A Novel Key Exchange Protocol using Logic Algebra for the Factorization Problem

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Abstract: Our current key exchange protocols are at risk of failing to keep private data secret due to advancements in technology. Therefore, there is a need to develop an efficient and secure key exchange protocol which can function in the new computing era to come. In this paper, we propose and develop a novel key exchange protocol based on logic algebra for the factorization problem. Both the security analysis and experimentation evaluation demonstrate promising results of our proposed approach.

1 INTRODUCTION

In 1976, the Diffie-Hellman public key exchange algorithm (Diffie and Hellman, 1976), which is known as the first asymmetric encryption algorithm, was proposed. A Key Exchange Protocol is a protocol in which a key is established by exchanging information between two parties. The scheme to accomplish this is as follows:

(1) Each party holds a private key and a public key.
(2) The encrypted message and public key are sent to the other party.
(3) The two parties use a series of mathematical methods to calculate the private key and public key in their hands and the other party's public key obtained from the communication to generate a common key that can encrypt or decrypt a message (Diffie and Hellman, 1976; Li, 2010).

The advantage of the public key cryptosystem is that the communicating parties do not need to have a shared secret key before communicating. The private keys held by the communicating parties are generated randomly by themselves, and the communicating parties cannot calculate the others' private key from the private key they each hold. While this method can prevent network data snooping to a certain extent, it cannot prevent the tampering of network data, which is known as the man-in-the-middle attack (MITM) (Kader and Hadhoud, 2009).

In order to address this problem, many suggestions have been made over the years. Most of them are based on the optimization of Diffie-Hellman key exchange protocol. Each includes a key exchange system that combine the Diffie-Hellman algorithm with various digital certificate algorithms such as RSA and DSA, which rely on signature algorithms for identity verification (Chen and Wang, 2019; Pal and Alam, 2017; Yusfrizal et al., 2018; Thayananthan and Albeshri, 2015; Bhavani and Krishna, 2021). Each also includes some algorithms that optimize the key generation based on the DH algorithm. For instance, the ECDH algorithm, which solves the "Discrete Logarithm Problem" in the DH protocol is replaced with solving the "Elliptic Curve Discrete Logarithm Problem". Another example includes an algorithm which changes the prime number in the DH algorithm to a group or a matrix (Vidhya and Rathipriya, 2020; Megrelishvili, 2018; Rudy and Monico, 2021; Bharathi et al., 2017). Gentile and Migliorato (2002) proposed cryptosystems that use hypergroupoids as keys for encryption on the issue of key structure. In (Shpilrain, 2008; Grigoriev and Shpilrain, 2014, 2019), Shpilrain proposed tropical algebra as the key structure based on the study of key exchange protocol using random natural number with exponential operations; the key exchange protocol using public non-commutative rings; and an analysis of a linear algebra attack. Ezhilmaran and Muthukumaran (2016) proposed the key exchange protocol using "The Decomposition Problem" in the near-ring scheme. These schemes have become the main trend of the current cryptosystem.

In this paper, we have further optimized the key construction scheme based on the ideas of using classical algebra and tropical algebra and chose to use

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logic algebra as the basis for key construction. The groupoid factorization proposed by Fayoumi (Fayoumi, 2020), splits a given groupoid into different factors according to the characteristics of logic algebra. A different operation, diamond (Fayoumi, 2020), is used instead of the usual mathematical operations. By optimizing key generation, we can greatly mitigate the man-in-the-middle attacks. Section 2 discusses the background and related work. Our proposed key exchange protocol is described in Section 3, followed by its security analysis in Section 4. The implementation and evaluation of the proposed protocol are covered in Sections 5 and 6. Finally, we conclude and propose our future work in Section 7.

2 BACKGROUND

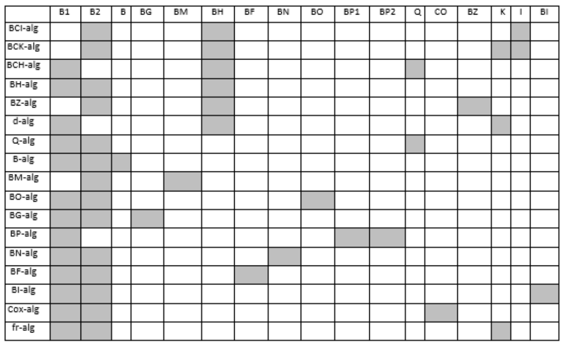
2.1 Algebras

In this paper, a groupoid is the main component of the key. To obtain the required groupoid efficiently and to make the groupoid strong and diverse, different types of logic algebras, such as *BCK*-algebra, *BCI*-algebra, *BCH*-algebra, and *d*-algebra, will be used. Several of their properties, which are used as axioms to define each algebraic structure, are in the following list (Fayoumi, 2020). Let $(X, \bullet, 0)$ be an algebra, for any $x, y, z \in X$:

