

A Longitudinal Model for Song Popularity Prediction

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Abstract: Usage of new generation music streaming platforms such as Spotify and Apple Music has increased rapidly in the last years. Automatic prediction of a song's popularity is valuable for these firms which in turn translates into higher customer satisfaction. In this study, we develop and compare several statistical models to predict song popularity by using acoustic and artist-related features. We compare results from two countries to understand whether there are any cultural differences for popular songs. To compare the results, we use weekly charts and songs' acoustic features as data sources. In addition to acoustic features, we add acoustic similarity, genre, local popularity, song recentness features into the dataset. We applied Flexible Least Squares (FLS) method to estimate song streams and observe time-varying regression coefficients using a quadratic program. FLS method predicts the number of weekly streams of a song using the acoustic features and the additional features in the dataset while keeping weekly model differences as small as possible. Results show that the significant changes in the regression coefficients may reflect the changes in the music tastes of the countries.

1 INTRODUCTION

The music industry is expanding every day with new artists, songs, and listeners. The growth of the music industry has been increasing since 2014 with the impact of music streaming services on preventing piracy (Stone, 2020). In 2020, half of the revenue of the industry is generated by these services and at the end of 2020 over 70 million songs were available in the leading music stream service, Spotify, with a market coverage of 170 countries. (Spotify, 2021) Increasing popularity of online music streaming services allows listeners to access newly released songs around the world besides the old ones. Hence the increasing variety of artists, song genres, and songs generate a vast amount of data with the help of the digitalization of the music industry

Music streaming services expanded their customer base with the increasing trend of paid media services. Increasing number of users creates higher number of streams for the songs which helps the music economy to grow. Meanwhile some of these services losing their share in the market, Apple Music, Spotify, and YouTube Music are becoming

more popular than ever. At the beginning of 2020, Spotify got 32-34% of the market which makes Spotify the market leader with 345 million users. (Mulligan, 2020).

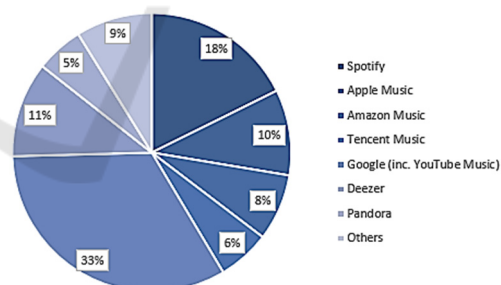


Figure 1: Market share of the music streaming services in 2020.

These platforms collect and store usage information about the users. Besides, some acoustic numerical features are generated out of the songs by these platforms. All this collected information allows developers/researchers to analyze the music listening habits, detect the hit songs/genres, and develop musical insights for producers, listeners, and artists. To increase user engagement and satisfaction, it is

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crucial to analyze music tastes. The music taste of the audience differs between countries and changes over time. Acoustic features, historical events, seasons, holidays, and social influence are factors that may affect a song's popularity. Naturally, the effects of these factors are dynamic and culture-specific. In different countries, song popularity may be affected by these factors differently over time. Geographical and cultural distances between countries affect the popularity of the songs. (Buda and Jarnowski, 2015)

In this paper, we used a time-varying penalized regression model to predict the number of song streams using acoustic features gathered using Spotify API and monitor the regression parameters over a period to discover the effects of the acoustic features on the music taste. Our approach allows us to smooth the extreme changes of the regression parameters over time so that the shifts in the musical preferences are reflected realistically. We also compared the results for two countries; Turkey and the U.S. to observe the cross-cultural differences in the music tastes.

The rest of the paper is organized as follows. Section 2 provides a review of related literature. Section 3 describes the problem that we have worked on and explains the methodology that we have used. Section 4 explains the data scraping and preparation steps and the results of the regression model. Finally, Section 5 concludes the paper and discusses future works.

2 RELATED WORK

Our motivation is to examine cross-cultural music preferences and their shifts over time by using the charts data and the acoustic features of songs. Previous works that focused on similar problems exist in the literature and the methodologies that they follow in these works vary. There are applications of predictive algorithms and survey-based approaches for understanding the Spatio-temporal music preferences of different populations.

