broader definition, many TBS games allow players to operate multiple units in each turn (i.e., whether and how to move each unit). Many TBS games allow players to play multiple units in one turn. In addition, many TBS games offer various initial states, for example, pieces(units)’ initial positions may be different in each play. This is different from Chess and Go, which have only one initial state. As the number and placement of pieces(units) changes, players need to think flexibly about their strategies. For example, Fire Emblem (Nintendo, 2019), Battle for Wesnoth (White, 2003) and Nintendo Wars (Nintendo, 1988) are famous TBS games with such rules.

To make the discussions clearer, TBS games in the rest of this paper exclude classical board games. For classical board games, state-of-the-art AI players are strong enough as humans’ opponents. Particularly,

AlphaGo defeated top professional Go players Ke Jie and Lee Sedol (Silver et al., 2017), and the successor AlphaZero also achieved superhuman levels in Chess and Shogi (Silver et al., 2018). In contrast, AI players for TBS games are still weak, mainly because of the massive number of move combinations in each turn, coming from the fact that a player can move many units in any order in one turn.

In this research, we employ an academic TBS game platform called TUBSTAP (Fujiki et al., 2015), in which several types of units exist, and each type has different attack damage on other types. The maps also contain different terrains (for example, plain, forest, and sea), which influence the mobility of units and damage. Despite the simplified rules of TUBSTAP, it is challenging to create strong AI players that can deal with many different situations well. A well-known method for creating evaluation functions of game states is to learn from game records, such as AlphaZero (Silver et al., 2018). The goal of our research is to find out what kind of network can be used to create a better state evaluation function for TUBSTAP.

In the case of AlphaZero, the state evaluation function is called a value network, and its input, i.e.,

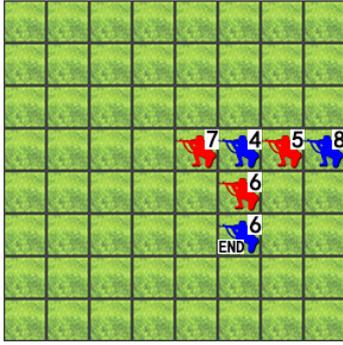


Figure 1: Game state 1.

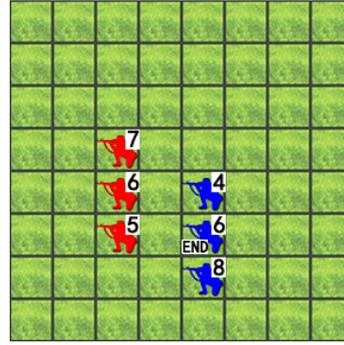


Figure 2: Game state 2.

the board, is represented as an “image” (for example, 19×19 planes representing black and white stones for Go). And value networks are based on convolutional neural networks (CNN) (Silver et al., 2018). Even though TBS maps are also image-like, image input may not be the best approach since the relationships between units are more important for decision making. Instead of image-like representation, we think it is promising to treat the states as graphs in order to adequately represent the relationships. So we propose to represent game states by graphs. Each node is for one unit in this graph, and each directed edge indicates that one unit can attack another. In addition, graph convolutional neural networks (GCN) has been proposed to deal with graph-based data better than CNN (Wu et al., 2020), so we employ GCN to evaluate state values.

There are several potential advantages of graph representation to image representation. Firstly, essentially similar unit arrangements have the same representation even the relative distance between two units is not the same. Such an advantage can make the learning efficient. Secondly, learning does not depend much on the size of the map or the terrain but most on the relationships between units. Therefore, the trained value network which employs graph representation is expected to be more reusable/applicable than the case of image representation.

In our experiments, we confirm that our proposed method increases the accuracy of the state evaluation and thus the playing strength. At first, we employ supervised learning to train value networks that evaluate game states and test them by prediction accuracy. The input of networks is the game state, and the output is game results (win/loss/tie). We collected the training and test data (states and game results) by Monte Carlo tree search (MCTS) (Browne et al., 2012) player vs. MCTS player. We compare image input with graph input, and CNN with GCN. The best estimation accuracy is obtained with the combination of graph input and GCN. We then combine the value networks into

minimax search to make the TBS game player. The GCN player obtains the highest winning ratio among all tested players.

