Fast and Efficient Union of Sparse Orthonormal Transform for Image Compression

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Abstract: Sparse coding has been widely used in image processing. Overcomplete-based sparse coding is powerful to represent data as a small number of bases, but with time-consuming optimization methods. Orthogonal sparse coding is relatively fast and well-suitable in image compression like analytic transforms with better performance than the existing analytic transforms. Thus, there have been many attempts to design image transform based on orthogonal sparse coding. In this paper, we introduce an extension of sparse orthonormal transform (SOT) based on unions of orthonormal bases (UONB) for image compression. Different from UONB, we allocate image patches to one orthonormal dictionary according to their direction. To accelerate the method, we factorize our dictionaries into the discrete cosine transform matrix and another orthonormal matrix. In addition, for more effective implementation, calculation of direction is also conducted in DCT domain. As expected, our framework fulfills the goal of improving compression performance of SOT with fast implementation. Through experiments, we verify that proposed method produces similar performance to overcomplete dictionary outperforms SOT in compression with rather faster speed. The proposed methods are from twice to four times faster than the SOT and hundreds of times faster than UONB.

1 INTRODUCTION

For the past decades, sparse coding, which express the input image with a small amount of information minimizing the loss of original information as much as possible, has been an important tool and is widely used in many signal and image processing applications (Zhang et al., 2015). In compression, they have not yet been applied in standards, but many works to design transform for compression or transform coding scheme to be used for compression standards (Sezer et al., 2015) have been proposed. Because sparse coding can increase coding efficiency by rate-distortion optimization and quantization as in (Kalluri et al., 2019), a set of sparse coding transforms can replace conventional transforms, such as DCT.

Sparse coding techniques are generally based on an overcomplete dictionary (Aharon et al., 2006), (Elad and Aharon, 2006), indicating a dictionary with larger number of columns than the number of rows. Therefore, its atoms are generally nonorthogonal with redundant properties and can make better representative dictionary to input signal. Owing to this characteristic, it has been a powerful tool in fields. Mathematically, given a dataset $X \in \mathbb{R}^{n\times N}$, it is formulated as:

$$\min_{D,A}\left\{\|X-DA\|_F^2+\lambda\|A\|_0\right\}, \tag{1}$$

where $D \in \mathbb{R}^{n\times m}$ is an overcomplete dictionary ($m > n$) and $A \in \mathbb{R}^{m\times N}$ is a sparse coefficient.

However, for sparse coding based on an overcomplete dictionary, finding an appropriate dictionary is generally a nondeterministic polynomial time-hard problem. This requires iterative optimization such as the method of optimal directions (MOD), the alternating direction method of multipliers (ADMM) and augmented Lagrange multipliers (ALM), with greedy algorithms, such as basis pursuit and orthogonal matching pursuit to estimate the approximate value (Zhang et al., 2015). They are sufficiently good methods to solve the sparse approximation problem, but they inevitably require considerable time and memory resources for learning. Therefore, it is one of main open problems in the field to design fast and efficient dictionary learning algorithms.

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To overcome this problem, (Lesage et al., 2005) proposed a sparse coding method. When a dictionary is square and orthogonal, the dictionary learning scheme becomes the orthogonal Procrustes problem. Therefore, this is solved by relatively simple and efficient singular value decomposition (SVD). (Lesage et al., 2005) and (Rusu and Dumitrescu, 2013) proposed methods implementing an overcomplete dictionary using unions of orthonormal bases (UONB). The basic formulation, which was proposed first by (Lesage et al., 2005), is formulated as:

$$\min_{D,A}\left\{ \|X-[D_1|D_2|...|D_L]A\|_F^2+\lambda\|A\|_0 \right\} \quad s.t. \quad D_i^T D_i = D_i D_i^T = I_n, \tag{2}$$

where $D_i \in \mathbb{R}^{n\times n}$ is an orthogonal sub-dictionary and $i = 1,...,L$.