A regression-based approach is used by (Suh, 2019) with Spotify's acoustic features to predict songs' success on the charts. They use OLS regression for prediction and analyse variable significances for 6 different countries. (Pınarbaşı, 2019) analyse music popularity characteristics of Turkey for a 6-month period by using decision tree algorithm. They also cluster the acoustic features gathered from the Spotify and concluded with 3 different clusters with similar acoustic features. (Yadati et al., 2017) focuses on the change of the

musical preferences when the mood/activity change. Their findings show that the acoustic features of the songs and the genre/instrument information are not sufficient for predicting the mood/activity change. Classification models are also applied to predict whether a song is a hit or not. A similar work from (Al-Beitawi et al., 2020) shows that the musical attributes from Spotify may help clustering the songs and discovering the acoustic features that have influence on the song popularity such as high danceability and low instrumentality. (Herremans et al., 2014) compare classification methods such as SVM, Naïve Bayes, logistic regression, and decision tree for hit song prediction. Their dataset includes Billboard's Hot 100 charts with the acoustic features from The Echo Nest which is owned by Spotify. Same acoustic features used for analyzing the music popularity by (Sciandra and Spera 2020). They applied a Beta GLMM to detect the features that have effects on song popularity. They found out that energy, valence, and duration features affect the song popularity positively. (Ni et al. 2011) discovered that the hit songs having higher tempo and getting louder over time as a result of their binary classification study. However, their findings showed that over a 50-years period harmonically simple songs are more likely to be hit.

There are also works that are not data driven. (LeBlanc et al., 2000) tested the music listening preferences by surveying young listeners around 5 countries. They found that the tempo of the song, listener's age and country affect the music preference. (Rentfrow and Gosling, 2003) collected over 3500 samples from different geographical regions and discover 4 music preference dimensions such as Reflective and Complex, Intense and Rebellious, Upbeat and Conventional and Energetic and Rhythmic. They explained and related the music preferences with the personal characteristics, political views, and cognitive abilities.

Time-varying coefficient models such as Kalman filters, smoothing spline methods and time-varying coefficient regressions are widely used to analyse longitudinal data in different domains.

Ordinary Least Squares (OLS) to estimate continuous values by using several independent variables. An alternative for the OLS is Flexible Least Squares (FLS) which is proposed by (Kalaba and Tesfatsion, 1989) to solve time-varying linear regressions. The method minimizes the difference between coefficients of consecutive weeks in addition to the sum of squared regression errors. FLS smooths the regression coefficient changes over time. FLS is used in different domains. (He, 2001) used the FLS to

compare sensitivities between stock markets in different countries. (Wood, 2000) mentioned the fact that OLS coefficients may vary too much across time thus they use the FLS for the presidential approval data of the U.S. Finally, (Lütkepohl and Herwartz, 1996) expand the FLS method with a generalized version of it by adding a seasonality term to the regression equation.

In this study, we aim to explore patterns in musical features over time while predicting the number of streams for each song using a time-varying regression model so that we can discover the dynamics of the music popularity in Turkey and the U.S. Our work proposes a unique approach to solve the FLS and its application on analysing the changes in the musical trends in different countries.

3 PROBLEM DEFINITION & METHODOLOGY

In this paper, our purpose is to predict weekly song streams via a regression model and monitor the change of the regression parameters to make inferences about the determinants of music popularity in Turkey and the U.S. We apply regression analysis using the acoustic features and the stream data provided from Spotify and some other temporal features. However, OLS regression does not take account of the dynamic behavior of the time-varying regression coefficients. For this reason, we use the FLS regression to smoothen the changes of the regression coefficients because in real life we do not expect extreme changes in the coefficients over time. In this section, we explain the methodology that we use to solve this type of regression problem.

3.1 Mathematical Programming

We used a quadratic programming (QP) model to minimize the SSE and the dynamic errors which are the difference between a regression parameter and its value in the previous period. Sets, parameters, decision variables that we used in our mathematical model are listed below.

h: week, $h=1, 2, \dots, H$

s: song, $s=1, 2, \dots, S$

m: feature/independent variable, $m=1, 2, \dots, M$

A_{smh} : Numerical value of feature m, for song s, for week h.

