Genetic Optimization of Excitation Signals for Nonlinear Dynamic System Identification

Volker Smits[1][a] and Oliver Nelles[2][b]
[1]DEUTZ AG, Ottostr. 1, Cologne, Germany
[2]Department of Mechanics and Control - Mechatronics, University of Siegen, Paul-Bonatz-Str. 9-11, Siegen, Germany

Keywords: Design of Experiment, Genetic Algorithm, System Identification of Nonlinear Dynamic Systems, Optimal Excitation Signals, APRBS, GOATS.

Abstract: Two new methods for optimization of passive step-based excitation signals for system identification of nonlinear dynamic processes via a genetic algorithm are introduced - an optimized Amplitude Pseudo Random Binary Signal ($\text{APRBS}_{\text{Opt}}$) and a Genetic Optimized Time Amplitude Signal (GOATS). The investigated optimization objectives are the evenly excitation of all frequencies and the uniform data distribution of the space spanned by the system's input and output. The results show that the GOATS optimized according to the uniform data distribution outperform the state-of-the-art excitation signals standard ARPBS ($\text{APRBS}_{\text{Std}}$), Optimized Nonlinear Input Signal (OMNIPUS), Chirp and Multi-Sine in the achieved model quality on three artificially created Single-Input Single-Output (SISO) nonlinear dynamic processes. However, the $\text{APRBS}_{\text{Opt}}$ only exceeds the Chirp, Multi-Sine and $\text{APRBS}_{\text{Std}}$ in the achievable model quality. Additionally, the GOATS can be used for stiff systems, supplementing existing data and easy incorporation of constraints.

1 INTRODUCTION

System identification refers to a process of building mathematical models of a dynamic or static system based on the relation between measured input-output data of a given system (Isermann, 1992; Hartmann, 2013). The quality of such data-based models is mainly influenced by the information which are gathered in the data for the model training (training data) (Hartmann, 2013; Heinz and Nelles, 2017; Heinz et al., 2017; Tietze, 2015). A well-known and validated methodology for the maximization of the amount of information of the training data is the Design of Experiment (DoE) (Hartmann, 2013). The DoE for the training of dynamic models (dynamic DoE) differs from the DoE for training of stationary models (static DoE) regarding the kind of information needed to be collected during the experiment. Both the dynamic and the stationary models need the information about the stationary nonlinearity (equilibrium), whereas the dynamic model needs additional information about the frequency and the transient behaviour of the systems.

[a] https://orcid.org/0000-0001-8004-7957
[b] https://orcid.org/0000-0002-9471-8106

In general, two classes of DoE can be distinguished: The passive and the active DoE. The passive DoE, defines the offline development of an experiment design, whereas the active DoE describes the online approach of a DoE (Heinz and Nelles, 2017). We assume that the optimization task of dynamic DoE's of complex nonlinear dynamic systems is too difficult to solve properly online in the limited time range. To address this problem, simplifications of the optimization task often have to be chosen such as simpler model structures or less computational demanding loss functions. This, however, results in only optimal solutions for the chosen simplifications. For this reason, the current paper aims to develop two new passive excitation signals to increase the modeling quality of nonlinear dynamic processes.

Chirp, Multi-Sine and Amplitude Pseudo Random Binary Signal (APRBS) are widely used passively designed excitation signals (Baumann et al., 2008; Hoagg et al., 2006; Nelles, 2013; Pintelon and Schoukens, 2012; Rivera et al., 2002; Tietze, 2015). Step-based excitation signals like an APRBS show a better capability to cover the space spanned by the system's input u and output y compared to sinusoid-based signals such as Chirp and Multi-Sine (Heinz

Smits, V. and Nelles, O.
Genetic Optimization of Excitation Signals for Nonlinear Dynamic System Identification.
DOI: 10.5220/0010545501380145
In *Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2021)*, pages 138-145
ISBN: 978-989-758-522-7

and Nelles, 2017). In the last decade, a variety of optimizations and modifications of the APRBS have been developed (Deflorian and Zaglauer, 2011; Heinz and Nelles, 2016; Nouri et al., 2018).
To the best of our knowledge, the study of Nouri et al. is the first study which optimized an APRBS via a genetic algorithm (GA) (Nouri et al., 2018). They have optimized the APRBS according to an information criterion to minimize the uncertainty for a parameter estimation of a predefined white box model structure. In their study, the optimization is used to improve the parameter identification instead of system identification. Another modern step-based signal is the Optimized Nonlinear Input Signal (OMNIPUS), which is proposed in (Heinz and Nelles, 2017; Heinz et al., 2017). It aims to optimize the coverage of the space spanned by u and y. However, the optimization of OMNIPUS is incrementally, which can lead to suboptimal designs, because earlier designed sequences of the optimization cannot be changed in the later process of the optimization.
The present paper aims to add to the current literature by developing two new passive excitation signals for system identification of nonlinear dynamic systems which will be compared to four state of the art excitation signals on three artificially created processes. Our approach differs from the current studies regarding the optimization of a step-based excitation signals by introducing new loss functions for optimizing the coverage of the space spanned by u and y and the evenly excitation of all frequencies in a global fashion via a GA.

