Training Machine Learning Models to Detect Group Differences in Neurophysiological Data using Recurrence Quantification Analysis based Features

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Keywords: Mathematical Skills, Cognitive Task, Machine Learning, Complex Systems, Recurrence Quantification Analysis.

Abstract: Physiological data have shown to be useful in tracking and differentiating cognitive processes in a variety of experimental tasks, such as numerical skills and arithmetic tasks. Numerical skills are critical because they are strong predictors of levels of ability in cognitive domains such as literacy, attention, and understanding contexts of risk and uncertainty. In this work, we examined frontal and parietal electroencephalogram signals recorded from 36 healthy participants performing a mental arithmetic task. From each signal, six RQA-based features (Recurrence Rate, Determinism, Laminarity, Entropy, Maximum Diagonal Line Length and, Average Diagonal Line Length) were extracted and used for classification purposes to discriminate between participants performing proficiently and participants performing poorly. The results showed that the three classifiers implemented provided an accuracy above 0.85 on 5-fold cross-validation, suggesting that such features are effective in detecting performance independently from the specific classifiers used. Compared to other successful methods, RQA-based features have the potential to provide insights into the nature of the physiological dynamics and the patterns that differentiate levels of proficiency in cognitive tasks.

1 INTRODUCTION

Numerical skills have shown to be strong predictors of attention, literacy, and decision-making (Merkley & Ansari, 2016), as well as of socioeconomic status and planning skills (Fernandez & Liu, 2019). Therefore, for being able to identify an individual's performance on numerical skills – and consequently other cognitive skills and abilities – it is important to reliably track processes connected to the development of numerical skills and their related performance. Tracking such processes might allow us to detect when an intervention is needed, helping individuals who have difficulties in tackling numerical problems, as well as improving socio-economic status, unemployment, and other skills connected to numeracy (Fernandez & Liu, 2019).

Past research has shown that performance in several skill domains can be effectively tracked using physiological signals such as electrocardiograms, galvanic skin response, and electroencephalograms (Sharma et al., 2020). Processes involved in mathematical tasks can be effectively monitored using electroencephalograms (EEG; Río et al., 2019). EEG tracks the electrical activity of specific electrodes placed on the subject's scalp, the signal is used to extract features using linear methods such as the time-frequency distribution, the fast Fourier transform, and the autoregressive method (Al-Fahoum & Al-Fraihat, 2014). Furthermore, EEG signals have been used for classification tasks using deep learning models such as long short-term memory neural networks (Ganguly et al., 2020). Deep learning models overall yield high accuracy but tend to not provide insights into the nature of the signal and the patterns differentiating groups.

In the current study recurrence quantification analysis (RQA) was used to extract features from the EEG signal of participants who either performed well or poorly on a mental arithmetic task. RQA is robust

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Guglielmo, G., Wiltshire, T. and Louwerse, M.
Training Machine Learning Models to Detect Group Differences in Neurophysiological Data using Recurrence Quantification Analysis based Features.
DOI: 10.5220/0010832200003116
In *Proceedings of the 14th International Conference on Agents and Artificial Intelligence (ICAART 2022) - Volume 3*, pages 428-435
ISBN: 978-989-758-547-0; ISSN: 2184-433X

to noise and has the added advantage that it is based on recurrences and self-similarity. Consequently, it does not require data transformations or mathematical assumptions (Zbilut, Thomasson, & Webber 2002). Using RQA, instead of the time-frequency distribution, fast Fourier transform, autoregressive method, or long short-term memory neural networks, might not only result in extracting effective features for machine learning purposes but also in obtaining insights about the nature of the signal itself and the patterns it contains.