B1: $x \bullet x = 0$,
B2: $x \bullet 0 = x$,
BG: $x = (x \bullet y) \bullet (0 \bullet y)$,
BH: $x \bullet y = 0 \; and \; y \bullet x = 0 \Rightarrow x = y$,
BF: $0 \bullet (x \bullet y) = y \bullet x$,
K: $0 \bullet x = 0$,
B: $(x \bullet y) \bullet z = x \bullet (z \bullet (0 \bullet y))$,
BM: $(z \bullet x) \bullet (z \bullet y) = y \bullet x$,
BN: $(x \bullet y) \bullet z = (0 \bullet z) \bullet (y \bullet x)$,
BO: $x \bullet (y \bullet z) = (x \bullet y) \bullet (0 \bullet z)$,
BP1: $x \bullet (x \bullet y) = y$,
BP2: $(x \bullet z) \bullet (y \bullet z) = x \bullet y$,
Q: $(x \bullet y) \bullet z = (x \bullet z) \bullet y$,
CO: $(x \bullet y) \bullet z = x \bullet (y \bullet z)$,
BZ: $((x \bullet z) \bullet (y \bullet z)) \bullet (x \bullet y) = 0$,
I: $((x \bullet y) \bullet (x \bullet z)) \bullet (z \bullet y) = 0$,
BI: $x \bullet (y \bullet x) = x$.

By combining two or more of these axioms, we can get different types of algebras we need to generate the groupoid (Table 1).

Table 1: Axioms of different types of algebras (Fayoumi, 2020).



	B1	B2	B	BG	BM	BH	BF	BN	BO	BP1	BP2	Q	CO	BZ	K	I	BI
BCI-alg		▨				▨										▨	
BCK-alg		▨				▨									▨	▨	
BCH-alg	▨					▨						▨					
BH-alg	▨	▨				▨											
BZ-alg		▨				▨								▨			
d-alg	▨					▨									▨		
Q-alg	▨	▨										▨					
B-alg	▨	▨	▨														
BM-alg		▨			▨												
BO-alg	▨	▨							▨								
BG-alg	▨	▨		▨													
BP-alg	▨									▨	▨						
BN-alg	▨	▨						▨									
BF-alg	▨	▨					▨										
BI-alg	▨	▨															▨
Cox-alg	▨	▨											▨				
fr-alg	▨	▨													▨		

For example, if we want to generate a *fr*-algebra, based on the axioms in Table 1, we know that the groupoid should satisfy B1, B2 and K. A groupoid $(X, \bullet)$ satisfies axiom B1 means for any $x \in X$, we can always get $(x \bullet x) = 0$. If $(X, \bullet)$ satisfies B2, then for any $x \in X$, $(x \bullet 0) = x$. Finally, if $(X, \bullet)$ satisfies property K, means for any $x \in X$, t $(0 \bullet x) = 0$. As for this, we can first generate an algebra frame as follows:

$\bullet$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	x	x	x
2	2	x	0	x	x
3	3	x	x	0	x
4	4	x	x	x	0

And then, we can fill up the groupoid using random method or brute-force method to get a complete groupoid with *fr*-algebra.

2.2 Factorization

Factorization is an important part in our experiment. As we mentioned in Section 1, we used a different type of binary operation, the diamond operation ($\diamond$), to calculate two groupoids. Corresponding to this, we use factorizations to divide a groupoid into two factor groupoids (Fayoumi, 2020). In this section, we present two different types of factorizations.

2.2.1 Similar-Signature Factorization

Similar-Signature Factorization is a unique factorization of a given groupoid in which two factors are derived from itself and from the left-zero-semigroup (Fayoumi, 2020).