2 METHOD

2.1 Design of Experiment

The first investigated optimization objective is the evenly excitation of all frequencies f_f which aims to the excitation of the relevant bandwidth of system without over-emphasizing specific frequencies. The second investigated optimization objective is the space-filling coverage f_i of the space spanned by the system's input u and output y or more precisely the input space of a Nonlinear AutoRegressive with eXogenous input (NARX) system. Since the regressors of the regression matrix $\underline{X}$ e.g. of a first order NARX structure are the delayed sequences of input $u(k-1)$ and output $y(k-1)$, the optimization of the space spanned by these regressors seems to be purposeful to improve the modeling quality.
In Fig. 1 the first order NARX input space of a nonlinear dynamic process separately excited with a standard $\text{APRBS}_{\text{Std}}$, Chirp and Multi-Sine is shown. As Heinz and Nelles have shown and also is illustrated in Fig. 1 step-based signals have a better space coverage compared to sinusoidal-based signals in the first order NARX input space (Heinz and Nelles, 2017). Sinusoidal-based signals like the Multi-Sine and Chirp signal are not able to fill the areas in the upper left and lower right corner. Step-based signals like the APRBS and OMNIPUS are able to cover the upper left and lower right corner as well as the center due to their piecewise constant sequences and their steps (Heinz and Nelles, 2017). Due to this reason, the signal type of step-based signals is considered for the two new excitation signals which are optimized via a GA.

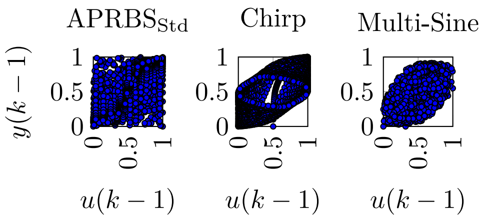

Figure 1: First order NARX input space point distribution of an APRBS, a Chirp and Multi-Sine.

The first new signal type is an optimized APRBS ($\text{APRBS}_{\text{Opt}}$) with an optimized amplitude permutation $\underline{p}_p$. An APRBS is based on a sequence which controls the duration of the constant phases and the time dependent occurrence of the steps. This sequence is generated by a pseudorandom binary sequence (PRBS). The minimum hold time T_h allows to adjust the APRBS to a specific frequency range (Isermann, 1992; Nelles and Isermann, 1995). The different amplitude levels $\underline{A} = N_a \times d$ ($N_a :=$ amount of amplitude levels, $d :=$ input dimension) could be chosen prior e.g. by a static DoE method and then modulated to the PRBS sequentially (Isermann, 2010). The permutation of these amplitudes $\underline{p}_p$ will define the amplitude order of the $\text{APRBS}_{\text{Opt}}$ which influences the coverage of the input space and the amplitude spectrum, whereby it is a promising parameter for the optimization. The second new signal type is inspired by the APRBS as well. For this signal type not only the amplitude order $\underline{p}_p$ is optimized via a GA, but also the sequence $\underline{p}_s$. Therefore, it is an independent new signal type and is named Genetic Optimized Amplitude Time Signal (GOATS).

2.2 Genetic Algorithm

A GA is a metaheuristic algorithm which belongs to the family of evolutionary algorithms (EA). The basic concept is to imitate the Darwinian principle of evo-