In the rest of this paper, Section 2 briefly introduces TUBSTAP and GCN. Section 3 presents the graph representation of TUBSTAP states, how to train value networks, and how to do minimax searches with the value networks. Section 4 shows the experiment results, and Section 5 makes concluding remarks and introduces our future work.

2 BACKGROUND

In this Section, we will explain the background of this research. In Section 2.1, we will introduce the experiment platform TUBSTAP. In Section 2.2, we will explain the graph convolutional networks (GCN).

2.1 TBS Game and Research Platform: TUBSTAP

In this paper, we focus on TBS games that allow players to move multiple pieces in one turn and have multiple initial states. There are many such TBS games, and their features make them differently challenging from Chess and Go.

TUBSTAP is a two-player complete information TBS game platform developed for academic research (Fujiki et al., 2015). It is a TBS game that do not contain internal politics elements (for example, gaining resources and using resource to create units, or upgrading units to improve their ability). TUBSTAP is a convenient platform, where some researchers have introduced and tested their algorithms on it (Fujiki et al., 2015; Sato and Ikeda, 2016; Kimura and Ikeda, 2020; Pipan, 2021). We consider that TUBSTAP has rules of moderate complexity and is suitable for comparing different methods, explained as follows. Too-simple rule will make it difficult to

Table 1: The characteristic of different units.

	attack						Mobility	Shooting Range
	Fighter	Attacker	Panzer	Cannon	anti-aiR	Infantry		
F	55	65	0	0	0	0	9	1
A	0	0	105	105	85	115	7	1
P	0	0	55	70	75	75	6	1
C	0	0	60	75	65	90	5	2-3
R	70	70	15	50	45	115	6	1
I	0	0	5	10	3	55	3	1

propose new methods because it is easy to create a strong player using existing methods. Too-complex rule will make it difficult to compare methods purely because various elements of the rules need to be addressed to create a strong computer player. Hence we consider using TUBSTAP as our test environment.

Figure 1 and Figure 2 show examples of this game platform. TUBSTAP's rule is based on Advance Wars Days of Ruin (Nintendo, 1988). One player owns the red units and the other player the blue units. The hit point (HP) of each unit is shown on the top-right of the unit. Different types of units have different characteristics, such as attack ability, which will be explained soon later. Victory condition of the game is to destroy all enemy's units or to have the sum of HP higher than that of enemy after some certain turns.

There are 6 types of units in TUBSTAP game: Fighter, Attacker, Panzer, Cannon, Anti-air and Infantry. Their characteristics such as attack power, mobility, and shooting ranges are shown in Table 1. We can find some predator-prey relationships; for example, Fighter(F) exploits Attacker(A), A exploits Panzer(P), P exploits anti-aiR(R), and R exploits F. The mobility decides how far a unit can move. For example, Infantries have the mobility of 3. When moving on the plain terrain (moving cost 1), Infantries can move at most three grids counted by Manhattan distances. And a unit cannot move through enemy units. The shooting range decides how far a unit can attack. Because of the differences between the units, how to use them cleverly is the core issue that players need to consider and is also the most interesting part of the game.

As for the battle system, the damage of an attack action is determined by the attacker's attack power, the attacker's HP, the defender's HP, and a terrain factor, as shown in Eq. 1 The terrain factor will be explained in Appendix.

$$\frac{70 + (Attack \times ATK_{unit}'sHP)}{100 + (DEF_{unit}'sHP \times TerrainFactor)} \quad (1)$$

When attacking neighbor units, after the attacker's damage has been applied to the defender, a counterattack is launched. For example, an Infantry with 10 HP

attacks another Infantry with 10 HP on the Plain; the attack will give $(70 + (55 \times 10)/100 + (10 \times 1)) = 5.6$ damage. And the counter-attack will give $(70 + (55 \times 4.4)/100 + (10 \times 1)) = 1.2$ damage¹.