Let $(X, \bullet)$ be a groupoid of finite order n. Then $d^{\bullet}$ is the diagonal function of $(X, \bullet)$ such that

$$d^{\bullet}: \mathbb{N} \rightarrow X \; where \; d^{\bullet}(i) = x_i \bullet x_i,$$

$$i = 1, 2, \ldots, n \; for \; all \; x_i \in X.$$

2.2.2 Orient-Skew Factorization

Orient-Skew Factorization is another unique factorization which can be applied to groupoids with the orientation property OP: $x * y \in \{x, y\}$ for all $x, y \in X$. We can derive the orient-factor of a groupoid such that all its elements are the same as those of a left-zero-semigroup except elements belonging to the anti-diagonal (Fayoumi, 2020). Similarly, the skew-factor is derived from the groupoid by letting its anti-diagonal change to its skew-diagonal, and the other elements are kept the same as the original groupoid.

2.3 Binary Operations

Given a binary operation "$*$" on a non-empty set X, groupoid $(X,*)$ is then considered a generalization of the very well-known structure of a group. Consider the collection of all groupoids defined on X, call it $Bin(X)$. Let $(X,*)$ and $(X,\circ)$ be two random groupoids in Bin(X), define a groupoid product operation "$\diamond$" where $(X,\diamond) = (X,*) \diamond (X,\circ)$ such that $x \diamond y = (x * y) \circ (y * x)$ for all x, y $\in$ X. This turns $Bin(X,\diamond)$ into a semigroup with identity $(x * y = x)$, the left-zero-semigroup, and an analog of negative one in the right-zero-semigroup. One can naturally observe that we can always get such a product if we know groupoids$(X,*)$ and $(X,\circ)$. However, reversing this action is not so straightforward. There are four cases we need to note:

Case 1 Distinct Factors:
$(X,\bullet) = (X,*) \diamond (X,\circ)$, such that $(X,*) \neq (X,\circ)$.

Case 2 Uniqueness:
$(X,\bullet) = (X,*) \diamond (X,\circ)$, such that if $(X,\cdot) = (X,*) \diamond (X,\circ)$, then$(X,\cdot) = (X,\bullet)$.

Case 3 Relatively prime to (X,•):
$(X,\bullet) = (X,*) \diamond (X,\circ)$, such that $(X,*) \neq (X,\bullet)$ and $(X,\circ) \neq (X,\bullet)$.

Case 4 Commutative:
$(X,\bullet) = (X,*) \diamond (X,\circ)$, such that if $(X,\bullet) = (X,\circ) \diamond (X,*)$, then $(X,\circ) \diamond (X,*) = (X,*) \diamond (X,\circ)$.

Of course, one can consider the subcases of each of the above four cases or even the combinations of two or more of them. We should notice that not all groupoids can satisfy case 4, in other words, not all groupoids commute. It is important to be able to find a collection of groupoids that do commute, and logic algebras using Similar-Signature Factorization and Orient-Skew Factorization can more likely generate commuting groupoids under our special diamond operation. It means, all groupoids can be factorized using the Similar-Signature Factorization and the Orient-Skew Factorization into two factor groupoids, but not all pairs of factor groupoids commute. In our experiment, an important step is to obtain groupoids which can be factorized into two commuting factor groupoids. Here is an example of using the Similar-Signature Factorization to generate two commuting factors.

Example. Let $(X,\bullet) = (Z_5,\bullet,0)$ be a groupoid with 0, and binary operation defined by:

$\bullet$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	3	0
3	3	3	2	0	3
4	4	4	1	1	0

Then we can find factors $(X,*)$ and $(X,\circ)$ which commute, as follows:

$$\begin{array}{c|ccccc} * & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 2 & 3 & 0 \\ 3 & 3 & 3 & 2 & 3 & 3 \\ 4 & 4 & 4 & 1 & 1 & 4 \end{array} \diamond \begin{array}{c|ccccc} \circ & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 2 \\ 3 & 3 & 3 & 3 & 0 & 3 \\ 4 & 4 & 4 & 4 & 4 & 0 \end{array} = \begin{array}{c|ccccc} \bullet & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 & 3 & 0 \\ 3 & 3 & 3 & 2 & 0 & 3 \\ 4 & 4 & 4 & 1 & 1 & 0 \end{array}$$

and,

$$\begin{array}{c|ccccc} \circ & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 2 \\ 3 & 3 & 3 & 3 & 0 & 3 \\ 4 & 4 & 4 & 4 & 4 & 0 \end{array} \diamond \begin{array}{c|ccccc} * & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 2 & 3 & 0 \\ 3 & 3 & 3 & 2 & 3 & 3 \\ 4 & 4 & 4 & 1 & 1 & 4 \end{array} = \begin{array}{c|ccccc} \bullet & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 & 3 & 0 \\ 3 & 3 & 3 & 2 & 0 & 3 \\ 4 & 4 & 4 & 1 & 1 & 0 \end{array}$$

2.4 OpenSSL

OpenSSL is an open-source software library written in the C programming language and has a wide variety of cryptographic functions to implement secure features. It is a powerful tool that implements the popular SSL and TLS protocols (Ruiter, 2016). Some of the features implemented by the OpenSSL library include symmetric encryption, asymmetric encryption, certificate handling and hash functions. Apart from that, OpenSSL also supports a wide range of cipher suites.