On the other hands (Schütze et al., 2016), (Sezer et al., 2008) and (Sezer et al., 2015) proposed efficient dictionary learning methods based on an orthogonal dictionary. Orthogonal sparse coding techniques are mathematically simple and fast because they also require iterative optimization, but there are closed-form solutions of dictionary and coefficient matrix for each iteration. They cannot only compute the orthogonal dictionary via singular value decomposition, but the coefficients are also easily computed by inner products and thresholding.

In addition, this fits well in image compression. Many image compression methods depend on analytic transforms. Transform is similar in purpose to sparse coding in that it represents the input signal with minimal basis. Because of this reason, there have been many attempts to substitute analytic transform with sparse coding-based ones. However, the overcomplete dictionary is not a square matrix and does not have to be orthogonal. Then, it does not contain the same bases as the transform and has no inverse transform. Because of these problems, these methods cannot replace the existing analytic transforms regardless of its capabilities. Unlike this, the existing orthogonal transforms such as discrete cosine transform (DCT) and Karhunen-Loeve transform (KLT) have been widely used in image compression field and the orthogonal dictionaries, since orthogonal sparse coding have similar properties with the transforms, such as it is invertible and satisfies Parseval's theorem. Sparse orthonormal transform exploited the orthogonal sparse coding scheme and proposed an orthogonal transform for image compression. This is theoretically reduced to the KLT in Gaussian process and superior in non-Gaussian processes with higher computation speed than that of overcomplete dictionary-based methods.

In this paper, we expand the SOT to outperform the performance of orthonormal dictionary learning based on the union of orthonormal bases. However, different from (Lesage et al., 2005), which use block coordinate relaxation or greedy orthogonal matching pursuit in coefficient update, we adapt hard threshold method of inner product, used in orthogonal sparse coding. To this end, we classify input data and allocate each orthogonal dictionary and its coefficients to each classified input data. When the number of orthogonal dictionaries increases, the compression performance also increases, but the computational time is also proportional to the number. To prevent this, we use a double-sparsity structure proposed in (Rubinstein et al., 2010). We use a DCT matrix as a fixed base dictionary of double-sparsity structure. In this way, we design a transform which outperform the performance of the SOT, but this algorithm is rather faster.

2 SPARSE ORTHONORMAL TRANSFORM

For the past few decades, there have been many attempts to make data-driven transforms using sparse coding to achieve better performance than analytic transforms (Sezer et al., 2015), (Ravishankar and Bresler, 2013). Especially the orthogonal dictionary-based sparse coding does not only give more compact representations of input data than existing analytic transforms, but also decorrelates the data like analytic transforms. Also, the orthonormal dictionary can not be only applied as a dictionary form, but also as a transform. It is because the inverse matrix of orthonormal dictionary is its transpose, $\|X-GA\|_F^2 = \left\|G^T X - G^T GA\right\|_F^2 = \left\|G^T X - A\right\|_F^2$. The first is a dictionary form and the last is equal to transform, which makes data sparse by product. In this section, we introduce a recent work based on an orthogonal sparse coding called sparse orthonormal transform (SOT).

In (Sezer et al., 2008) and (Sezer et al., 2015), the basic idea of SOT is simple. It was designed based on an orthogonal sparse coding methodology. Sezer et al. formulated a transform with an orthonormal matrix and an L_0 norm constraint to the transform coefficients:

$$\min_{G,A}\left\{ \|X-GA\|_F^2+\lambda\|A\|_0 \right\} \quad s.t. \quad G^T G = GG^T = I_n, \tag{3}$$

where A is the sparse transform coefficient, G is the SOT matrix, and I_n is $n \times n$. They use an iterative optimization methods to find two variables, a dictionary and a coefficient matrix. This problem is solved by following algorithm:

Given the dataset $X = \{x_1, x_2, ..., x_m\} \in \mathbb{R}^{n \times N}$, $\lambda > 0$ and initial orthonormal matrix G_0,

Initialization:

$$G = G_0 \ .$$

Iterations:

Iterate until the stopping criterion is met:

1. Update the coefficients:

$$A = \mathcal{T}(G^T X, \lambda^{1/2}) \ .$$

2. Find the optimal dictionary:

(a) Compute the singular value decomposition:

$$XA^T = U \Sigma V^T \ .$$

(b) Update new dictionary:

$$G = UV^T \ .$$

$\mathcal{T}(\cdot, \alpha)$ is a hard-threshold operator, which zerorizes when the absolute value is smaller than α. U and V are left and right singular vector matrix respectively. They verify that SOT is the principled extensions of KLT because this transform is theoretically reduced to KLT in Gaussian processes. It is well-known that the KLT is optimal in Gaussian process and it shows that the optimal dictionary in Equation (3) has same structure with KLT in Gaussian process. In other words, the SOT is also optimal in Gaussian process, and is superior to KLT in non-Gaussian process. They experimentally show the transform is superior to DCT and KLT in image compression.

3 PROPOSED METHOD

3.1 Motivation

(Sezer et al., 2015) proposed the SOT, which improves the existing analytic transforms in image compression and has the same properties of analytic transforms. In addition, it is relatively efficient than overcomplete dictionary-based methods because it eliminates time-consuming greedy algorithms by orthogonal sparse coding schemes. However, the orthogonal dictionary is well fit to compress images, but restricts its performance because of its size. To the best of our knowledge, there are some attempts to achieve close performance to an overcomplete dictionary with orthogonal dictionary, but an orthogonal dictionary generally has poorer performance than the one of overcomplete dictionary. Dictionaries with larger size generally have more redundant representation, so they produces sparser representation of the input data.

In this paper, we propose an extension of SOT. To overcome the limitation of orthogonal dictionary, we construct the dictionary as several orthogonal dictionaries such as UONB in (2). However, because large number of dictionaries require more computations and time, we propose another technique to mitigate the problem.

3.2 Algorithm Explanation

We used the method in (Lesage et al., 2005) to construct overall dictionary and modified this for fast implementation. (Lesage et al., 2005) exploits a greedy algorithm to update coefficients A in Equation (2). Different from UONB, coefficients update scheme of orthogonal sparse coding exploits simple hard threshold method (Schütze et al., 2016). To extend the orthogonal sparse coding algorithm in Section 2 to our algorithm, we assign each input data to an orthogonal dictionary. In other words, we classify input data to generate more optimal orthogonal dictionary for each group. We discern input image patches by their directions. We assume that when the patches with similar direction are gathered, the more optimal dictionary is generated. The assumption comes from DCT, which gives optimal performance for horizontal or vertical directional patches in image compression, but poor performance for arbitrary directional patches (Pavez et al., 2015).

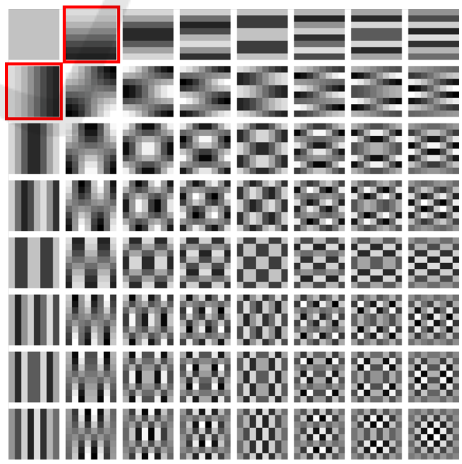


Figure 1: The basis of two-dimensional discrete cosine transform. Each basis include horizontal or vertical directional information.

In this paper, we exploit the DCT matrix because of its characteristics for improving performance and reducing time spent for the SOT. As the DCT is popular in image compression fields, it has been important to analyze and extract information of images in the compressed domain for fast implementation. In particular, (Shen and Sethi, 1996) designed an edge model in the DCT domain, based on the characteristics of the DCT. As mentioned earlier, the DCT gives optimal performance for horizontal and vertical directional data. It is because the bases of the DCT represent the horizontal and vertical directions or the diagonal directions made by their combinations. The two directional bases have the same edge complexity, according to their order. This is well illustrated in the bases in Figure 1. The bases in the red box show the same complex edge information in different directions. (Shen and Sethi, 1996) directly extracted low-level features, such as edge orientation, edge offset, and edge strength, from DCT compressed images. (Shen and Sethi, 1996) suggested four metrics for edge orientation, with coefficients based on the 8x8 block DCT, and we use and introduce one of the metrics.