lution (variation, reproduction and selection) to technical environment to iteratively solve optimization problems (Holland, 1975; Sivanandam S.N., 2008). Therefore, a GA is suitable to optimize the introduced parameters of the last subsection the permutation $\underline{p}_p$ and the sequence $\underline{p}_s$ without information about the derivatives.
The used GAs for single objective optimization (SOO) and multi objective optimization (MOO) of the objectives f_f and f_i in this paper are a combination of different methods for selection, recombination and mutation of popular genetic algorithms due to their good performance. The Tournament Selection is commonly used and very popular method for selection due to its efficiency and simple implementation (Goldberg and Deb, 1991; Razali and Geraghty, 2011). In this paper, it is used in selection of recombination candidates and the candidates for the next generation. In the SOO, the fitness of the individuals is directly compared with a set of four individuals. The MOO uses the tournament selection of the Nondominated Sorting Genetic Algorithm II (NSGA-II) proposed in (Deb et al., 2000).
The two parameter types of the optimization are the permutation $\underline{p}_p \in \mathbb{N}_a^N$ of the APRBS and GOATS and the sequence $\underline{p}_s \in \mathbb{N}_a^N$ of the GOATS. The permutation $\underline{p}_p$ defines the order of the different amplitude levels of the ARPBS and the GOATS. The sequence $\underline{p}_s$ is represented as a sequence of integers between two limits which are defining the minimum and maximum duration of an amplitude level of the GOATS.
As crossover operator of the permutation parameter the Order Crossover 1, Order Crossover 2, Partially Map Crossover and Position Based Crossover are used (Davis, 1985; Goldberg et al., 1985; Syswerda, 1991). Which specific crossover method in a crossover situation is chosen, is depending on a uniform random distribution. This concept of a uniform selection is analogous implemented for the the mutation operators, whereby every method can contribute with its advantages. The mutation operators of the permutation are the Reverse-, Interchanging- and One-Point-Slide-Mutation[1] (Sivanandam S.N., 2008). The mutation operator used for the sequence parameter type is the Power-Mutation (Deep et al., 2009). It is used to produce new genes for the sequence. As crossover operators of the sequence parameter type the Uniform-, SBX- and Two-Point-Crossover are taken (Deb and Agrawal, 1995; Hartmann, 1998). The SBX-crossover is slightly adapted by a rounding function, so that after the crossover the sequence only contains integers.
Additionally, the mutation rate λ_m and crossover rate λ_c are adaptively changed during optimization by rating the normalized relative improvement of the fitness caused by the mutation or crossover. This approach is inspired by the work of Lin et al. (Lin et al., 2003).

2.3 Modeling Approach for Nonlinear System Identification

Besides a good space-filling of the training data and good coverage of frequency spectra, the question arises how the quality of an excitation signal can be quantified. A straightforward and reasonable approach is to quantify the quality of an excitation signal for nonlinear system identification whilst a model is trained based on the data which is gathered by the excitation signal. While it is too computational expensive and impractical to use this directly in a GA, for rating the results of the optimization it is well suited.
A deterministic model training is preferable, because a nondeterministic training would impede the analysis due to a more complicated distinction of the reasons of the change of the model performance. One model architecture which is easy to train by the usage of a deterministic training method and yields good model performances, is the architecture of local model networks (LMN) (Hartmann, 2013; Nelles, 2013). For the optimization of the LMN the hierarchical local model tree (HILOMOT) is used (Nelles, 2006). The HILOMOT is an incremental tree construction algorithm which divides the input space in an axis-oblique manner and estimates local models in the created subspaces. The overall model output $\hat{y}$ is calculated by the weighted sum of the sub-models $\hat{y}_i(\underline{x})$ and the validation functions $\Phi_i(\underline{z})$ with the subsets $\underline{x}$ and $\underline{z}$ of all inputs $\underline{u}$ (Nelles, 2006).

$$\hat{y}(\underline{x},\underline{z}) = \sum_{i=1}^{M} \hat{y}_i(\underline{x}) \cdot \Phi_i(\underline{z}) \text{ , where} \sum_{i=1}^{M} \Phi_i(\underline{z}) = 1 \quad (1)$$

3 LOSS FUNCTIONS AND OPTIMIZATION PROBLEMS

For rating the space-filling property of a point distribution, loss functions are required to quantify the coverage of the points in a space-filling sense. For the quantification of the coverage of the input space three different loss function are investigated. The considered input space in this survey is the NARX input space $\underline{X}$. For the later usage and simplicity they will

[1] Slides a subtour for one position, Example: Parent: [7, 10, 5, 3, 4, 2, 8, 9, 6, 1]; Subtour: [3, 4, 2]; Child:[7, 10, 3, 4, 2, 5, 8, 9, 6, 1]

be gathered under the term *input space*-loss functions f_i.

- *Audze Eglais* (AE) (Audze and Eglais, 1977)

$$L_{AE} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{1}{L_{ij}^2}, \text{ where } L_{ij} = \left\| \underline{X}_i - \underline{X}_j \right\|_2 \quad (2)$$

- *Maximum Projection* (MP) (Joseph et al., 2015)

$$L_{MP} = \left\{ \frac{1}{\binom{N}{2}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{1}{\prod_{l=1}^{p} (x_{il} - x_{jl})^2} \right\}^{1/p} \quad (3)$$

- *Fast and Simple Dataset Optimization* (FA) (Peter and Nelles, 2019)

$$L_{FA} = \frac{1}{N} \sum_{i=1}^{N} |\underline{1} - \hat{q}(\underline{X}(i))|, \quad (4)$$

$$\text{where } \hat{q}(\underline{X}(i)) = \frac{1}{N} \sum_{i=1}^{N} \frac{e^{-\frac{1}{2}[\underline{X}-\underline{X}(i)]^T \underline{\Sigma}^{-1} [\underline{X}-\underline{X}(i)]}}{\sqrt{(2\pi)^n |\underline{\Sigma}|}},$$

$$\underline{\Sigma} = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$$

The AE and MP belong to the maximin distance designs which mainly penalize close points in the input space. The MP is based on the AE criterion and tries to extend its projection properties in the subspaces of the given input space but is computational more expensive (Joseph et al., 2015). In comparison, the FA loss function is suited to adjust a data distribution to a specific probability distribution (Peter and Nelles, 2019). In this study the FA is used to quantify the similarity of the data distribution of the NARX input space to a uniform distribution.