One of the main reasons for choosing OpenSSL platform is that OpenSSL has all the features already built into it to test a new cryptographic protocol. Researchers trying to test the functionality of a cryptographic algorithm or a protocol in a full-fletched application can do so by implementing that module in the OpenSSL library and using the already available applications like HTTP to test its performance. However, to integrate the module into the OpenSSL, we need to understand the libraries the two main libraries of OpenSSL – libssl and libcrypto. Libssl, as the name suggests is a library that implements SSL and TLS functions like opening and closing a connection, forming packets, managing handshakes, creating certificates, and using lib-crypto library to perform cryptographic operations. The libcrypto library has a wide variety of symmetric, asymmetric, and key-exchange protocols, which provides the low-level implementation of cryptographic algorithms to the libssl library.

Another reason is that since OpenSSL is an open-source software, all of its software packages rolled out in the past is easily accessible. In addition, the older cipher suits, which have been retired by the IETF, can be modified in a specific way to add a new cryptographic algorithm easily. Modifying the older cipher suites like AECDH-NULL-SHA allows us to integrate our new key exchange algorithm by modifying the already present Elliptic Curve Diffie-Hellman key exchange without having to write a symmetric algorithm that goes with it, as indicated by NULL in the middle. Moreover, the "A" in AECDH means anonymous, i.e., the key exchange algorithm will not have to use an Ephemeral Key for changing the keys periodically. However, we should be careful not to use those cipher suites on their own as these generally have security vulnerabilities. This important feature allows us to test the functionality of our key exchange algorithm in isolation without having to implement a Pseudo Random Function for generating symmetric keys as well as making the keys ephemeral. It is important to note that these two components are the foundation of modern cryptography and should not be excluded from any cryptographic implementation. However, these components have been left out from our implementation as it gives us more flexibility to test the raw performance of our key exchange protocol.

3 PROPOSED PROTOCOL

3.1 Basic Scenario

The basic scenario of our proposed key exchange protocol is as follows:

1. Picking commuting groupoid pairs
   - Alice:
     Randomly picks two private groupoids $A_1 = (X,*)$ and $A_2 = (X,\circ)$.
     Sends Bob the shared groupoid $u = A_1 \diamond A_2$.
   - Bob:
     Randomly picks two private groupoids $B_1 = (X,\circledast)$ and $B_2 = (X,\odot)$.
     Sends Alice the shared groupoid $v = B_1 \diamond B_2$.
2. Generating the decryption key:
   - Alice computes
     $K_A = A_2 \diamond (v) \diamond A_1$
     $= (X,\circ) \diamond [(X,\circledast) \diamond (X,\odot)] \diamond (X,*)$
     $= [(X,\circ) \diamond (X,\circledast)] \diamond [(X,\odot) \diamond (X,*)]$
     $= (X,\circledast) \diamond (X,\circ) \diamond (X,*) \diamond (X,\odot)$
   - Bob computes
     $K_B = B_1 \diamond (u) \diamond B_2$
     $= (X,\circledast) \diamond [(X,*) \diamond (X,\circ)] \diamond (X,\odot)$
     $= (X,\circledast) \diamond (X,\circ) \diamond (X,*) \diamond (X,\odot)$

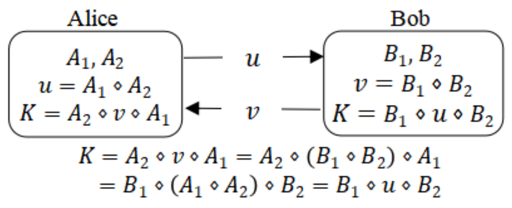

Alice
Bob
A_1, A_2
$u = A_1 \diamond A_2$
$K = A_2 \diamond v \diamond A_1$
u
v
B_1, B_2
$v = B_1 \diamond B_2$
$K = B_1 \diamond u \diamond B_2$
$K = A_2 \diamond v \diamond A_1 = A_2 \diamond (B_1 \diamond B_2) \diamond A_1$
$= B_1 \diamond (A_1 \diamond A_2) \diamond B_2 = B_1 \diamond u \diamond B_2$

Figure 1: Proposed key exchange protocol.