Y_{sh} : Output variable value (stream) of song s, for week h.

λ : Penalization term for dynamic errors.

W_{mh} : Regression coefficient for feature m, for week h.

B_h : Intercept of the regression equation for week h.

$Pred_{sh}$: Predicted output for song s, for week h.

D_{sh} : Difference between predicted output and the Y_{sh} for song s, for week h.

P_{mh} : Difference between consecutive weeks' coefficient value for feature m.

$$\min \sum_s \sum_h D_{sh}^2 + \sum_s \sum_h \lambda P_{hm} \quad (1)$$

$$s. t. \sum_m A_{smh} W_{mh} + B_h = Pred_{sh} \quad \forall s, h \quad (2)$$

$$Pred_{sh} - Y_{sh} = D_{sh} \quad \forall s, h \quad (3)$$

$$W_{m,h-1} - W_{mh} \leq P_{mh} \quad \forall m, h = 2, \dots, H \quad (4)$$

$$W_{mh} - W_{m,h-1} \leq P_{mh} \quad \forall m, h = 2, \dots, H \quad (5)$$

The objective function (1) minimizes the SSE and the weighted sum of the residual dynamic errors over songs and the weeks. Constraint (2) is the regression equation for each song in each week and it calculates the variable $Pred_{sh}$. Constraints (3) defines the prediction error. Constraints (4) and (5) calculates the P_{mh} as the absolute value of the residual dynamic error.

3.2 Bootstrapping

Optimization problem provides the optimal values for regression parameters. However, the model does not generate the confidence intervals for these parameters because of the unknown distribution of the estimates generated by FLS. Bootstrapping is proposed by (Efron, 1992) to generate sample statistics by randomly selecting samples from the dataset with replacement. We used bootstrapping to generate the confidence bounds for FLS estimates and to see whether variable is statistically significant or not.

4 RESULTS

In this section we discuss the results from our approach and the data clean-up/preparation steps that we applied to the dataset before solving our optimization model.

4.1 Data Preparation

We applied various data pre-processing steps to our dataset. In this section, we explain these steps in detail and compare the results from our approach with OLS and the FLS. www.spotifycharts.com keeps the record of the daily song streams for all markets that Spotify operates. We collected Spotify Top 200 charts from the web for 102 weeks and 2 countries. In addition to the chart data, we also scraped the acoustic features of each song with the help of the Spotify API for Python. Each song has acoustic features such as danceability, energy, key, loudness, etc. generated by The Echo Nest which is a music intelligence and data platform that was owned later by Spotify in 2014 (Spotify for Developers, 2014). These high-level acoustic attributes are created by Spotify's algorithms and allow developers to build applications such as recommender systems and predictive models.

After collection of 2 years of top 200 songs data from Turkey and the U.S.'s charts, we applied data pre-processing operations to use datasets in our regression model. Songs' release dates are converted to daily basis and these days are discretized using equal probability intervals. The lower bounds of these intervals are 13th, 30th, 66th weeks for Turkey and 6th, 16th, 60th weeks for the US.

List of unique songs in the US and Turkey dataset with their acoustic features are used for clustering and creating groups with similar acoustic features. We used the k-means clustering method for this and we decided the number of clusters using silhouette and elbow methods. We decided on 5 acoustically similar clusters for both Turkey and the US.

We checked the Pearson correlations between variables to see if there are highly correlated variables. We also calculated the variance influence factor (VIF) scores for the same reason. To avoid multi-collinearity, we eliminated some of the correlated variables in the datasets.

We also generated and modified some variables. If the song is a new release, the new variable "Previous Stream" created for the song is zero for that week. Popularity variable separated into two variables as Local Popularity and Popularity. Local songs' popularity value is switched to Local Popularity and the Popularity value of these songs became zero. On the other hand, non-local songs' Popularity values stayed same, but they had the Local Popularity value as zero. (Schedl and Hauger, 2012) suggested to use cosine similarity metric in their work. We calculated the average cosine similarity of a song with other songs in that week and created a new Similarity variable.