The approach to quantify the evenly excitation of the frequencies is done by describing the mean value and standard deviation of a normalized single sided amplitude spectrum U_n. Three different combinations are investigated and will be gathered under the term *frequency* - loss functions f_f.

- *Mean Value of Normalized Amplitude Spectra* (MAP)

$$L_{MAP} = -\overline{U}_n = -\frac{1}{N} \sum_{i=1}^{N} U_n(i) \quad (5)$$

- *Standard Deviation of Normalized Amplitude Spectra* (SAP)

$$L_{SAP} = \sigma_{U_n} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (U_n(i) - \overline{U}_n)^2} \quad (6)$$

- *Mean Value and Standard Deviation of Normalized Amplitude Spectra* (MSAP)

$$L_{MSAP} = -\overline{U}_n + 2\sigma_{U_n} \quad (7)$$

$$= -\frac{1}{N} \sum_{i=1}^{N} U_n(i) + 2\sqrt{\frac{1}{N} \sum_{i=1}^{N} (U_n(i) - \overline{U_n})^2}$$

The factor 2 in (7) is used to scale the loss function into the interval $[0, 1]$. The normalized single sided amplitude spectrum is calculated as follows:

$$U(k) = \sum_{n=1}^{N} u(n) \cdot e^{-i \frac{2\pi}{N} kn}$$

$$U_+(f) = \begin{cases} 2U(k) \;, & \text{for } 0 < k < N/2 - 1 \\ U(k) \;, & \text{for } k = 0 \\ 0 \;, & \text{for } k < 0 \end{cases} \quad (8)$$

$$U_n = \frac{U_+}{\max(U_+)}.$$

It is to note that all loss functions are constructed as a minimization problem. Each loss function gathered under the terms f_i and f_f first is optimized in a SOO. After that the best of the f_i loss functions is combined with every loss function of f_f and investigated via a MOO.

$$\text{single-APRBS} : \min_{\underline{p}_p} (f_{i/f}(\underline{X}(\underline{p}_p))) \quad (9)$$

$$\text{single-GOATS} : \min_{\underline{p}_p, \underline{p}_s} (f_{i/f}(\underline{X}(\underline{p}_p, \underline{p}_s))) \quad (10)$$

$$\text{multi-APRBS} : \min_{\underline{p}_p} (f_i(\underline{X}(\underline{p}_p)), f_f(\underline{X}(\underline{p}_p))) \quad (11)$$

$$\text{multi-GOATS} : \min_{\underline{p}_p, \underline{p}_s} (f_i(\underline{X}(\underline{p}_p, \underline{p}_s)), f_f(\underline{X}(\underline{p}_p, \underline{p}_s))) \quad (12)$$

4 EXPERIMENT AND DESIGN OF TRAINING AND TEST SIGNALS

4.1 Artificial Processes

The following three artificially created nonlinear processes are considered:

- *First order Hammerstein* (hamm^{1st})

$$y(k) = 0.2f(u(k-1)) + 0.8y(k-1) \quad (13)$$

- *First order Wiener* (wiener^{1st})

$$y(k) = f(z(k)), \quad (14)$$
$$\text{where } z(k) = 0.2u(k-1) + 0.8z(k-1)$$

- *Second order Hammerstein* (hamm^{2nd})

$$y(k) = 0.2f(u(k-1)) + 0.5y(k-1) + 0.3y(k-2) \quad (15)$$

The nonlinear static function $f(x)$ of the Hammerstein- and Wiener-systems is calculated as follows:

$$f(x) = \frac{\text{atan}(8x-4) + \text{atan}(4)}{2\text{atan}(4)} \quad (16)$$

The optimization of the coverage of the NARX-space requires information of the process output. Therefore, a first model of each process is needed. In this study, a simple linear model is estimated which generates an approximation of the information without much effort.