Thus, Alice and Bob have a shared secret key $K = K_A = K_B$. For this protocol to work, we need the following pairs of groupoids to commute:

1. $(X,*)$ and $(X,\circ)$
2. $(X,\circledast)$ and $(X,\odot)$
3. $(X,\circ)$ and $(X,\circledast)$
4. $(X,\odot)$ and $(X,*)$

Additional conditions considering Eve interception:

- If she intercepts the key exchange, then she can either compute:
  $u \diamond v = [(X,*) \diamond (X,\circ)] \diamond [(X,\circledast) \diamond (X,\odot)]$
  $= (X,*) \diamond (X,\circledast) \diamond (X,\circ) \diamond (X,\odot)$
- Or she can compute:
  $u \diamond v = [(X,\circledast) \diamond (X,\odot)] \diamond [(X,*) \diamond (X,\circ)]$
  $= (X,\circledast) \diamond (X,*) \diamond (X,\odot) \diamond (X,\circ)$
- For this to not recover the shared secret key K, we would need both of the following pairs of groupoids to not commute:

5. $(X,*)$ and $(X,\circledast)$
6. $(X,\circ)$ and $(X,\odot)$

Hence, by ensuring 1-6 hold, we have our public key exchange systems.

3.2 Groupoid Generation

Our protocol is based on a series of groupoid operations. Thus, how to generate a useful groupoid as the exchange key is important. A useful groupoid first means that it can use the Similar-Signature Factorization to derive a pair of commuting factor groupoids or it can be decomposed into two commuting factor groupoids using the Orient-Skew

Factorization. After factorization, we must then ensure that these factor groupoids meet the four "commute" conditions and the two "not commute" conditions.

In this paper, we decided to use two different groupoids as shared groupoids for both parties. It means while Alice and Bob communicate with each other, they send the different shared groupoids.

Step 1: Read the input value of the groupoid length "L" and the type of logic algebra t_1, t_2 or 0.

- Type " t_1 " will use the Similar-Signature Factorization.
- Type "t_2" will use the Orient-Skew Factorization.
- Input "0" will choose a random type of logic algebra.

Step 2: Based on the properties of the "t_1" algebra shown in Table 1, create one groupoid frame (a two-dimensional array) "G_1" with dimensions L *by* L. Fill up the rest of the groupoid table with random integers greater than or equal to 0 and less than the length of the groupoid "L".
Step 3: Factor groupoid "G_1" using the Similar-Signature Factorization to get factor groupoids $A(X,\bullet,0)$ and $U(X,\bullet,0)$. If they commute, we store them and continue to the next step. Otherwise, go back to step 2.
Step 4: Based on the properties of the "t_2" algebra shown in Table 1, create one groupoid frame "G_2" with dimensions $L\ by\ L$. Fill up the rest of the groupoid table with random integers greater than or equal to 0 and less than the length of the groupoid "L".
Step 5: Factor groupoid "G_2" using Orient-Skew Factorization and we can get factor groupoids $O(X,\bullet,0)$ and $J(X,\bullet,0)$. If they commute, we store them and continue to the next step. Otherwise, go back to step 4. We can define a count number and a maximum number of times. If it cannot find the required groupoid G_2 in the number of times, go back to step 2 to find a new groupoid G_1.
Step 6: Use check functions to check if the four factor groupoids satisfy the four commute conditions and the two non-commute conditions. If not satisfied, go back to Step 4 or Step 2 depending on the number of times.
Step 7: Assign $A_1 = U(X,\bullet,0)$, $A_2 = O(X,\bullet,0)$, $B_1 = J(X,\bullet,0)$, $B_2 = A(X,\bullet,0)$. The private groupoids of Alice are A_1 and A_2; while her shared groupoid is $u = A_1 \diamond A_2$. Similarly, the private groupoids of Bob are B_1 and B_2; while his shared groupoid is $v = B_1 \diamond B_2$.