RQA-based features have shown to be effective for hypothesis testing purposes and for training machine learning models (Hou et al., 2019; Lyby et al., 2019). For example, RQA-based features, combined with machine learning models, have been used successfully to detect drowsiness and epileptic seizures (Gruszczyńska et al., 2019; Shabani, Mikaili, & Noori, 2016). Taken together, the evidence supports the idea that RQA might effectively capture the complexity of biological processes, which are often not linear (Zbilut & Weber 2008). Nevertheless, RQA-based features, combined with machine learning, have so far not been used yet to detect differences in performance on a cognitive task.

The current work aims to explore the possibility of using RQA-based features to track performance in cognitive skills within the domain of numeracy. The hypothesis is that the recurrent structures in the EEG signals, reflected by the RQA-based features, will differ between participants performing proficiently and participants performing poorly in the task. We will make use of the RQA-based features extracted to perform a binary classification task to discriminate proficient and non-proficient participants.

2 BACKGROUND AND THEORY

2.1 Neurophysiology and Numeracy

Mathematical skills are rooted in human capabilities to deal with space, numbers, and time. These skills are argued to stem from a non-linguistic ability that appeared during the late Palaeolithic and underwent development throughout human history (Amalric, & Dehaene, 2016; Wildgen, 2020).

Evidence from cognitive neuroscience shows that brain regions involved in mathematical problems such as the bilateral intraparietal and prefrontal areas are present not only in humans but also in non-human animals, such as monkeys (Cantlon & Brannon, 2007). Similar brain areas seem to be activated by mathematical tasks belonging to different domains such as topology, analysis, algebra, and geometry. These brain areas include the bilateral inferior temporal regions, bilateral intraparietal sulci, cerebellum, and several regions of the prefrontal cortex (dorsolateral, bilateral, superior, and mesial) (Amalric, & Dehaene, 2016). Different mathematical tasks involve high activity in the prefrontal and parietal areas during their execution. Therefore, EEG signals obtained from these areas are likely to be of interest when investigating the levels of mathematical abilities.

2.2 Recurrence Quantification Analysis

2.2.1 RQA and Its Specifications

Performing RQA requires a phase space reconstruction (PSR) that is used to unfold the dynamics of the signal. PSR is based on the setting of a few parameters, such as the delay and the number of embedding dimensions.

2.2.2 Phase Space Reconstruction

Phase space reconstruction is needed to define the temporal evolution and behavior of the signals before one can proceed with the use of RQA on continuous data. One method to reconstruct the time-series behavior in a multidimensional phase-space form is to use the time-delay embedding (Takens, 1981) that is based on four main parameters: the delay (τ), the number of embedding dimensions (D), the radius (r), and the rescaling norm (Wallot & Leonardi, 2018).

The delay specifies the number of time lags to shift the copies of the signal, while the number of embedding dimensions refers to the number of dimensions (i.e., time-delayed copies) needed to unfold the higher-dimensional dynamics that characterize the time-series (Wallot, 2017). The radius and the rescaling norm refer respectively to the interval that defines two points as recurrent and to the phase-space rescaling of the distance matrix. The choice of the aforementioned parameters depends on the time-series typology, its characteristics, and the use of specific methods to obtain the optimal values when considering the embedding dimensions and the delay parameters.

The optimal delay value is calculated using the average mutual information function, which provides the lag representing the first local minima after which the average mutual information remains generally quite constant (Wallot, 2019). The number of embedding dimensions is defined using the false nearest-neighbor function, which computes the optimal number of dimensions considering the number of delays selected. Conversely, the radius and the norm are chosen according to the level of noise in the data and the magnitude of the values composing the time series. Generally, the radius is set between 0.01 and 0.05 while the norm has three possible options: Euclidian, Supremum, and Manhattan (Marwan et al., 2007). Most important in setting the norm, however, is keeping the norm constant when comparing different time-series (Wallot & Leonardi, 2018).

2.2.3 Recurrence Plot and Features

The parameters listed in the previous sections are used as input to create the Recurrence Plot (RP; Figure 2 and 3). The RP provides a visual presentation of the patterns, repetitions, and dynamics contained in the time-series under analysis.