Some researchers have employed TUBSTAP in their research on TBS games. Fujiki et al. introduced a depth-limited Monte-Carlo tree search and tested their method on TUBSTAP (Fujiki et al., 2015). Sato and Ikeda proposed forward-pruning techniques to minimax search variants and evaluated them on TUBSTAP (Sato and Ikeda, 2016). Kimura and Ikeda used an AlphaZero-like method on the TBS game and introduced their method on TUBSTAP, and their main idea is to use the network multiple times. Their selection unit is in the input layer, as opposed to the usual implementation where the unit selection is in the output layer. Their main purpose is to reduce the complexity of the output layer and improve learning efficiency (Kimura and Ikeda, 2020). Pipan implemented three MCTS-based game-playing agents in his modified TUBSTAP and compared their performance; the main purpose is to apply and understand how MCTS variations work with hidden information (for example, fog-of-war) in TBS game (Pipan, 2021). In our case, the main purpose is to find out what kind of network can be used to create a better board evaluation function for TUBSTAP.

2.2 Graph Convolutional Network

AlphaZero uses CNN to build the value network to evaluate the state of the game of Go (Silver et al., 2018). CNN is being used not only for games, but also in other fields, such as image recognition (LeCun et al., 2015). Usually, CNN deals with the image or image-like data well; however, many data in the real world are represented by graphs, such as social networks (Meqdad et al., 2020), information networks (Xiong et al., 2021), and knowledge

¹The example rounds off the HP to one decimal place; we used float-type to calculate the HP in our program, which is different from the original TUBSTAP calculation method that rounds off to integers.

graphs (Yang et al., 2020). CNN may not be suitable to deal with graph data. Kipf and Welling (Kipf and Welling, 2016) proposed graph convolutional networks (GCN) to deal with graph data in neural networks “directly”. Unlike CNN, the graph convolution’s process is similar to matrix dot calculation, which would not change the size of the input graph.

GCN model can make use of the graph structure (and of features extracted from the graph structure at later layers) (Kipf and Welling, 2016). GCN has been successfully employed in some applications where the input is essentially one or some graphs. For example, DeepMind used GCN to boost the accuracy of the Google maps estimated time of arrival (Derrow-Pinion et al., 2021). Their method succeeded in reducing the estimation error by about half in cities such as Berlin, Sydney, Tokyo and Washington DC (Derrow-Pinion et al., 2021).

3 APPROACHES

In this Section, we will introduce the purposed methods. In Section 3.1, we will introduce how we represent the game state as graph. In Section 3.2, we will introduce the training method on state evaluation. In Section 3.3, we will introduce minimax search with GCN.

3.1 Graph Representation for Game State

There are many possible ways to represent a stage of TUBSTAP. One way is to represent TUBSTAP game states as image-like data, which uses several matrices to represent the units’ position, HP, attack, color (side), type and other information separately. For simplicity of discussions, we will explain using the HP matrix as an example. The matrix representations of the HP information of the state in Figure 1 is shown in the left of Figure 3, and state in Figure 2 is shown in the right Figure 3. Each element in the matrices is the HP of the unit on the corresponding grid.

As we can tell from the figure, the two matrices are entirely different, i.e., image representation will treat these two states as different states. However, all units’ relationships (which units can attack which units) are essentially similar; for example, both the red units with 6 HP in Figure 1 and 2 can move and then attack all the blue units. Using image representation may cause training inefficiently. Besides, the image representation requires different sizes of network inputs when the map size is changed. It is hard

to apply a trained network directly to a map of a different size². It is usually necessary to prepare a new network of that size and train it again.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7 & 4 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 3: HP matrix of game states 1(left) and 2(right).

To solve these problems, we propose to represent a TUBSTAP state as a graph. Each node in this graph represents one unit in the current map. A directed edge from node-X to node-Y means that unit X can attack unit Y. The absolute placement provides a rich amount of information, but we believe that considering whether units can attack each other is often sufficient to estimate the goodness of a state. By representing many similar states with similar evaluation in the same graph, learning is expected to be more efficient. If we consider the two examples before, their representation of graphs is like Figure 4.

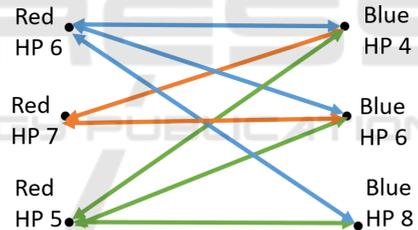


Figure 4: Graph for game state in Figure 1.