For simple and efficient implementation, the proposed method exploit DCT matrix in two ways. First, we discern the patches in DCT domain with a formulation below:

$$\theta = \begin{cases} \tan^{-1}(\left|\frac{C_{01}}{C_{10}}\right|), , & \text{where} \quad C_{01}C_{10} \geq 0 \\ 90^\circ - \tan^{-1}(\left|\frac{C_{01}}{C_{10}}\right|), & \text{where} \quad C_{01}C_{10} < 0, \end{cases} \tag{4}$$

where C_{01} and C_{10} are DCT coefficients in (0, 1) and (1, 0), which are corresponding to the bases in the red box in Figure1. We restrict the range of direction from 0° to 90°.

Then, we quantize the θ in L levels. L is the number of orthogonal dictionaries used in the proposed method. As in (Lesage et al., 2005), we construct the dictionary as a set of several orthonormal dictionaries, i.e. $D = [D_1|D_2|...|D_L]$, where D_is are orthogonal square matrix. Then, we classify input data into L groups and assign each to an orthogonal dictionary. When the value of L increases, it is generally natural that the performance of compression also improves. We verify this via experiments in Section 4.

However, with the performance improvement, the number of orthogonal dictionaries is also increased. Because this leads to increasing computation time, it significantly impairs the strength of orthogonal sparse coding. To prevent this, the DCT matrix is used again. We exploit the so-called double sparsity method proposed in (Rubinstein et al., 2010). (Rubinstein et al., 2010) proposed the method to bridge the gap between analytic approach and learning-based approach. They factorize a dictionary as two matrices, which are a prespecified base dictionary and an atom representation matrix.

We construct our dictionary by the product two-dimensional DCT matrix as a fixed base dictionary and another dictionary, which is only computed via optimization procedure.

In a mathematical formulation:

$$D = TH, \tag{5}$$

where T is a DCT matrix in $\mathbb{R}^{n\times n}$ and H is an orthonormal matrix in $\mathbb{R}^{n\times n}$.

As mentioned above, the object of using DCT matrix as base dictionary is to reduce the convergence time of algorithm. Because it is well-known that the DCT matrix produces quite good sparsity in advance, it accelerates the algorithm with fewer iterations than the case, which constructs dictionary with only a dictionary. Based on equations (4) and (5), our proposed method can be formulated in detail:

For an input data $X = [X_1|X_2|...|X_L] \in \mathbb{R}^{n\times N}$, where the X_i is the patch of which direction is between $(90^\circ/L)(i-1)$ and $(90^\circ/L)i$, the dictionary is $D = T[H_1|H_2|...|H_L] \in \mathbb{R}^{n\times Ln}$ and the sparse coefficient matrix is $A = [A_1^T|A_2^T|...|A_L^T]^T \in \mathbb{R}^{Ln\times N}$,

$$\min_{H_i,A_i} \sum_{l=1}^{L} \left\{ \|X_i - TH_iA_i\|_F^2 + \lambda\|A_i\|_0 \right\} \quad s.t. \quad H_i^T H_i = H_i H_i^T = I_n, \tag{6}$$

where $i = 1,...,L$.

For efficient implementation, all data is processed in DCT domain during all procedure. First, input image patches are transformed in DCT domain by product with two-dimensional DCT matrix. Second, the patches are classified through Equation (4). Since a DCT matrix is orthonormal matrix, the Frobenius norm of the DCT matrix, $\|T\|_F$, is 1. Then the Equation (6) is transformed as follows:

$$\min_{H_i,A_i} \sum_{l=1}^{L} \left\{ \left\|\hat{X}_i - H_iA_i\right\|_F^2 + \lambda\|A_i\|_0 \right\} \quad s.t. \quad H_i^T H_i = H_i H_i^T = I_n, \tag{7}$$

where $\hat{X}_i = T^T X_i$, the transformed data in DCT domain.