4.2 Training Signals

First the design of $APRBS_{Std}$, $APRBS_{Opt}$ and GOATS will be described. Then the design of the sinusoid-based signals will be delineated. The design of the OMNIPUS is described in (Heinz and Nelles, 2017). All excitation signals are set up for different durations t_{stop} for a better analysis of their properties and the influences of the loss functions. In example to answer the question if an optimization of the input space coverage is more important for short signals compared to longer signals. In the design of an APRBS or a GOATS where all amplitudes should be modulated, the duration of the signal is defined by the sequence and the different amount of amplitude levels. Therefore, first the step-based signals will be compared to their amount of amplitude levels. Later they will be juxtaposed to the sinusoid-based signals with similar durations. Four different amounts of levels in this paper are investigated $(26, 51, 101, 167)$. For the $APRBS_{Opt}$, the $APRBS_{Std}$ and GOATS the same amplitude levels are considered. The $APRBS_{Opt}$ and the $APRBS_{Std}$ even share the same sequence. The hold time of 0.5 s is identified by step experiments based on an assumed sampling period of 0.1 s and the suggestions of Nelles to choose the minimum hold time approximately to the dominant time constant of a system (Nelles and Isermann, 1995; Nelles, 2013). In this study, the amplitude levels can be simply generated by equidistant points, because a Single-Input Single-Output (SISO)- System is investigated.
The following settings of the GA have been chosen: maximum generation $n_{max,gen} = 2500$, population size $n_{pop} = 220$, $\lambda_{c,ini} = 0.5$ and $\lambda_{m,ini} = 0.5$. The rates λ_c and λ_m are adapted with $\Delta = 0.005$ in each generation during the optimizations according to their performance of the last 10 generations.

The Chirp and Multi-Sine signal are generated for several durations t_{stop} in the interval $[20\,\text{s}, 200\,\text{s}]$ with a step size of 10 s. The Chirp has a linear frequency modulation (f = $[1/500\,\text{Hz} - 1\,\text{Hz}]$). Each Multi-Sine signal contains the $t_{stop}/2$ amount of sine-waves with equidistant frequencies in the interval $f = [1/500\,\text{Hz} - 1\,\text{Hz}]$ and an optimized Schroeder Phase. In addition, the system output of each process, generated by the different excitation signals, is disturbed with white Gaussian noise with $\sigma = 0.05$ and $\mu = 0$.

4.3 Test Signals

The test signal in this survey is a concatenation of an $APRBS_{Std}$, a Ramp (Tietze, 2015), a Chirp and a Multi-Sine. Each signal has the same duration. The following itemize summarizes the parameter settings of the signal creation.

- $0\,\text{s} - 500\,\text{s}$: $APRBS_{Std}$: sample period = 0.1 s, 50 random amplitudes, hold time = 0.5 s
- $500.1\,\text{s} - 1000\,\text{s}$: Ramp: sample period = 0.1 s, 50 random amplitudes, hold time = 0.5 s
- $1000.1\,\text{s} - 1500\,\text{s}$: Chirp: linear frequency modulation, f = $[1/500\,\text{Hz} - 1\,\text{Hz}]$
- $1500.1\,\text{s} - 2000\,\text{s}$: Multi-Sine: 51 sine-waves, f = $[1/500\,\text{Hz} - 1\,\text{Hz}]$, optimized Schroeder Phase

5 COMPARISON AND ANALYSIS OF THE TRAINING SIGNALS

The model performances achieved by the different excitation signals on the test data is indicated by the Normalized Root Mean Squared Error (NRMSE).

$$NRMSE = \sqrt{\frac{\sum_{i=1}^{N}(y(i) - \hat{y}(i))^2}{\sum_{j=1}^{N}(y(j) - \bar{y})^2}} \quad (17)$$

In this analysis, an amount of 10000 APRBSs ($APRBS_{10000}$) with space-filling amplitude levels and random permutation are created for each of the two amounts of amplitude levels 26 and 51 in order to improve the comparability of the optimized step-based excitation signals to a $APRBS_{Std}$. All $APRBS_{10000}$ are used to train 10000 models for each of the three artificial processes. Figure 2 illustrates the histograms of the achieved NRMSE of the models on the test data trained by $APRBS_{10000}$ for the amounts of amplitude levels 26 and 51 for the hamm^{1st}. Table 1 summarizes all NRMSE values of the different optimizations. First the loss functions AE, MP and FA belonging to the category of f_i are compared in a SOO for the hamm^{1st}. The results show that the AE and MP have a comparable effect on the model performance. The effect of optimizing the input space coverage weakens for more amplitude levels, since the space will be covered good enough, if just enough amplitude levels are modulated. For the hamm^{2nd} the better projection feature of the MP does not come into play. The FA loss function performs better when more data is available. However, with more data the computational demand of the FA loss function is quite high and therefore the loss function becomes inappropriate. In the next step, the influence of the loss func-

Table 1: Summary of optimization results indicated by the NRMSE.