The RQA-based features are directly extracted from the patterns present in the RP. For the current study we extracted those features also used in previous studies (Gruszczyńska et al., 2019; Shabani, Mikaili, & Noori, 2016; Turianikova et al., 2015):

- *Recurrence Rate (RR)*: The likelihood of recurrence of a specific state in the signal. The recurrence rate is obtained by dividing the number of recurrent elements, represented by the points in the plot, by the RP size.
- *Determinism (%DET)*: The percentage of diagonal recurrent points lying adjacently.
- *Laminarity (%LAM)*: The percentage of the number of recurrent elements arranged vertically on the RP.
- *Average Diagonal Line (ADL)*: The mean length across all the diagonal lines present in the RP.
- *Maximum Diagonal Line (MDL)*: The length of the longest diagonal line present in the RP.
- *Entropy (ENT)*: A feature based on the frequency distribution of the diagonal lines. The value obtained in this feature is directly proportional to the complexity of the signal analyzed. For example, uncorrelated noise has a low value of ENT.

Since RQA considers self-similarity within a single time-series, these features concern the points on one side of the line of identity (the diagonal line dividing the RP in two). The features extractable from the RP are not limited to the ones listed above. For example, other features include, but are not limited to, trapping time and trend. Webber & Marwan (2015) provide further information about additional features and detailed explanations of the RQA equations.

3 METHODS

3.1 Dataset

For our study, we used the publicly available dataset on Physionet, the "Electroencephalograms during Mental Arithmetic Task" dataset (Zyma et al., 2019). This dataset contains 36 healthy participants that performed a mental arithmetic task for 4 minutes.

According to their performance, Zyma et al. (2019) assigned the participants to two different groups: participants who performed well were assigned to group "G" (standing for good) and those who performed poorly were assigned to group "B" (standing for bad). According to the dataset on Physionet, 10 participants were assigned to group "B" ($M_{calculations} = 7$ per minute, *SD* = 3.6) while group "G" had 26 participants ($M_{calculations} = 22$ per minute, *SD* = 7.3).

The data were recorded using a 23 EEG channel system where the recording sites were defined according to the international 10/20 scheme; each channel had a 500 Hz sample rate. The signal was filtered with a low pass filter (45 Hz) and a power notch filter (50 Hz). The data are artifact-free and ready for analysis purposes. More information about the sample and the task can be found in the original work by Zyma et al. (2019).

3.2 Workflow

The workflow followed in this work is comparable to the one used in other works that extracted RQA-based features from physiological signals, and specifically from EEG signals (Shabani, Mikaili, & Noori, 2016).

In order to proceed with the RQA-based features extraction, we focused on four electrodes for our analyses purposes: the F7, Pz, P4, and Fp1. These electrodes were adopted in a previous study using the same dataset to train a long short-term memory neural network and provided the highest accuracy on a classification task to detect the signal specific to the arithmetic task (Ganguly et al., 2020). Furthermore, the use of pre-selected electrodes, instead of all the ones present on the EEG cap, was successfully adopted in other studies using RQA-based features combined with machine learning techniques (Gruszczyńska et al., 2019).

After having selected the electrodes of interest, the RQA-based features were extracted to train a Support Vector Machine (SVM), a Random Forest (RF), and a Gradient Boosting Classifier (GBC). Before training the classifiers, we selected the five most relevant features, using the Extra Trees method (Sharma, Giri, Granmo, & Goodwin, 2019), and resolved the class imbalance present in the dataset using the Synthetic Minority Oversampling Technique (SMOTE) (Chawla et al., 2002). These processes were implemented to reduce the likelihood of overfitting (Ying, 2019).

Figure 1 gives the overview of the workflow followed in this study, which is similar to the one adopted by Borowska et al. (2018).