4 SECURITY ANALYSIS

Different from the simple addition and multiplication operations of matrices, the groupoid of this encryption algorithm adopts the $\diamond$ operation of multiple coordinate values, by taking values of (x, y) and (y, x), and then taking the values according to the value of (x, y) and (y, x). This means that the length of one diamond operation is $O(L^2)$, which is efficient for generating groupoids of size L. It not only ensures the complexity of the operation, but also makes the encryption algorithm simple, operable, and advanced in operation. However, to ensure the operability of the encryption key, it is essential to generate an effective groupoid. In this protocol of the encryption algorithm, we have analyzed the attacks that our proposed algorithm may suffer.
Brute Force Attack. It is known that the most important factor affecting the brute force attack is the length of the key. In our key exchange protocol, the key is actually an $L * L$ groupoid. Hence, given a nonempty set X with $|X| = L$, we can deduce the complexity of the groupoids in our protocol (which are elements in *Bin(X)*) to be $L^{(L^2)}$. Therefore, while we use this protocol, a sufficiently large enough "L" can ensure that the key exchange protocol will not be cracked within an effective period of time through brute force attack.
Man-in-the-Middle Attack. When the man-in-the-middle or the eavesdropper obtains the shared groupoids sent by both parties, the closest calculation method to the encryption key is to perform the obtained two groupoids $\diamond$ product operations or factor them into factor groupoids. However, as demonstrated in Section 3, after ensuring the generated private groupoids satisfy the four "commute" conditions, we can ensure that both parties can obtain the same key. We added two "non-commute" conditions to ensure that the man-in-the-middle cannot get the key by using the obtained two shared groupoids. Besides it, due to the particularity of the operation, it is very difficult for the attacker to generate another pair of groupoids that meet the six conditions for the groupoids of one or both parties and equal to the key. We can also make sure that the groupoids will never be the left-zero-semigroup, which is the identity in *Bin(X)*, so that man-in-the-middle cannot use any of the two shared groupoid directly as the key. Moreover, we assign groupoids U and O to Alice and groupoids J and A to Bob. The shared groupoid of each party will not be the same and will not be factored into any of the four factor groupoids, in which way we can say that this protocol is irreversible, that makes our protocol more secure.

5 IMPLEMENTATION OF PROTOCOL

All the implementations and experiments are performed on a PC workstation with the following specifications:

- Operating System: Windows 10 Home
- Processor: Intel i5-9600k
- Processor Speed (Base): 3.70 GHz
- Storage Type: SSD
- System Memory (RAM): 32 GB

5.1 Algorithm Implementation

Our main groupoid generation algorithm and factorization algorithm are written in JAVA language. At the beginning, according to the study of different algebras, we present an array framework represent the groupoid which basically satisfies the axioms of selected algebra, and then we use brute force algorithm and random function to fill up the array. We have defined checkAU and checkOJ functions, while the array can be factorized to a pair of commutative factor groupoids *A* and *U* and a pair of commutative factor groupoids *O* and *J*, the factor groupoids *A, U, O* and *J* will be stored. Then we use commute function to check if the four factor groupoids meet the four commute and two not commute conditions. If all the conditions are met, we store the array as the result and use it in OpenSSL to see how it performs. Otherwise, we restart the groupoid generation process.

5.2 OpenSSL Implementation

Key exchange is a vital technique used by the SSL/TLS protocol to secretly exchange keys between the communicating parties, to obtain the final session keys by some form of conversion using Pseudo Random Function and other methods.

The application layer sits on the top of the protocol stack and has applications like ciphers and genpkey. This component is responsible for high-level implementation of ciphers. Below that component is TLS component, which is in charge of handling the connections. It has a state machine, a record layer, packet, and buffer formation mechanisms and so on. The crypto component below it implements all the encryption and key exchange algorithms needed for TLS in low-level. The engine is a dynamically loadable module that uses the available hooks to provide cryptographic algorithm implementations.

OpenSSL operates everything in a state machine. The full operation of the state machine in SSL/TLS has been explained in (Ruiter, 2016). Using state machines makes it easier to get the current state of operation and makes packet management easier. An SSL/TLS connection can have states like ClientHello, ServerHello, ServerKeyExchange and so on. The next state of the connection depends on the previous states and the type of packet. Since the world of Internet is unpredictable, any of these packets could be dropped on its way. Hence, this way of message-passing handles the situation by keeping track of every state and packet to manage them more easily.

6 PROTOCOL EVALUATION

6.1 Evaluation of Groupoid Generation

In this section, we evaluate the performance of the groupoid generation algorithm. We ran the groupoid generation program by choosing the size as “5”, and the algebra t_1 and t_2 as “3”, the *BH*-algebra. We changed the variable count, the maximum number of times of groupoid generation to exit, as 10000000. Then, we ran the check function and finally got 5173 pair of groupoids as the result with one example:

•	0	1	2	3	4
0	0	0	0	2	0
1	1	0	2	2	4
2	2	0	0	3	4
3	3	0	2	0	2
4	4	0	2	0	0

And

•	0	1	2	3	4
0	0	1	0	0	4
1	1	0	1	3	4
2	2	2	0	2	2
3	3	1	3	0	1
4	4	4	2	2	0

We can use the Similar-Signature Factorization to derive the first groupoid and use Orient-Skew factorization to derive the second groupoid to get four factor groupoids. Next, by computing, we can get the same result from K_A and K_B, and we can find that the assumed man-in-the-middle attacks will not get the same Key groupoid. This means the groupoid is ready to be the key composition of our protocol.