$$\begin{matrix} \text{HP} & & \text{type} & & \text{mobility} \\ \begin{pmatrix} 0 & 0 & 0 & 6 & 6 & 6 \\ 0 & 0 & 0 & 7 & 7 & 0 \\ 0 & 0 & 0 & 5 & 5 & 5 \\ 4 & 4 & 4 & 0 & 0 & 0 \\ 6 & 6 & 6 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 5 & 5 & 5 \\ 0 & 0 & 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 5 & 5 & 5 \\ -5 & -5 & -5 & 0 & 0 & 0 \\ -5 & -5 & -5 & 0 & 0 & 0 \\ -5 & 0 & -5 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$\begin{matrix} \text{moved or not} & & \text{distance} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 2 & 2 & 1 & 1 & 3 \\ 2 & 0 & 2 & 1 & 3 & 3 \\ 2 & 2 & 0 & 1 & 3 & 1 \\ 1 & 1 & 1 & 0 & 2 & 2 \\ 1 & 3 & 3 & 2 & 0 & 4 \\ 3 & 3 & 1 & 2 & 4 & 0 \end{pmatrix} \end{matrix}$$

Figure 5: Graph representation for game state in Figure 1.

For the two game states in Figure 1 and Figure 2, the units’ relationships (can attack another) are essen-

²As CNN cannot be applied directly to larger maps, our idea (GCN, shown next) cannot be applied directly to maps with more units than it was trained.

tially similar, so they can be represented by the similar graph. Adjacency matrices are one way to represent a graph, and GCN usually expects one or more adjacency matrices to be input. Figure 5 shows the adjacency matrix of HP information for the graph in Figure 4. Each directed edge is weighted by the HP of the attacking unit, indicated by the parent node. Naturally, the size of the adjacency matrix depends on the number of ALIVE units, but we fix it to $x \times x$, where x is the number of total units in the initial states. So it is also possible to apply our graph presentation to maps with different sizes. And we can reuse the trained network for these different maps.

In addition to the HP matrix, several matrices are input to the value network. The type, mobility matrix are adjacency matrices but have different edge weights to represent different information. The other two matrices represent whether moved or not and the distance information. The example of different matrices are shown in Figure 5; it should be noted that the blue unit with 6 HP has already been moved.

1. type: similar to the HP matrices, we fill the position with unit type number, and type numbers are positive for red player, negative for blue player.
2. mobility: similar to the HP matrices, we fill the matrix with units mobility.
3. whether moved or not: if we have n units for each side, if Red i -th unit can move, then in i -th row, $(i, n+1)$ to $(i, 2n)$ are filled with 1; if Blue j -th unit can move, then in j -th row, $(j, 1)$ to (j, n) are filled with 1; with this information, the network can determine the current turn player.
4. distance: if we have n units for each side, in this matrix, for each i -th row, $(i, 1)$ to $(i, 2n)$ are filled with the distance between the i -th unit and all units no matter they can attack each other or not. We use the Manhattan distance between two units i and j , i.e., $|x_i - x_j| + |y_i - y_j|$ where x_i and x_j are the row coordinates of i and j , y_i and y_j are the column coordinates.

With the adjacency matrix's help, we can ameliorate the input data of the neural network to make one representation correspond to the different unit arrangements and different map sizes. To sum it up, we will convert the game state into a graph structure representation, use an adjacency matrix to represent it, and finally send it to the neural network to learn features.

3.2 Supervised Learning on State Evaluation

Kimura et al. tried AlphaZero-like learning to train the value network for the TBS game (Kimura and Ikeda, 2020), but AlphaZero learning is more complex than pure supervised learning because game records are created simultaneously, and policy network is also trained. Since the output representation of the policy network is another issue that greatly affects the performance, we decided to separate it and use simple supervised learning to train value networks. We prepare training data as follows. We randomly generated various opening situations, let existing AI players play from these opening situations, and collected the initial/intermediate states and results. As for why using different opening situations, in TBS games, opening situations are often designed by the game designers and differ much. We want to train a good value network that can handle several kinds of opening situations, so the variety of opening situations is important.

For each state in a game record, we label the value as the final result (win/loss/tie) of that game from the view of the player to move. In more detail, if Red player wins a game, we label 1 for all Red turn states and -1 for all Blue turn states and vice versa. And if the game ends as a tie, we will mark all states to 0. By learning state-outcome pairs in this way, we expect the neural network to be able to estimate the value (how advantageous the player is) of the given game state.