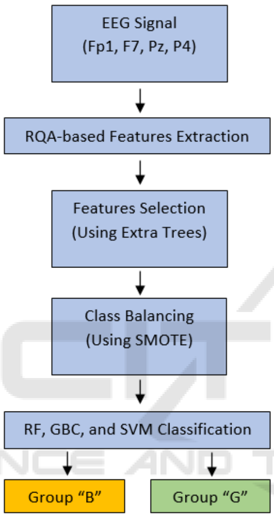

Figure 1: An overview of the workflow adopted for this study.

3.3 RQA-based Features Extraction

Before extracting the RQA-based features, the mutual average information function and the false nearest-neighbor function were used to define the optimal number of dimensional embedding and delays. The functions were run in R using the Tserieschaos package (Di Narzo, 2019) and the nonlinearTseries package (Garcia, 2021). Such functions were applied to a few subjects across the two groups and to different electrodes to verify if there was an approximate constant optimal value across electrodes and subjects. The signals analyzed had a number of values for delay generally ranging between 4 and 5, while the embedding dimensions had a value between 6 and 8. Therefore, the delay value was set to 5 and the number of embedding dimensions to 7 when performing RQA on all the data.

These parameters were used to create the RP together with a radius of 0.05, which is generally used for physiological data (Wallot, 2017), and Supremum as norm, which is the default parameter in the Pyunicorn library (Donges et al., 2015). The Pyunicorn library, in Python, was used to extract the RQA-based features and to visualize the RPs. To slightly reduce the computational power required by RQA, we used the initial 30,000 data points out of 31,000 composing the original dataset (Zyma et al., 2019); 30,000 data points correspond approximately to 3.87 minutes of recording out of a total of 4 minutes.

As conveyed in Figure 2 and Figure 3, the RP offers preliminary visual information of the differences between participants belonging to the two groups.

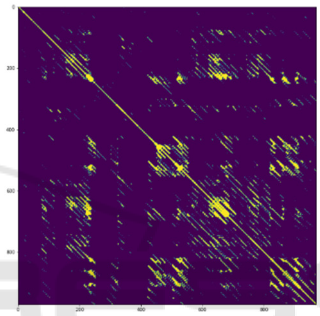

Figure 2: RP illustrating the F7 electrode signal for a participant of group "G" (1,000 data points). The x and y axes represent the data points composing the signal.

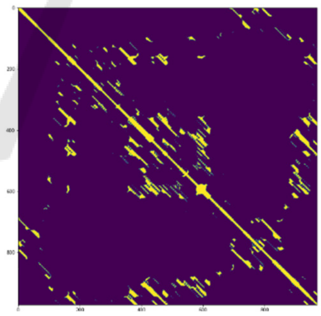

Figure 3: RP illustrating the F7 electrode signal for a participant of group "B" (1,000 data points). The x and y axes represent the data points composing the signal.

Upon visual inspection, participants who performed well on the task have an RP characterized by a higher degree of complexity, compared to those performing poorly. This visual information might provide early insights into the differences between groups and how their physiological signals may affect RP's outlook.

3.4 Features Selection

Each electrode selected for this work (F7, Pz, Fp1, and Pz) was used to extract the six RQA-based features (RR, %DET, %LAM, MDL, ADL, ENT). The final dataset contained 24 features obtained by multiplying the six RQA-based features times the four electrodes. To avoid overfitting and to select the most important features, the Extra Trees method (Sharma, Giri, Granmo, & Goodwin, 2019) was implemented for features selection. The Extra Trees method was also used to obtain more insights into which electrodes and features are likely to be the most important to track cognitive performance and differences between groups. After having performed the Extra Trees method on the data, we selected the top 5 features out of the 24 initial ones that were extracted. More specifically, as shown in figure 4, the features used as input for the classifiers were ADL for electrode F7, RR for electrode F7, %LAM for electrode F7, %LAM for electrode Pz, and RR for electrode Fp1. The features selection process was performed to reduce potential overfitting especially considering the limited size of the dataset used for this work.