We applied 2 different transformations to song streams (response) and the independent variables. Box-Cox transformation proposed by (Box and Cox, 1964) is applied to the response variable. We also used Yeo-Johnson transformation (Yeo and Johnson, 2000) to transform the predictor variables. Yeo-Johnson transformation is a similar method to the Box-Cox model, but it can accommodate predictors with zero and/or negative values.

$$\psi(\lambda, y) \begin{cases} \frac{(y+1)^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0, y \geq 0 \\ \log(y+1) & \text{if } \lambda = 0, y \geq 0 \\ -\frac{[(-y+1)^{2-\lambda} - 1]}{(2-\lambda)} & \text{if } \lambda \neq 2, y < 0 \\ -\log(-y+1) & \text{if } \lambda = 2, y < 0 \end{cases} \quad (6)$$

Finally, we normalized the predictor variables by subtracting the mean and dividing by the standard deviation. At the end we got a dataset of 102 weeks each has 200 rows and 18 columns which concludes songs' number of streams, acoustic features, and the other generated features.

The first notable difference between the two datasets is the total number of streams. Since the number of Spotify users are higher in the U.S., it is not unexpected. In Turkey, decrease in the song stream when rank increase is sharper. This shows that higher ranked songs have distinctly higher stream values than the lower ranked songs. In Figure 2 we plot the average number of streams for each rank (1-200) for both countries.

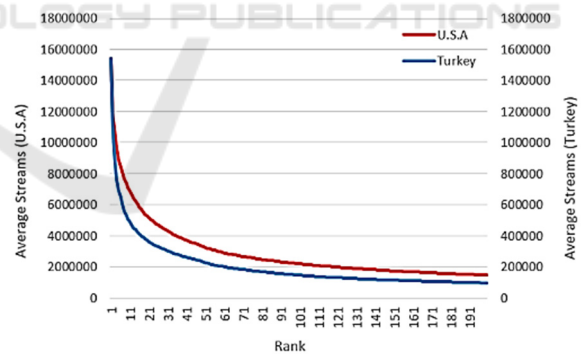


Figure 2: Comparison of the number of average streams over weeks.

Figure 3 shows the total weekly streams of the Turkish songs "Heyecanı Yok", "Beni Sen İnandır" and "Let Me Down Slowly" until the last appearance in the Top 200 charts. Each of the three songs have followed different patterns for 2 years which shows us that appearing in the charts longer does not imply the higher number of streams and vice versa.



Figure 3: Comparison of the streams of 3 songs in Turkey's charts.

Each song has different first lives in the charts which shows its number of weeks that it shows up in the charts after first release. In Figure 4, histogram of the first lives in the Turkey is shown.

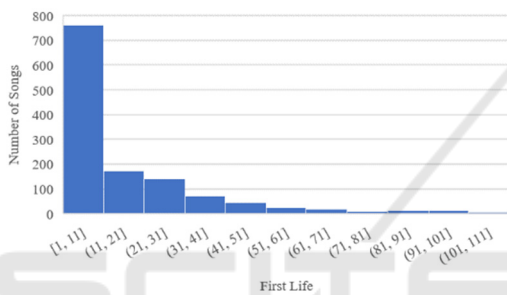


Figure 4: Histogram of the first lives of the songs in Turkey.

During the two years, the total number of streamed songs has increased due to the increase in the number of users and the spread of music streaming platforms. This increase can be seen even in the change of the stream of the 200th songs over 2 years in Figure 5.

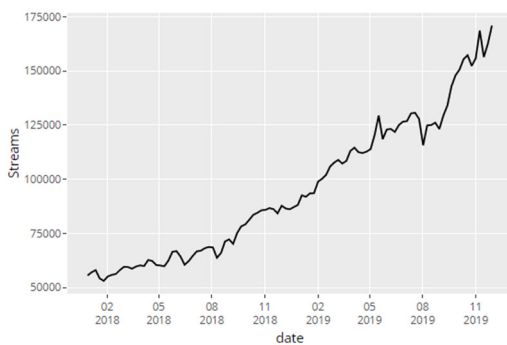


Figure 5: Number of streams for 200th song in Turkey.

Each 200 rank have been shared by different number of unique songs in 2 years. Figure 6 shows that the appearing in the higher ranks could be achievable for a couple of successful songs.