3.3 Minimax Search with GCN

The minimax algorithm is a classical tree search algorithm for zero-sum two-player games, one player is called the maximizer, and the other player is a minimizer. If we assign an evaluation score to game state, one player tries to choose a game state with the maximum score, while the other chooses a state with the minimum score (Russell and Norvig, 2010). The ideal way of minimax tree search is searching the game state until wins/losses/ties are obtained, but it is infeasible for most cases. Thus, evaluating states by some function is practical.

Different from classical board games, players in TUBSTAP (and other TBS games) can operate multiple units in one turn. If one "action" includes all unit's moves, the number of possible actions is considerably large. Taking three Infantries versus three Infantries (situation in Figure 2) without considering unit interaction as an example, each player has roughly $(3!) \times 25 \times 25 \times 25 = 93,750$ moves per turn,

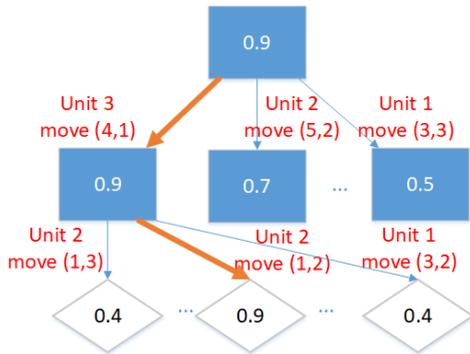


Figure 6: An example of MAX-MAX search with principal variation (orange bold arrow).

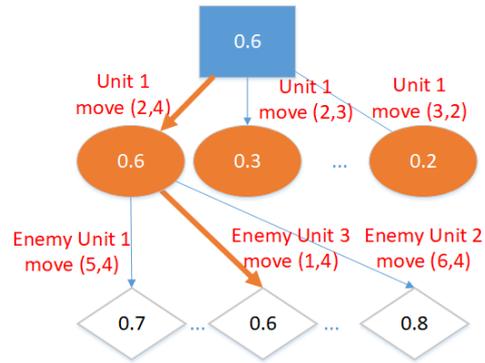


Figure 7: An example of MAX-MIN search with principal variation (orange bold arrow).

where $3!$ is for the different order of moving three Infantry and 25 for possible movements of an Infantry. When doing a minimax search considering only two turns, about $93,750^2 = 8,789,062,500$ states will be evaluated, which is extremely costly.

Therefore, we break down one turn-action into several unit operations and modify the minimax search a little (Muto and Nishino, 2016). As long as it is the player’s turn, the tree nodes are MAX-nodes; similarly, when it is the opponent’s turn, the tree nodes are MIN-nodes. Taking the same example of three Infantry, assume to look ahead for two operations (depth 2). When 0 or 1 unit has been moved in that turn, a MAX-MAX search is executed, which is shown in Figure 6. When two units have been moved, a MAX-MIN search is executed because after the first move, all three units complete their moves then the opponent’s turn begins, which is shown in Figure 7. The depth-2 search may not be promising in strength, but we can get an affordable search cost and a simple way to compare different value networks’ abilities. When doing such a unit-based search, the leaf node is not necessarily at the beginning of the turn but after some units have moved. To allow the value networks to evaluate such intermediate states, we include “whether moved or not” in the input (described in Section 3.1) and also include the intermediate states in the training data (described in Section 3.2).

4 EXPERIMENT AND ANALYSIS

In this Section, we present the results of our evaluation experiments. The purpose of our experiments is to compare graph representation with image-like representation, and to compare GCN with CNN. Section 4.1 introduces the maps used for training and evaluation. Section 4.2 describes the settings and results of supervised learning of the value network. Sec-

tion 4.3 shows the results when the trained network is combined with Minmax tree search.

4.1 Map Generation

In TBS games, a complete game (or story) often consists of multiple scenes that happen in different maps with various sizes, terrains, and units’ types, numbers, and locations. In our experiments, the TUB-STAP games consisted of one single 8×8 map, and the terrain was fixed to plains only, but various patterns of units were used for learning and evaluation. The generated maps can be divided into three main categories: random maps, symmetric maps, and local-search maps.

The victory conditions are the same for all categories. The player who destroys all of his/her opponent’s units wins. If the HP of any unit of both sides is not reduced for 20 turns, the game is judged as a tie.