Then, our overall proposed algorithm is indicated below:

Given the input $\sqrt{n} \times \sqrt{n}$ image patches, the dataset $X = \{x_1, x_2, ..., x_m\} \in \mathbb{R}^{n\times N}$, $\lambda > 0$ and the number of dictionaries, L,

Initialization:

1. The input data X_is are transformed into DCT domain.

2. Classify the transformed data $\hat{X}_i$s into L groups via Equation(4).

Iterations:
Iterate until the stopping criterion is met:

1. Update the coefficients: for each coefficient matrix $l = 1, \ldots, L$,

$$A_i = \mathcal{T}(H_i^T \hat{X}_i, \lambda^{1/2}).$$

2. Find the optimal dictionary: For each orthogonal dictionary $l = 1, \ldots, L$,

(a) Compute the singular value decomposition:

$$\hat{X}_i A_i^T = U_i \Sigma_i V_i^T.$$

(b) Update dictionary by the inner product:

$$H_i = U_i V_i^T.$$

4 EXPERIMENTS

4.1 Experimental Environment

We experimented with our methods using images in Figure 2. To create equivalent environments for comparison, we resized the image to 256×256 pixels and segmented it to 256×256 or 8×8 patches before applying the algorithm. To measure the performances of different algorithm, we focused on image compression, comparing PSNR (dB) with the number of used coefficients. Sparse coding-based algorithms are dependent on the value of λ in their formulation, Equations (3) and (6), and this decides the best sparsity. In the compression sense, the optimal value is varied from the number of coefficients or bases used in compression scheme. In this paper, we heuristically try to find the value for each level of sparsity before these experiments for each method. Then, in experiments, we only use the optimal values of λ as prior information and do not consider the computation time to find it.

All the algorithms used in these experiments are conducted with the same stopping condition. We set the stopping condition to the difference of objective functions between present and 10 iterations past.

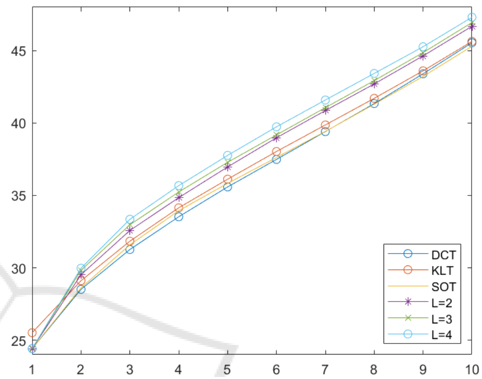
For equivalent comparisons, each algorithm and all experiments were implemented using MATLAB R2021a in Windows 10 Education on a same computer, equipped with an Intel i7-9700 CPU and 32-GB RAM.

4.2 Experimental Results

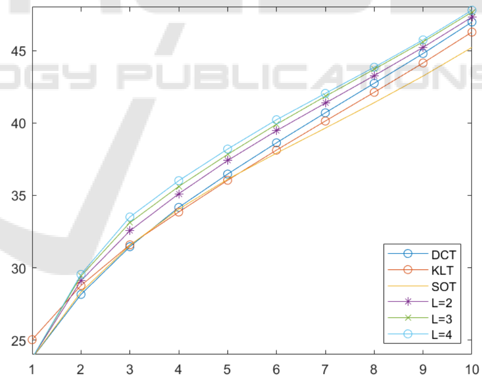
In this subsection, we first compare the SOT and the proposed method with different number of dictionaries for energy compaction. The SOT is achieved by

(a) Lena. (b) peppers.

Figure 2: The test image. We experiment and verify proposed method with Lena and peppers image.



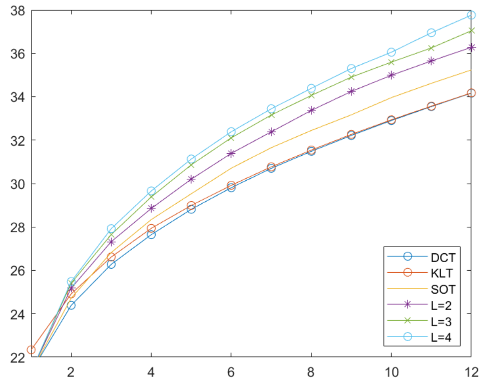
(a) Lena.