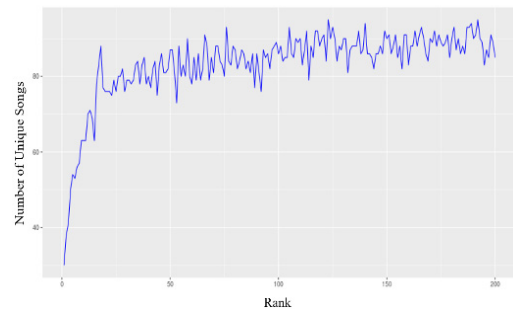


Figure 6: Number of unique songs in each rank.

4.2 Computational Results

We solved FLS model on a PC with 8 GB RAM and Intel Core i7-8550U 1.80 GHz processors using CPLEX 12.8 solver on Python 3.7. Before starting to solve the model, we decided the value of the penalization parameter. To determine the penalization parameter λ , we applied 5-fold cross validation by fitting the regression model for 10 weeks of data with different λ values. We found the best λ value both for Turkey and the U.S. Figure 7 shows the cross-validation results for train and test sets for the U.S dataset and the selected value for the penalization parameter. Selected values for Turkey and the U.S. datasets are 16 and 5, respectively.

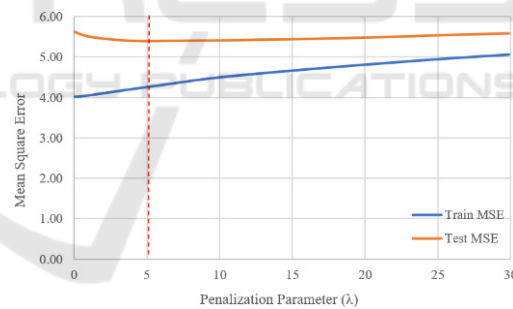


Figure 7: Change of the dynamic and residual errors with the change of the penalization parameter.

Each run of the model takes 4.51 second in average and it takes 11297 minutes for completing 2500 bootstrap samples for Turkey's dataset. In Detailed statistics for computational times can be seen in Table 1.

Table 1: Run time statistics (seconds) of the optimization model across bootstrap samples.

Country	Min	Median	Max
Turkey	3.31	4.51	15.68
U.S.	3.94	6.01	16.48

As mentioned in the Section 3, we used bootstrapping to generate confidence bounds for the regression parameters. The number of bootstrap samples is selected to be 2500. These samples were generated with Monte Carlo cross validation method by selecting 120 samples from the common songs that appears in both the previous and the next weeks' charts. In Figure 8, we show the change of the 90% confidence bounds and the estimations generated by bootstrapping.



Figure 8: FLS estimate of acousticness variable in Turkey with bootstrap 90% confidence intervals.

FLS method creates smoother transitions between consecutive weeks' coefficients by penalizing the dynamic error in the objective function. However, this causes a worse fit for the overall model and the accuracy. Cross-validation allowed us to choose the best lambda values for the datasets so that we can achieve smooth shifts between the weeks while keeping the regression errors as small as possible. In Figure 9 the change of the objective function value with the OLS and FLS for both datasets can be seen. Dynamic errors decreases with the increase of the λ so that we had smooth transitions between weeks while increase in the SSE is small.

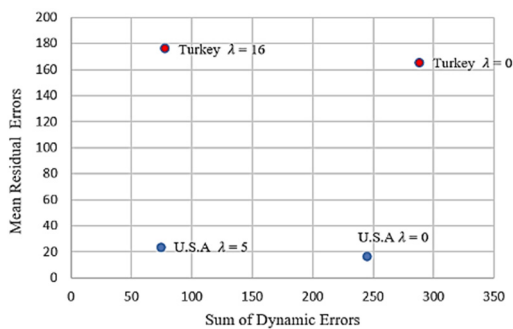


Figure 9: Train and test errors of the cross-validation sets.