6.2 Evaluation based on the OpenSSL Platform

In this section, we will test the time it takes for each type of messages to be received. Each of these tests records the amount of time taken to form a ClientHello, ServerHello, ServerKeyExchange and ClientKeyExchange messages. That will essentially test the message passing time of the sender. We also need to test the amount of time the receiver takes to process the message. Finally, the times of the new key exchange algorithm will be compared with the ECDH protocol.

We modified the message passing in ECDH algorithm's built-in extension to make it work as a new key exchange protocol. The Elliptic Curve Diffie-Hellman sends two extensions in a ClientHello message. The first extension selects the client's preference for the type of EC point format to use like compressed or uncompressed. The most common type of EC point format has an extension type 000b and is called Elliptic curve point formats uncompressed. The other extension that ECDH uses is for sending the preference on the EC curves. It has an extension of 000a and is called Elliptic curves. For our purpose, we will not need the extension 000b, and will only use 000a to send the preference on the type of matrix to use for our algorithm.

Custom extensions are treated differently in the OpenSSL library. Separate callback function needs to be written for these extensions and the performance of the protocol depends largely on the programmer. The first implementation is the unmodified ECDH key-exchange in OpenSSL. The second implementation modifies the built-in ECDH extension to piggyback our new key exchange data in OpenSSL. Finally, we used a custom extension using the private-use extension number provided by IETF. The packets formed by using our second and third implementation look very identical, the only difference is how OpenSSL treats these extensions. When built-in extension is used, the OpenSSL designers would have already set the properties of these extensions like extension number, maximum allowed data, information to be included in the certificate, etc. There is no flexibility in choosing the new functionality required for a new protocol. The gist is that although modifying built-in extensions for a new protocol is easy, it is impossible to add new functionality when it is needed later.

In this paper, we look at two criteria to evaluate the performance of each implementation. The construction time is the time taken by the OpenSSL library to construct a particular message. This includes the time taken to fill in the header and data portion of the messages. For a ClientHello message, a header includes handshake message type, message version, content length, and client hello protocol version. Similarly, the data portion includes a random value, session ID length, cipher suites length, a list of cipher suites and extension data.

The processing time is the time taken to process the message at the other end. Precisely, it is the time taken between receiving a message and deciding what to do next in the state machine. For a ClientHello message, that includes parsing the raw ClientHello message into ClientHello_MSG structure that has fields and variables to store each component of the message. Tables 2 and 3 compare the message construction time and processing time of the three implementations.

Table 2: Comparison of message construction time.

	Construction Time (Microseconds)		
Type of Message	ECDH	Built-in ext.	Custom ext.
ClientHello	337	116	582
ServerHello	91	107	304
ServerKeyExchange	1238	834	1540
ClientKeyExchange	1953	1748	1762

Table 3: Comparison of message processing time.

	Processing Time (Microseconds)		
Type of Message	ECDH	Built-in ext.	Custom ext.
ClientHello	56	44	195
ServerHello	94	267	331
ServerKeyExchange	34	26	37
ClientKeyExchange	943	532	819

As seen in Tables 2 and 3, the proposed key exchange algorithm implemented using built-in extension performed well compared to the ECDH algorithm. The use of built-in extensions for implementing a new key-exchange algorithm certainly limits the overall performance of the protocol because it is designed to work well with a particular algorithm. With that in mind, we also implemented the new protocol using custom extensions, and it showed a good performance.

7 CONCLUSION

In this paper, we proposed a novel key exchange protocol, which use logical algebra to solve the factorization problem, in order to greatly alleviate common attacks and deal with the challenges of cryptography brought by the rapid development of technology. We generated commutative groupoids based on different logical algebras by using two factorization functions, Similar-Signature Factorization and Orient-Skew Factorization, and used the diamond operation to perform operations on groupoids. We analyzed the brute force attack and man-in-the-middle attack on the proposed protocol and proved that it is secure against them. We also implemented a groupoid generation algorithm using

Java language and evaluated its efficiency. Finally, we implemented our key exchange protocol on the open-source OpenSSL platform and evaluated its runtime performance. Experimental results demonstrated our proposed protocol has comparable performance with the built-in ECDH key exchange algorithm in the OpenSSL platform.