opt. type		loss	APRBS$_{Opt}$ amplitude levels				GOATS amplitude levels			
			26	51	101	167	26	51	101	167
SOO	hamm^{1st}	MAP	0.156	0.105	0.114	0.098	0.137	0.112	0.115	0.114
		SAP	0.156	0.111	0.094	0.107	0.136	0.136	0.111	0.109
		MSAP	0.136	0.113	0.116	0.101	0.229	0.201	0.140	0.157
		AE	0.121	0.116	0.113	0.104	0.114	0.095	0.094	0.098
		MP	0.109	0.126	0.092	0.106	0.107	0.103	0.111	0.10
		FA	0.127	0.110	0.121	0.115	0.178	0.135	0.084	0.076
	wiener^{1st}	AE	0.147	0.150	0.126	0.138	0.142	0.112	0.106	0.101
	hamm^{2nd}	AE	0.181	0.175	0.156	0.163	0.156	0.144	0.158	0.170
MOO	hamm^{1st}	AE+MAP	0.097	0.111	0.098	0.103	0.120	0.119	0.105	0.105
		AE+SAP	0.146	0.121	0.104	0.110	0.311	0.267	0.214	0.183
		AE+MSAP	0.150	0.105	0.104	0.107	0.136	0.175	0.119	0.181

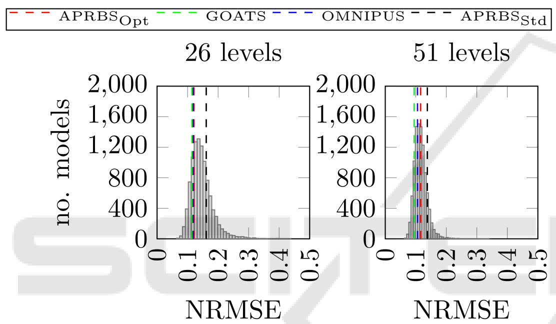

Figure 2: Comparison of the step-based signals with the histogram of test errors for APRBS$_{10000}$ for hamm^{1st}.

tions f_f is investigated. The performance of the excitation signals which are optimized according to f_f for 26 amplitude levels, is not better than the mean value of the NRMSE for the APRBS$_{10000}$ illustrated in Fig. 2 and Table 1. It is even sometimes worse than the mean value of the APRBS$_{10000}$ which leads to the assumption, that f_f does not have the main effect on the model performance. Another point which underpins this assumption is that the GOATS optimized according to f_f result in quite volatile and sometimes relatively bad model qualities. The reason for this is the degree of freedom in the sequence of the GOATS in contrast to the APRBS$_{Opt}$. Due to the degree of freedom the f_f can drive the sequence of the GOATS to short durations so that the equilibrium is not covered sufficient. For a higher amount of amplitude levels, the achieved model quality for the APRBS$_{Opt}$ and GOATS optimized according to the f_f becomes better. This can be explained since the input space will be covered good enough, if just enough amplitude levels are modulated.

In addition, all f_f also are optimized together with the AE loss function in a MOO to prove if it can contribute additional information which are not considered by f_i. For these optimizations the modeling performance is in the same range as the modeling quality of the SOO with f_i. Therefore, the optimization according to f_f does not lead to an improvement of the modeling quality. The explanation for this is given by the structure of the step-based signals which limits the degree of freedom of optimization of the evenly excitation of all frequencies resulting in too similar amplitude spectra of the different step-based signals. For this reason, only the SOO of f_i is further considered for the optimization of the step-based excitation signals.

Figure 2 shows the comparison of the APRBS$_{Opt}$ and GOATS optimized with the loss AE in SOO and the two state of the art step-based signals OMNIPUS and APRBS$_{Std}$. The modeling quality achieved by the different step-based excitation signals is indicated by the dashed lines. Consider that the APRBS$_{Std}$ and the APRBS$_{Opt}$ share the same sequence. Figure 2 indicates that an optimization of the permutation of the APRBS$_{Opt}$ compared to the APRBS$_{Std}$ leads to an improvement of the modeling quality. Although this improvement is limited due to the degree of freedom of the APRBS$_{Opt}$. This can be analyzed through the comparison of the achieved model quality of APRBS$_{Opt}$ to the quality of the GOATS. The GOATS exceeds the APRBS$_{Opt}$ in all investigated cases, because it can better cover the space due to its degree of freedom in the duration of each amplitude level. Consider that similar results are obtained for the wiener^{1st} and hamm^{2nd} so they are omitted in Fig. 2 to conserve space. Figure 3 illustrates the comparison of step-based and sinusoid-based signals over different signal durations for the achieved model