Random Map. A random map contains three Red Infantry and three Blue Infantry. Their positions and HPs were randomly determined. The HP of the initial player ranges from 1 to 10, where the HP of the second player ranges from 1 to 9. Maps are often unfair because of the randomness of their placement, rather than the impact of different HP ranges. It is usually easy to predict the outcome of such an unfair map.

Symmetric Map. Symmetric unit arrangement at the beginning of a game are often used in traditional turn-based board games such as Chess and Shogi, so we also consider this kind of maps. A symmetric map has two Infantry and one Panzer for each side. The Infantry’s HP is fixed to 10, and the Panzers’ HP is fixed to 5. We firstly randomly decide positions for the red side units in the upper half of the game map.

Table 3: The network layers.

Network name	layer 1	layer 2	layer 3	layer 4	weight number
CNN _i	<i>cnn</i> 32 × 3 × 3	<i>cnn</i> 64 × 2 × 2	<i>FC</i> 512	<i>FC</i> 512	174,721
CNN _g	<i>cnn</i> 32 × 3 × 3	<i>cnn</i> 64 × 2 × 2	<i>FC</i> 256	<i>FC</i> 512	76,705
GCN ₄	<i>gcn</i> 16 × 6 × 6	<i>gcn</i> 4 × 6 × 6	<i>FC</i> 144	<i>FC</i> 512	76,433
GCN ₃₂	<i>gcn</i> 16 × 6 × 6	<i>gcn</i> 32 × 6 × 6	<i>FC</i> 1152	<i>FC</i> 512	596,225

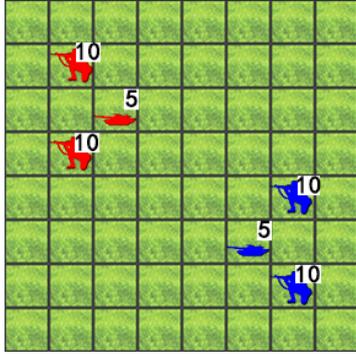


Figure 8: An example of symmetric maps.

Then we set the blue side units rotationally symmetrically at the lower right of the game map. The symmetric maps may be still unfair because of an initial player advantage or disadvantage. The example of a symmetric map is shown in Figure 8.

Local-search Map. A local-search map contains two Infantries and one Panzer for each side. At first, the units' position is randomly set, and the Infantries' HP is 10, the Panzers' HP is 5. We then employ two MCTS players with 6400 rollouts to play this map against each other. If one player wins, we decrease one unit's HP by 1 for the winning side or increase one unit's HP by 1 for the losing side. Until there is a tie or the winning side changes, we save the map as a local-search map. We expect such maps to be relatively fair for the two players. The example of a local-search map is shown in Figure 9.

4.2 Supervised Learning for Value Network

In this experiment, we are going to train value networks and test their prediction accuracy. We led MCTS players play against each other, then collected the result (win/loss/tie). All states were labeled by 1/-1/0 for win/loss/tie, as mentioned in Section 3.2. We collected 83,060 (state, result) pairs for training from 200 random map games, 274 symmetric map games, and 1,900 local-search map games. We also collected 3,859 pairs for testing from another 15 random maps, another 25 symmetric maps, and another

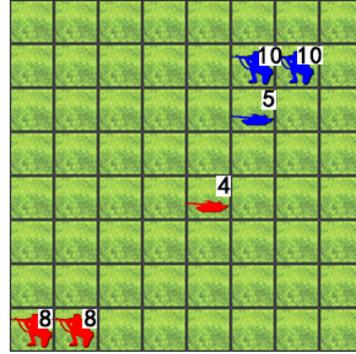


Figure 9: An example of local-search maps (red first move).

80 local-search maps. The number of states for each label in training data and test data is shown in Table 2.

Table 2: The number of states for each label.

	win	loss	tie	total
Training data	34,863	19,768	28,429	83,060
Test data	1,783	1,186	890	3,859

The collected (state, result) pairs are used for supervised learning, where the state is input to networks, and networks output the result. Different network structures and weight numbers may cause different prediction abilities. We designed four networks shown in Table 3, CNN's structure is based on Le-Net (Kayed et al., 2020). And in this table *gcn* $n \times m \times k$ means n -dimensional graph convolution layer with $m \times k$ kernel; *cnn* $n \times m \times k$ means n -dimensional convolution layer with $m \times k$ kernel; *FC* n means a fully-connected layer containing n neurons. All output layers consist of one node, which expresses $[-1, 1]$, representing the input state's goodness.