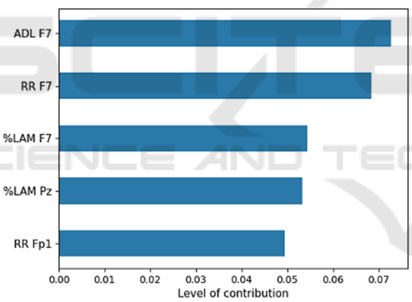

Figure 4: The features selected according to their level of contribution.

After having selected the most relevant features, the class imbalance present in the dataset (10 subjects labeled as "B" and 26 labeled as "G"), was solved using the SMOTE function. Eventually, the final dataset fed to the classifiers was composed of 5 features and 52 instances of which 36 were original and 16 synthetically created.

3.5 Classifiers Specifications

Multiple classifiers were used to confirm the effectiveness of using RQA-based features to detect cognitive performance. For this reason, an SVM, an RF and, a GBC were used for classification purposes. The targets of the classification were group "G" and group "B", respectively encoded as 1 and 0.

The hyperparameters selection was performed using a randomized search on the 3 classifiers. The hyperparameters adopted after having performed the randomized search can be found in the following link: https://osf.io/wtxpv/?view_only=ab98b469151a48a1a91d221dc6596429.

4 RESULTS

The results obtained using the 3 classifiers show a performance above 0.85 accuracy using 5-fold-cross validation. In order to verify that the performance was not due to the presence of synthetic data, the classification task was also performed on the imbalanced dataset containing 36 instances. The results suggest that, even in the case of an imbalanced dataset, the classifiers managed to perform reasonably well on this specific task. A more detailed overview of the performance obtained by each single classifier using both the imbalance and balance datasets can be visualized in Table 1.

Table 1: The accuracy scores obtained using the original imbalanced dataset and the balanced dataset after resolving the class imbalance.

	Imbalanced dataset accuracy	Balanced dataset accuracy
RF	0.77 (SD = 0.07)	0.89 (SD = 0.09)
SVM	0.75 (SD = 0.13)	0.90 (SD = 0.06)
GBC	0.85 (SD = 0.12)	0.87 (SD = 0.04)

The use of the classifiers on the imbalanced dataset seemed to confirm that the three classifiers adopted in this study still performed above chance given the information provided by the RQA-based features.

5 DISCUSSION

This work aimed to investigate whether RQA-based features could be used to successfully detect group differences in a mental arithmetic task. We hypothesized that the dynamics of the EEG signals can differentiate participants with different levels of numerical proficiency. The obtained results confirm the hypothesis that the RQA-based features extracted from the signal could discriminate effectively between the two groups in a machine learning binary classification task. These results are in line with other studies that combined machine learning and RQA-based features to detect epilepsy, drowsiness, and preterm birth (Borowska et al., 2018; Gruszczyńska et al., 2019; Shabani, Mikaili, & Noori, 2016). Given the results of our study, it is reasonable to think that RQA has the potential to detect or differentiate performance in other cognitive domains. For example, RQA-based features might be adopted in the context of training and when comparing experts and novices on a domain-specific task. To this extent, there may be features showing convergence between novices and experts after a period of training. Future studies might investigate if our findings can be extended to skills belonging to other cognitive domains.

The main contribution of this study consists in providing insights into the nature of the signal characterizing the two groups. Using RQA-based features, instead of other methods such as neural networks, provide information about how the signal differs in the two groups. Tracking changes in the extracted signal, and being able to quantify them, might be useful when considering the effect of training or to evaluate if a needed intervention to improve proficiency had a beneficial outcome. The results of our study show that %LAM and RR are present two times among the features selected. The difference in RR between the two groups seems to be intuitively visualized where participants belonging to group "B" seem to have a much more deterministic structure in the RP compared to participants in group "G" (see Figure 2 and Figure 3). Interestingly, according to Zbilut and Webber (2008), %LAM seems a crucial feature of biological signals, and more specifically physiological signals given that it represents transitions such as those occurring between chaotic and periodic phases. High values of %LAM, in the context of a physiological signal, were associated with low flexibility, high stability, and more time needed for state transitions (Curtin et al. 2017). For example, experts showed lower %LAM than novices in an experiment involving eye-tracking when inspecting dermatological images (Vaidyanathan et al., 2014). ADL, the most important feature in our selection, might follow a similar pattern to %LAM where higher values might represent a more deterministic system. In the context of cognitive skills, a higher %LAM and a longer ADL might represent a more deterministic and less complex signal, which might affect the time needed to switch from a task to another resulting in poor performance.