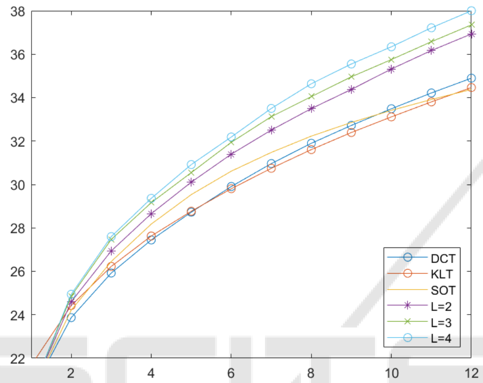
(b) peppers.

Figure 3: The object quality comparison: PSNR (dB) versus the number of retained coefficient for 4 x 4 patches between SOT and our methods.

the algorithm in Section 2. Figures 3 and 4 show the comparison of the objective qualities, in PSNR (dB), for each number of retained bases. All proposed methods with different number of dictionaries outperform the SOT method in PSNR (dB). As shown in Figures 3 and 4, our method has improvement from SOT, which is constructed by one orthonormal dictionary, in performance. We analyze the result rise from



(a) Lena.



(b) peppers.

Figure 4: The object quality comparison: PSNR (dB) versus the number of retained coefficient for 8 x 8 patches between SOT and our methods.

two conditions: (a) sparse coding algorithms work better based on the input data classified according to their structure than whole unstructured input data, and (b) the dictionary from small dataset is more adaptive and well representative than from large dataset. In addition, it is interesting that the difference of reconstruction error between proposed method and SOT in Figure 4 is less than the difference in Figure 3. Small patches have more simple and dominant directional information than large patches. Large patches usually have more complex and have diverse orientations. This makes the difference.

Figure 5 explains the assertion (a) is reasonable. We verify our proposed classification method for input data by Equation (4) works well. Figure 5 plots the difference between two different classification methods. We compare our classification method based on Equation (4) with simply grouping the data evenly in order. In Figure 5, 'cls-direction' indicates our classification method using direction and 'cls-order' indicates a sequentially grouping way. In all cases, our methods produce better performance. It

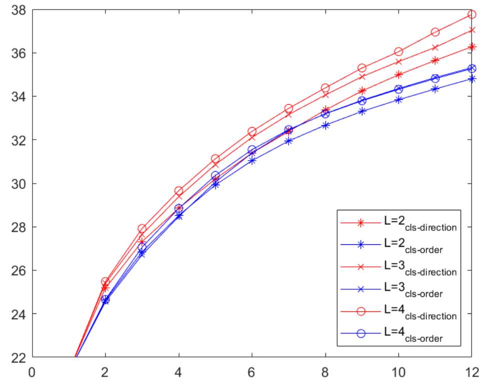


Figure 5: The object quality comparison: PSNR (dB) versus the number of retained coefficient for 8 x 8 patches between classification methods with different number of orthogonal dictionaries.

indicates that the dictionary made of data including similar structure is better at expressing than the dictionary made of irregular and unstructured data.

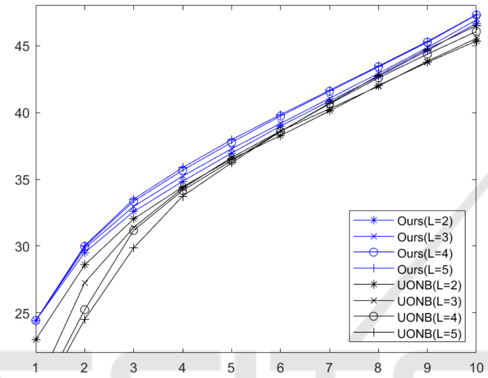
To compare with the overcomplete dictionary-based algorithm, we use UONB with optimization in (Lesage et al., 2005) and block coordinate relaxation algorithm, which is faster than the orthogonal matching pursuit. Experiment is conducted by the different numbers of dictionaries of proposed method and UONB from 2 to 5. Figure 6 shows the experimental result. We observe that the difference between our method and UONB in Figure 6. In Figure 6, proposed method is more powerful for small patches. In Figure 6-(b), performance graphs of UONB have the crossing point of performance at about 4 retained coefficients. At the two and three in X-axis, the UONB with two dictionaries gives better performance than others, but at more than four coefficients, large number of dictionaries make better performance. Different from UONB, our method results in an improved performance with the increasing number of dictionaries. Comparing the two methods, our method outperforms UONB when the number of coefficients is small. As the number of bases increases, the performance difference of UONB is greater, and when a large number of coefficients are used, UONB shows better performance. We infer that this result comes from difference between overcomplete and orthogonal dictionaries. Despite the number of dictionaries, our method is based on square orthogonal dictionaries. This leads to the difference of expressiveness when the number of basis are increased.