For the future work, we plan to improve the algorithm of groupoid generation by increasing the size and optimizing its structure. We also plan to continue to verify the possibility of generating groupoids based on different logic algebras. Moreover, we intend to optimize the code of our built-in extension and custom extension on the OpenSSL platform and aim to apply it to real-world network applications.

REFERENCES

Bharathi, M. B., Manivasagam, M. G., & Kumar, M. A. (2017). Metrics For Performance Evaluation of Encryption Algorithms. In *International conference on emerging trends in engineering, science and management*.

Bhavani, Y., & Krishna, B. J. (2021). Security Enhancement Using Modified AES and Diffie–Hellman Key Exchange. In *Advances in Computational Intelligence and Communication Technology* (pp. 173-183). Springer, Singapore.

Chen, C. M., Wang, K. H., Yeh, K. H., Xiang, B., & Wu, T. Y. (2019). Attacks and solutions on a three-party password-based authenticated key exchange protocol for wireless communications. *Journal of Ambient Intelligence and Humanized Computing*, *10*(8), 3133-3142.

de Ruiter, J. (2016, November). A tale of the OpenSSL state machine: A large-scale black-box analysis. In Nordic Conference on Secure IT Systems (pp. 169-184). Springer, Cham.

Diffie, W., & Hellman, M. (1976). New directions in cryptography. *IEEE transactions on Information Theory*, *22*(6), 644-654.

Ezhilmaran, D., & Muthukumaran, V. (2016). Key exchange protocol using decomposition problem in near-ring. *Gazi University Journal of Science*, *29*(1), 123-127.

Fayoumi H F. (2020). Groupoid Factorizations in the Semigroup of Binary Systems. *Scientiae Mathematicae Japonicae Online, e-2020-13* and to appear (2022) *Scientiae Mathematicae Japonicae, Vol.84-3*.

Gentile, G., & Migliorato, R. (2002). Hypergroupoids and cryptosystems. *Journal of Discrete Mathematical Sciences and Cryptography*, *5*(2), 119-138.

Grigoriev, D., & Shpilrain, V. (2014). Tropical cryptography. *Communications in Algebra*, *42*(6), 2624-2632.

Grigoriev, D., & Shpilrain, V. (2019). Tropical cryptography II: extensions by homomorphisms. *Communications in Algebra*, *47*(10), 4224-4229.

Kader, H. M., & Hadhoud, M. M. (2009). Performance evaluation of symmetric encryption algorithms. *Performance Evaluation*, 58-64.

Li, N. (2010, April). Research on Diffie-Hellman key exchange protocol. In *2010 2nd International Conference on Computer Engineering and Technology* (Vol. 4, pp. V4-634). IEEE.

Megrelishvili, R. (2018). New asymmetric algorithm for fast message transmission and tropical cryptography. In *Proceedings of the eleventh international scientific-practical conference INTERNET-EDUCATION-SCIENCE-2018, Vinnytsia, 22-25 May, 2018: 175-178.* ВНТУ.

Pal, O., & Alam, B. (2017). Diffie-Hellman Key Exchange Protocol with Entities Authentication. *International Journal Of Engineering And Computer Science*, *6*(4).

Rudy, D., & Monico, C. (2021). Remarks on a tropical key exchange system. *Journal of Mathematical Cryptology*, *15*(1), 280-283.

Shpilrain, V. (2008, June). Cryptanalysis of Stickel's key exchange scheme. In *International Computer Science Symposium in Russia* (pp. 283-288). Springer, Berlin, Heidelberg.

Thayananthan, V., & Albeshri, A. (2015). Big data security issues based on quantum cryptography and privacy with authentication for mobile data center. *Procedia Computer Science*, *50*, 149-156.

Vidhya, E., & Rathipriya, R. (2020). Key Generation for DNA Cryptography Using Genetic Operators and Diffie-Hellman Key Exchange Algorithm. *Computer Science*, *15*(4), 1109-1115.

Yusfrizal, Y., Meizar, A., Kurniawan, H., & Agustin, F. (2018, August). Key management using combination of Diffie–Hellman key exchange with AES encryption. In *2018 6th International Conference on Cyber and IT Service Management (CITSM)* (pp. 1-6). IEEE.