The current study also offers insights relevant for EEG and electrode selection, as it answers the question of which electrode signals are most relevant when extracting features using RQA. This study indicates that F7 alone might be relevant for classification purposes in this specific task. In fact, the three top features out of five were extracted from this electrode. Similarly, Mikaili and Noori (2016) found that F8 alone was effective in detecting subjects suffering from drowsiness.

The RQA-based features extracted to detect cognitive performance related to numeracy seem to provide high performance, especially once the class imbalance is resolved, independently of the classifier used. Ghosh and Saha (2021) employed a recurrent neural network and features extracted using power spectral density and correntropy spectral density, obtaining an accuracy of 0.89 in detecting proficiency in the same task used for this study. These results, comparable to the ones obtained in the current study, seem to provide further evidence about the effectiveness of using RQA-based features to detect performance in this domain. Future work might implement models combining RQA-based features with features extracted with other methods (e.g., spectral content) to verify if this approach might lead to higher accuracy in classifying tasks in the numerical domain.

More generally, RQA-based features have previously been shown to be effective in several domains to analyze numerous physiological signals ranging from the electrocardiogram (Zbilut & Webber, 2008) to the electrohysterogram (Borowska et al., 2018). RQA is generally noise-resistant and it does not require any linear transformation before performing the analysis (Zbilut, Thomasson, & Webber 2002). Furthermore, the extracted features offer interpretability giving insights into the nature of the signal. Such characteristics might encourage researchers to use this method in other contexts and domains exploring its potentiality combined with machine learning and deep learning models.

However, despite the advantages offered by this method, it is important to put our findings in context. The dataset used had a relatively small sample, which may have affected the results. This issue characterizes most of the recent studies involving physiological measurements, machine learning, and RQA-based features where the number of participants often tends to be small. Consequently, this issue posits limitations when applying machine learning models.

Another limitation affecting this study is the limited number of RQA-based features selected. RQA can be computationally expensive and it might require a lot of time, or computational power, to extract its features in case of long time-series and phase space reconstructions with several dimensions and high delay values. Therefore, the current study was limited to the extraction of 6 RQA-based features from just 4 electrodes. As a consequence, this work was not able to provide a wider overview of the relevance of other features and electrodes, and their effect on the machine learning models' performance.

Furthermore, the results obtained in our work do not provide a thorough comparison between the features extracted using linear methods on EEG data and those obtained using RQA. Future studies should apply RQA to larger datasets and accurately compare RQA-based features with features extracted using linear methods. Such efforts might provide more information about the effectiveness of using this non-linear method to extract features for machine learning purposes.

6 CONCLUSIONS

The RQA-based features extracted from EEG signals seem to provide adequate information to track cognitive performance. Such an approach might be implemented as an alternative to the classic linear methods used to analyze EEG data. Future research might provide insights into the effect of each single RQA-based feature on performance and compare the effectiveness of such features with the ones extracted using different methods.

ACKNOWLEDGMENTS

The research reported in this study is funded by the MasterMinds project, part of the RegionDeal Mid- and West-Brabant, and is co-funded by the Ministry of Economic Affairs, Region Hart van Brabant, REWIN, Region West-Brabant, Midpoint Brabant, Municipality of Breda, and Municipality of Tilburg awarded to MML.

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