The network would output the continuous value in float type. In order to calculate the prediction accuracy, we converted the output value into three labels (win/loss/tie) in this experiment. We set the label as "win" when the output value is higher than 0.5, and set the label as "loss" when the output value is lower than -0.5; otherwise, we set the label as "tie". We will compare the output label with the true label and get the prediction accuracy.

We compared GCN₄, GCN₃₂ with CNNs. The result in Figure 10 shows that the CNN's prediction accuracy was significantly better when using graph

Table 4: The results in prediction accuracy for GCNs and CNNs.

	Overall	Random map	Symmetric map	Local-search map
CNN _i	58.2%	74.5%	61.1%	56.8%
CNN _g	71.5%	81.1%	75.9%	70.6%
GCN ₄	77.5%	85.3%	81.2%	76.7%
GCN ₃₂	78.2%	85.4%	81.2%	77.5%

Table 5: Results of game matches between 4 players on symmetric maps. The winning ratios, win-loss-tie counts and avg. thinking times in the initial maps.

	vs GCN player	CNN _g player	CNN _i player	MCTS-200 player	avg. thinking time
GCN player	-	55.5% (199-144-157)	78.2% (349-67-84)	64.6% (304-158-38)	1.42s
CNN _g player	44.5%	-	71.0% (298-88-114)	42.7% (196-269-35)	1.78s
CNN _i player	21.8%	29.0%	-	20.2% (95-393-12)	1.89s
MCTS-200 player	35.4%	57.3%	79.8%	-	5.34s

input (71.5%) than image input (58.2%). And GCN₄ showed better accuracy (77.5%) than CNN_g (71.5%) even though they have similar numbers of weights. And GCN₃₂ is a much richer network than GCN₄, but the improvement in accuracy was about 0.8%. Table 4 also shows the evaluation results when applying individual map groups to test each value network. The tendency for graph input and GCN to outperform image input and CNN is the same for each map group. And as expected, when tested on unfair maps, it is easier for networks to predict the game results.

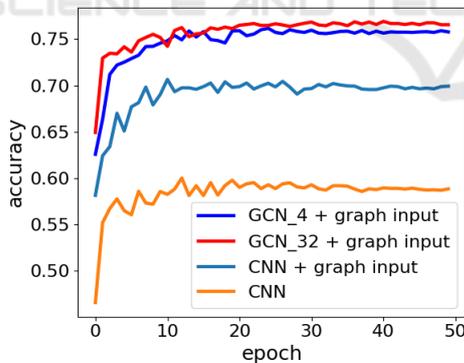


Figure 10: Accuracy transition of supervised learning for predicting win/loss/tie.

From the result, we can say that, CNN's prediction accuracy is significantly improved with graph input, and GCN shows better prediction performance than CNN. Our approach improved the value networks' prediction accuracy.

4.3 AI Players for TUBSTAP

Since we showed the superiority of graph input and GCN in terms of learning accurate value networks, next we evaluated the strength of the computer player when the value network was combined with tree search. We prepared four players as follows:

1. GCN player: Depth-2 minimax search with the GCN₄ value network.
2. CNN_g player: Depth-2 minimax search with the CNN_g value network, which use graph representation as input.
3. CNN_i player: Depth-2 minimax search with the CNN_i value network, which use image representation as input.
4. MCTS-200 player: MCTS with 200 rollouts.

There are six possible combinations of players. For each combination, 250 symmetric maps and 250 local-search maps were given. Each map was played twice for fairness, where the two players alternately played as the first player. For symmetric maps, we generated the map for each game. And for local-search maps, we prepared 250 newly created maps and shared by all combinations. The results of the four players playing against each other are shown in Table 5 and Table 6, denoted by winning ratio (win-loss-tie), where a tie was counted as a half win. Wins and losses are from the views of the players in the first column. The average computation time to decide the next move in the initial maps (when both sides had all three units) is also shown for reference.