There are some obvious patterns occurred in the coefficient changes of the Turkey's regression outputs. Especially in the 20th week of our dataset and the following weeks, there are significant rises and

falls in the various coefficients. In 2018, the rap/trap music rush started in Turkey. Even the old rap songs such as “*Neyim Var ki*” released in 2004, appeared and survived in the top 200 charts with the increasing popularity of this genre. Typical characteristics of the rap/trap songs are the high speechiness and low energy. Figure 10 shows the changes in the regression coefficients of the speechiness variable.

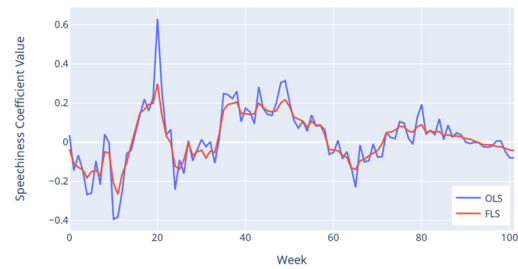


Figure 10: OLS and FLS estimates of Turkey's "Speechiness" coefficient.

Another coefficient that shows us a similar outcome is “Similarity” which is generated by calculating the cosine similarity of the songs in a week. Around the 20th weeks, increase of the similarity coefficient shows a similar pattern with the other rap related characteristics. Figure 11 shows the FLS output of the similarity coefficient over time.

We also wanted to compare the popularity of local and foreign artists, so we separated the popularity variable for Turkey's dataset as mentioned in the Section 4.1. However, we could not detect different patterns on these two variables.

In the timeline of the FLS coefficients of the U.S., “valence” seems to have an apparent temporal behaviour variable. Valence is a measure from 0.0 to 1.0 that describes the musical positiveness of a track. Higher valence means happier songs. At the end of the years, coefficient of the valence increases and then decreases in a couple of weeks. As the Christmas time arrives, people tend to listen old songs related the holiday. These Christmas songs are happier songs thus they have higher valence.



Figure 11: OLS and FLS estimates of Turkey's "Similarity" coefficient.

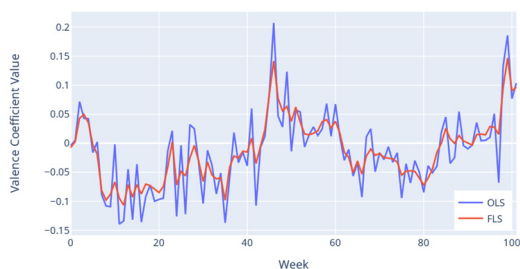


Figure 12: OLS and FLS estimates of the U.S.'s "Valence" coefficient.

Another similar pattern is observed in the coefficients of the U.S. is the "Previous Stream". As mentioned before, the "Previous Stream" variable is generated using a song's previous week stream. Negative peaks in the beginning of 2018 and at the end of the 2019 may show the relationship between the increasing popularity of the Christmas songs which has less previous streams than the new songs.



Figure 13: OLS and FLS estimates of the U.S.'s "Previous Stream" coefficient.

Another important motivation in this study is to reveal cross-cultural differences of the musical attributes. We expected that similar or different patterns may be observable in the regression coefficients of Turkey and the U.S. The coefficient of the "Previous Stream" variable follows a different pattern. As we mentioned before, coefficient value of the "Previous Stream" is decreasing with the Christmas in the U.S. It does not follow a similar pattern in Turkey's results. Coefficients of the variable have a decreasing pattern.

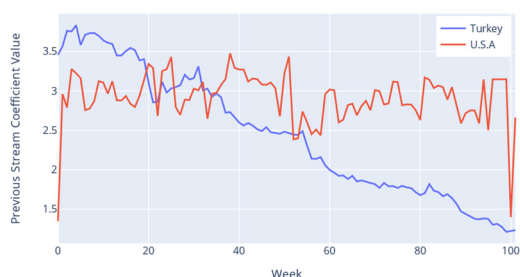


Figure 14: Comparison of two countries' FLS coefficient of the "Previous Stream" variable.

Speechiness is another variable that has an obvious pattern in Turkey's results which represents the rap songs' increasing popularity. However, we can not see such pattern change in the U.S.'s speechiness variable. In Figure 15, country comparison speechiness coefficients are shown.



Figure 15: Comparison of two countries' FLS coefficient of the "Speechiness" variable.