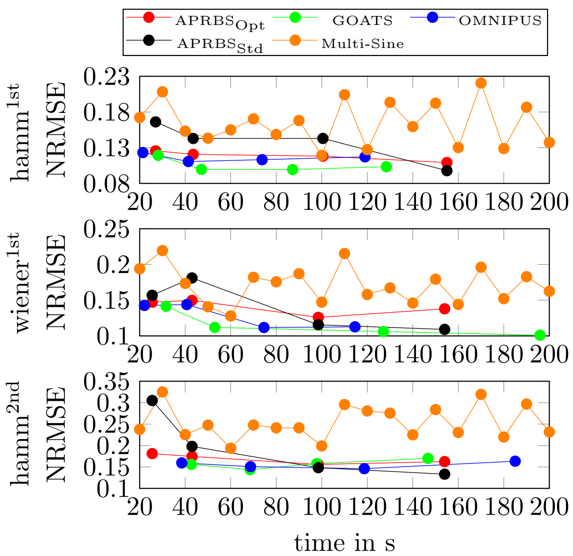

Figure 3: Comparison of test errors for step-based and sinusoid-based signals over signal duration.

quality on the test data. Comparing the OMNIPUS to the GOATS, the GOATS has a similar performance except for the $\mathrm{hamm}^{\mathrm{1st}}$ for the amounts of amplitude levels 51, 101 and 167 and for the $\mathrm{wiener}^{\mathrm{1st}}$ with 51 points where it surpasses the OMNIPUS more significantly. The GOATS is the only excitation signal which in all cases significantly outperforms the mean values of the APRBS_{10000} (17 % – 28 %).

The Chirp is omitted of Fig. 3 for better visibility, but it performs like the Multi-Sine. Figure 3 shows that the step-based signals significantly exceed the sinusoid signals in the achievable model quality. Furthermore, the model quality for the different durations of the sinusoid-based signals is quite volatile. In addition, the optimized signals like $\mathrm{APRBS}_{\mathrm{Opt}}$, GOATS and OMNIPUS clearly outperform all other excitation signals for short signal durations. For longer signal durations (approximately $2-4$ times) the $\mathrm{APRBS}_{\mathrm{Std}}$ can achieve a comparable model quality, because with enough amplitude levels the space will be covered good, when the amplitude levels which are modulated to an APRBS are selected with a space-filling criterion.

6 CONCLUSION AND OUTLOOK

The current paper proposes two novel approaches for the optimization of step-based excitation signals —$\mathrm{APRBS}_{\mathrm{Opt}}$ and GOATS — for nonlinear dynamic system identification. For this purpose, the coverage of the space spanned by the system's input and output and the evenly excitation of all frequencies of the step-based signals have been investigated as objectives for the optimization via a GA. The $\mathrm{APRBS}_{\mathrm{Opt}}$ and GOATS are compared with four state-of-the-art excitation signals ($\mathrm{APRBS}_{\mathrm{Std}}$, Chirp, Multi-Sine and OMNIPUS) on the three artificially created nonlinear dynamic processes in order to evaluate the expectable model quality.

Our results show that the optimization of the space-filling coverage of the step-based excitation signals leads to a significant improvement of the model quality compared to the usage of a $\mathrm{APRBS}_{\mathrm{Std}}$ for short signal durations. The reason for this is the avoidance of unexplored areas in the space spanned by the system's input and output. In contrast to our expectation, the results show that our optimization of an evenly distributed amplitude spectrum does not yield an improvement of the model quality. This can be explained by the given structure of the step-based signals which limits the degree of freedom for the optimization of the evenly excitation of all frequencies resulting in too similar amplitude spectra.

Therefore, the single objective optimization of the uniform coverage of the space is used for our newly developed excitation signals $\mathrm{APRBS}_{\mathrm{Opt}}$ and GOATS. The $\mathrm{APRBS}_{\mathrm{Opt}}$, leads to an improved model quality compared to the standard APRBS which shares the same PRBS basis. The improvement is, however, limited due to its degree of freedom constrained by the given PRBS. We have found that the GOATS leads to a significantly higher model quality compared to the state-of-art-excitation signals $\mathrm{APRBS}_{\mathrm{Std}}$, Chirp, Multi-Sine and a slightly higher model quality in comparison to the OMNIPUS on the three investigated artificial nonlinear dynamic processes. In addition, the GOATS is suitable for stiff systems, capable of supplementing existing data and easy incorporation of constraints.

The present results are limited to the three artificially created low order dynamical SISO systems. Therefore, in future research the GOATS has to be investigated for higher dimensional, higher dynamical order and real world systems. Another future research topic is the investigation of new loss functions for the optimization.

REFERENCES

Audze, P. and Eglais, V. (1977). New approach for planning out of experiments. *Problems of Dynamics and Strengths*, 35:104–107.

Baumann, W., Schaum, S., Roepke, K., and Knaak, M. (2008). Excitation Signals for Nonlinear Dynamic Modeling of Combustion Engines. In *Proceedings of* *the 17th World Congress The International Federation of Automatic Control*, pages 1066–1067.

Davis, L. (1985). Applying Adaptive Algorithms to Epistatic Domains. In *Proceedings of the 9th International Joint Conference on Artificial Intelligence*, pages 162–164.