In symmetric maps, from the result, we can find that the CNN_g player was stronger than the CNN_i player. GCN player's superiority is not obvious when

Table 6: Results of game matches between 4 players on local-search maps. The winning ratios, win-loss-tie counts and avg. thinking time in the initial maps.

	vs GCN player	CNN _g player	CNN _i player	MCTS-200 player	avg. thinking time
GCN player	-	59.4% (213-119-168)	78.3% (356-73-71)	64.0% (315-175-10)	3.64s
CNN _g player	40.6%	-	72.9% (306-77-117)	39.6% (182-286-32)	3.98s
CNN _i player	21.7%	27.1%	-	18.9% (93-404-3)	4.15s
MCTS-200 player	36.0%	60.4%	81.1%	-	5.54s

the opponent is a CNN_g player but still gets an over-half winning ratio. Even though the result of direct matching on GCN player and CNN_g player was not clear, when playing against other players, GCN player performed better, and the differences were clear. So we can say that GCN player had the best playing strength in these three players. The result is similar to prediction accuracy, where GCN is better than CNN_g, and CNN_g is better than CNN_i.

In local-search maps, results were similar to symmetric maps. The graph input improved CNN's playing strength, and GCN player showed the best performance in game playing. With the results from these two kinds of maps, we can say that our graph representation improved the CNN's playing strength, and GCN showed its advantage over CNN in our experiment.

4.3.1 Experiment on Different Map Size

As we mentioned in Section 3.1, it is also possible to apply our proposed methods to maps with different sizes. We tested the GCN player against the MCTS-200 player in larger (10×10) symmetric map. 250 maps were newly generated, and each map was played twice, where the two players alternately played as the first player. In these 500 games, the GCN player won 296 games, lost 89 games, and tied 115 games. The average thinking time in initial board for the GCN player was 1.65s, and the average thinking time for the MCTS-200 player was 5.76s. The results showed that our proposed methods could work well even in the untrained larger map size.

5 CONCLUSION AND FUTURE WORK

This paper studied TBS games through an academic platform TUBSTAP. We proposed to use graphs to represent game states and employed CNNs or GCNs to learn state evaluations (i.e., value networks) by supervised learning. We further combined the value net-

works into a modified minimax search that one edge represents one action of one unit. The experimental results showed that graph representation improved the prediction ability for CNN, and GCN obtained the best prediction accuracy. When incorporating trained networks into the minimax search with depth 2, the GCN player obtained the highest winning ratio among all tested players.

There are many experiments and implementations to be done in the near future. First, since only a limited variety of unit types and terrains were used in this paper, it would be valuable to investigate the performance of GCN on more complex maps. The next issue is that a minimax search with a depth of 2 is not enough, but it is also difficult to search any deeper. The reason is many possible moves, including bad ones, and prioritization using a policy network would be effective. Hence, thirdly, it would be promising to use graph input and GCN for AlphaZero-like learning, which learns value and policy simultaneously while playing against itself.

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Table 7: The terrain factors and moving cost in TUBSTAP.

	Plain	Fortress	Mountain	Forest	Sea	Road
terrain factor for F,A	0	0	0	0	0	0
terrain factor for P,C,R,I	1	4	4	3	0	0
moving cost for F,A	1	1	1	1	1	1
moving cost for P,C,R	1	1	-	2	-	1
moving cost for I	1	1	2	1	-	1

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APPENDIX

In this appendix, the detailed setting is described, which is a bit far from the central issue of this paper. The following two paragraphs introduce the terrain factor of TUBSTAP and the implementation of the MCTS player used in the experiments.

In TUBSTAP maps, there are different terrains, as listed in Table 7. In our experiments, only the plain terrain was employed. Two sets of rules relating to terrains are the terrain factor and the moving cost. The terrain factor is one of the elements deciding the damage of an attack, as shown in Eq. 1 in Section 2.1. A higher terrain factor means that it is better for defense. The moving cost, along with the mobility of a unit, decides how far the unit can go. With a higher moving cost, a unit takes more moves to travel the same distance.

The MCTS players in this research are MCTS with upper confidence bound (UCB) (Kocsis and Szepesvári, 2006). Where the coefficient for UCB is $\sqrt{2}$. It contains four steps, selection, expansion, simulation, and backpropagation. In the selection step, we use the UCB to choose which node to expand. In the expand step, the selected node would be fully expanded, which means all valid child nodes would be created. In the simulation step, we employ a random player, and the simulation ends till the game end. In the backpropagation step, nodes’ value is updated by the result in the simulation.