Popularity is one of the features that Spotify calculates for each artist. Higher popularity means being most popular. When we compare the popularity coefficients of Turkey and the U.S., it is apparent that the artist popularity affects the music stream in a different manner for these countries. In Turkey, Popularity had a positive effect on the song streams in the first quarter and this effect decreased in following 20 weeks. After a sudden change in 20th week, artist popularity became positively effective for the songs in Turkey. For the rest of the weeks, popularity does not follow a strict pattern which may show us; with the rise of the new trends/artists its effect decreases over time. In the U.S. artist popularity is not effective on a song's as Turkey's 2019 pattern. In Figure 16 comparison of the FLS coefficients for the popularity coefficients of two countries are shown.

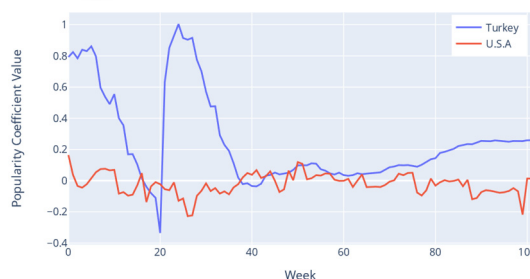


Figure 16: Comparison of two countries' FLS coefficient of the "Popularity" variable.

As mentioned before, the similarity metric has followed a pattern that may highlight the shifts in Turkey's music listening habits with the rap music trend. In the U.S., coefficients of the "Similarity" metric seems to be ineffective on the song streams

which shows that the influence of the songs with similar acoustic attributes did not affect songs' number of streams. Comparison of the "Similarity" coefficients of the countries is shown in Figure 17.

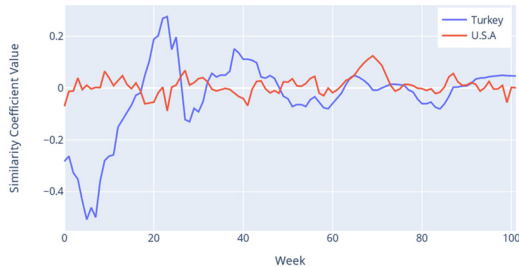


Figure 17: Comparison of two countries' FLS coefficient of the "Similarity" variable.

Another variable that we generated in addition to Spotify's acoustic features is "Release Date". We expected to see different effects of the song recency on the song streams. Release dates of the songs are discretized as equal probability intervals as mentioned in Section 4.1. The comparison of the coefficients of the newest songs is shown in Figure 18. In Turkey, the new songs have a positive coefficient during 2018. However, song recency became ineffective in 2019. In the U.S., there are no significant changes in the coefficient except the year's ends.

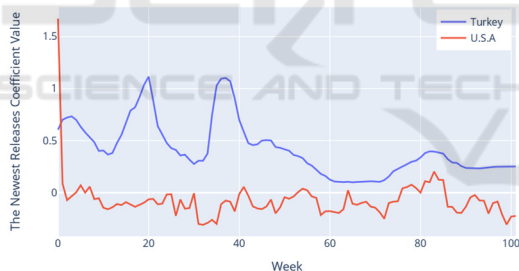


Figure 18: Comparison of two countries' FLS coefficient of the "The Newest Releases" variable.

As result, we observed that the music listening habits in Turkey change more than in the U.S. These changes can be seen in the coefficient changes over time more clearly. It can be said that political and economic dynamics in Turkey may affect the social influence on music listening habits differently.

5 CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed an application of the FLS method on music data to analyze shifts in music taste and popularity over different countries. We used

quadratic programming with bootstrapping to solve the model and generate the confidence bounds for the regression coefficients of the features. FLS method allows us to smoothen the coefficient changes in time so that the changes in the coefficients are reflected realistically. Our findings show that the acoustic features of the songs may have an effect on the song's popularity and there may be obvious patterns in the regression coefficients so that we can observe the shifts in the music tastes.

As future work, one should analyze/compare more countries' musical tastes with the help of new information and methods. Since our work consists of 2 countries' data in 2 years, it can be extended with more data from different countries and length of periods. Our methodology is also able to be diversified with a new constant parameter time series regression model and a smoothing method applied to the OLS coefficients for comparing the methods with the FLS.

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