Deb, K. and Agrawal, R. B. (1995). Simulated Binary Crossover for Continuous Search Space. *Complex Systems*, 9:115–148.

Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T. (2000). A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In *International conference on parallel problem solving from nature*, pages 849–858.

Deep, K., Singh, K. P., Kansal, M. L., and Mohan, C. (2009). A real coded genetic algorithm for solving integer and mixed integer optimization problems. *Applied Mathematics and Computation*, 212(2):505–518.

Deflorian, M. and Zaglauer, S. (2011). Design of experiments for nonlinear dynamic system identification. *IFAC Proceedings Volumes*, 44(1):13179–13184.

Goldberg, D. E. and Deb, K. (1991). A Comparative Analysis of Selection Schemes Used in Genetic Algorithms. *Foundations of genetic algorithms*, 1:69–93.

Goldberg, D. E., Lingle, R., and Others (1985). Alleles, loci, and the traveling salesman problem. In *Proceedings of an international conference on genetic algorithms and their applications*, volume 154, pages 154–159.

Hartmann, B. (2013). *Lokale Modellnetze zur Identifikation und Versuchs- planung nichtlinearer Systeme*. PhD thesis, University of Siegen.

Hartmann, S. (1998). A Competitive Genetic Algorithm for Resource-Constrained Project Scheduling. *Naval Research Logistics (NRL)*, 45:733–750.

Heinz, T. O. and Nelles, O. (2016). Vergleich von Anregungssignalen für Nichtlineare Identifikationsaufgaben. In Hoffman, F., Hüllermeier, E., and Mikut, R., editors, *Proceedings 26. Workshop Computational Intelligence*, pages 139–158. KIT Scientific Publishing.

Heinz, T. O. and Nelles, O. (2017). Iterative Excitation Signal Design for Nonlinear Dynamic Black-Box Models. *Procedia Computer Science*, pages 1054–1061.

Heinz, T. O., Schillinger, M., Hartmann, B., and Nelles, O. (2017). Excitation signal design for nonlinear dynamic systems with multiple inputs – A data distribution approach. In Röpke, K. and Gühmann, C., editors, *International Calibration Conference - Automotive Data Analytics, Methods, DoE*, pages 191–208. expertVerlag.

Hoagg, J. B., Lacy, S. L., Babuška, V., and Bernstein, D. S. (2006). Sequential multisine excitation signals for system identification of large space structures. In *Proceedings of the American Control Conference*, pages 418–423.

Holland, J. H. (1975). Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence.

Isermann, R. (1992). *Identifikation dynamischer Systeme 1*. Springer Verlag.

Isermann, R. (2010). *Elektronisches Management motorischer Fahrzeugantriebe*. Springer.

Joseph, V. R., Gul, E., and Ba, S. (2015). Maximum projection designs for computer experiments. *Biometrika*, 102(2):371–380.

Lin, W. Y., Lee, W. Y., and Hong, T. P. (2003). Adapting crossover and mutation rates in genetic algorithms. *Journal of Information Science and Engineering*, 19:889–903.

Nelles, O. (2006). Axes-Oblique Partitioning Strategies for Local Model Networks. In *IEEE International Symposium on Intelligent Control*, pages 2378–2383. IEEE.

Nelles, O. (2013). *Nonlinear system identification: from classical approaches to neural networks and fuzzy models*. Springer Science & Business Media.

Nelles, O. and Isermann, R. (1995). Identification of nonlinear dynamic systems - classical methods versus radial basis function networks. In *Proceedings of the American Control Conference*, volume 5, pages 3786–3790.

Nouri, N. M., Valadi, M., and Asgharian, J. (2018). Optimal input design for hydrodynamic derivatives estimation of nonlinear dynamic model of AUV. *Nonlinear Dynamics*, 92(2):139–151.

Peter, T. J. and Nelles, O. (2019). Fast and simple dataset selection for machine learning. *at-Automatisierungstechnik*, 67(10):833–842.

Pintelon, R. and Schoukens, J. (2012). *System identification: a frequency domain approach*, volume 478. John Wiley & Sons.

Razali, N. M. and Geraghty, J. (2011). Genetic algorithm performance with different selection strategiesin solving TSP. In *Proceedings of the World Congress on Engineering*, volume 2, pages 1–6.

Rivera, D. E., Lee, H., Mittelmann, H. D., and Braun, M. W. (2002). Constrained multisine input signals for plant-friendly identification of chemical process systems. *IFAC Proceedings Volumes*, 35(1):425–430.

Sivanandam S.N., D. S. (2008). *Introduction to genetic algorithms.* Berlin: Springer.

Syswerda, G. (1991). Scheduling optimization using genetic algorithms. *Handbook of genetic algorithms*.

Tietze, N. (2015). *Model-based calibration of engine control units using gaussian process regression*. PhD thesis, Technische Universität Darmstadt.