4.3 Processing Time

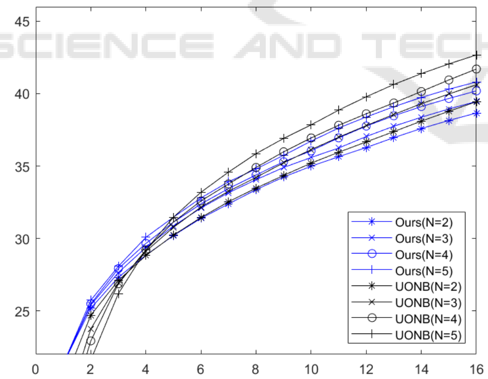
One of our main contributions is to reduce computation and time with improved performance. To reduce

Table 1: The processing time and number of iterations for each method. L indicates the number of orthogonal dictionaries. We compare each algorithm with λ optimal for two different number of retained coefficients. The bold texts indicate minimum results.

L	# of retained coefficients	SOT		UONB		Proposed	
		Iterations	Time (s)	Iterations	Time (s)	Iterations	Time (s)
1	3	2095	1.4553	-	-	-	-
	5	2741	1.9480	-	-	-	-
2	3	-	-	622	49.7882	**72**	**0.0498**
	5	-	-	536	82.9780	**107**	**0.0650**
3	3	-	-	628	343.3251	**183**	**0.1174**
	5	-	-	505	97.5815	**157**	**0.0872**
4	3	-	-	482	57.8052	**195**	**0.0920**
	5	-	-	754	101.3284	**192**	**0.1007**



(a) 4×4 patches.



(b) 8×8 patches.

Figure 6: The object quality comparison: PSNR (dB) versus the number of retained coefficient for different patch sizes of Lena image between UONB and our methods with different number of orthogonal dictionaries.

time, we tried to make the best use of DCT matrix. In Table 1, we compared SOT, UONB, and proposed method for the number of iterations and spent time in seconds until convergence. In this section, we experiment UONB and proposed method for three dictionary sizes, L = 2, 3, 4. Because SOT use only an orthogonal dictionary, it is only marked in L = 1. For designing equivalent experimental setting, we set λs optimal to two level of sparsity, 3 and 5. In this section, the time to search λ is not considered.

From the Table 1, we observe that our proposed method works the best at all cases. SOT works better than UONB in time. Although the number of iterations required in convergence for SOT is much larger than one required for UONB, SOT is much faster than UONB, because it does not use greedy algorithms. Comparing our method and UONB, the number of iteration for our proposed method is several times smaller than UONB. The degree of reduction is different, about twice to four times, but in all cases our algorithm requires less iterations. Comparing the computation time, the differences become larger than iterations. Because our method requires smaller number of iterations as mentioned and computation time for each iteration is much less than UONB like SOT, our method is hundreds times faster than UONB on average.

One of the interesting points of this section is the comparison between SOT and proposed method. Although proposed method tries to find more dictionaries and coefficients than SOT, we achieve reduction in the number of iterations by factorizing a dictionary into DCT matrix and an orthonormal matrix, and it leads to prevent from increase in time.

5 CONCLUSIONS

In this paper, we proposed a novel sparse coding-based image transform framework for efficient implementation as the form of extension of SOT. Overcomplete dictionary-based methods produce good sparse representation, but require a long time and many resources because of their iterative or greedy optimizations. Also, it does not fit to image compression as analytic transforms, which are invertible and satisfy Parseval's theorem. Orthogonal sparse coding is proposed to overcome the time-consuming algorithm of overcomplete dictionary-based algorithm. The orthogonal sparse coding also has a lot in common with analytic transforms such as DCT and KLT. Because the dictionary is square and orthonormal, this is invertible and conserves the energy of data. Thus, orthogonal sparse coding-based transforms for image compression have been proposed for past decades.

One of these transforms is SOT. SOT is theoretically proved to outdo KLT (Sezer et al., 2015). We extend the SOT based on unions of several orthonormal dictionaries such as UONB. Although the number of variables to be computed increases, we prevent from increasing computational time by making the best use of DCT matrix for classification of input data and factorization of dictionaries. As the result of these efforts, the proposed method outperforms the SOT with reduction of computation time. The section 4 verifies that our method satisfies the object of this paper through PSNR graphs and a table of processing time.

In this paper, we only proposed sparse coding-based transform scheme for image compression. In the future works, we attempt to design the overall transform coding scheme for better image compression as in (Sezer et al., 2015).

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REFERENCES

Aharon, M., Elad, M., and Bruckstein, A. (2006). K-svd: An algorithm for designing overcomplete dictionaries for sparse representation. *IEEE Transactions on Signal Processing*, 54(11):4311–4322.

Elad, M. and Aharon, M. (2006). Image denoising via sparse and redundant representations over learned dictionaries. *IEEE Transactions on Image Processing*, 15(12):3736–3745.

Kalluri, M., Jiang, M., Ling, N., Zheng, J., and Zhang, P. (2019). Adaptive rd optimal sparse coding with quantization for image compression. *IEEE Transactions on Multimedia*, 21(1):39–50.

Lesage, S., Gribonval, R., Bimbot, F., and Benaroya, L. (2005). Learning unions of orthonormal bases with thresholded singular value decomposition. In *Proceedings. (ICASSP '05). IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005.*, volume 5, pages v/293–v/296 Vol. 5.

Pavez, E., Egilmez, H. E., Wang, Y., and Ortega, A. (2015). Gtt: Graph template transforms with applications to image coding. In *2015 Picture Coding Symposium (PCS)*, pages 199–203.

Ravishankar, S. and Bresler, Y. (2013). Learning sparsifying transforms. *IEEE Transactions on Signal Processing*, 61(5):1072–1086.

Rubinstein, R., Zibulevsky, M., and Elad, M. (2010). Double sparsity: Learning sparse dictionaries for sparse signal approximation. *IEEE Transactions on Signal Processing*, 58(3):1553–1564.

Rusu, C. and Dumitrescu, B. (2013). Block orthonormal overcomplete dictionary learning. In *21st European Signal Processing Conference (EUSIPCO 2013)*, pages 1–5.

Schütze, H., Barth, E., and Martinetz, T. (2016). Learning efficient data representations with orthogonal sparse coding. *IEEE Transactions on Computational Imaging*, 2(3):177–189.

Sezer, O. G., Guleryuz, O. G., and Altunbasak, Y. (2015). Approximation and compression with sparse orthonormal transforms. *IEEE Transactions on Image Processing*, 24(8):2328–2343.

Sezer, O. G., Harmanci, O., and Guleryuz, O. G. (2008). Sparse orthonormal transforms for image compression. In *2008 15th IEEE International Conference on Image Processing*, pages 149–152.

Shen, B. and Sethi, I. K. (1996). Direct feature extraction from compressed images. In Sethi, I. K. and Jain, R. C., editors, *Storage and Retrieval for Still Image and Video Databases IV*, volume 2670, pages 404 – 414. International Society for Optics and Photonics, SPIE.

Zhang, Z., Xu, Y., Yang, J., Li, X., and Zhang, D. (2015). A survey of sparse representation: Algorithms and applications. *IEEE Access*, 3:490–530.