MMR: Multiple Majority Rule Model with Bias

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Abstract: The Galam Majority Rule model describes how an opinion can spread in a network of nodes with no pre-existing edges. The model is used to describe opposing opinions that are represented in two states (Susceptible or Infected). This paper introduces a Multiple Majority Rule model (MMR) that improves the cons surrounding the Galam Majority Rule model by allowing opinion bias and adding a third state (undecided). The paper presents a comparative study between both models' behavior and performance. Lastly, the paper analyzes the impact of the bias in the consensus of a majority.

1 INTRODUCTION

In the last few decades, opinion dynamics models have been developed to model the spread of opinion in a population. People's opinion can change based on those they interact with. The Galam Majority Rule model does not consider social constructs that make people biased to what they believe (Rossetti, 2017). Furthermore, it represents a two-state system where every node in the population has an initial state: *Susceptible* and *Infected*. This model gives a clear advantage to the *infected state* skewing results towards an *infected* majority.

Our Multiple Majority Rule model (MMR) can be used as a two-state system: *Adopter* and *Rejector*, and as a three-state system: *Adopter*, *Rejector*, and *Undecided* which classifies a population that has not formed an opinion yet. Unlike the Galam Majority Rule model, the MMR model includes a modifiable opinion bias which considers the impact of social constructs on opinion dynamics. While a population may have two equal majorities, the bias can be adjusted so that one population has a higher probability of becoming prominent.

An experimental analysis has been carried out with the MMR model, to understand the effect of the bias. We analyze how changing the bias parameter will impact reaching a consensus and its effectiveness in comparison to the inherent bias used in the Galam Majority Rule model.

The paper is organized as follows: In Section 2, we go through the Galam Majority Rule model, how it works, and the cons that come with the model. In Section 3, we go through the MMR model, how it works, and its cons. Section 4 analyzes the differences between the Galam and MMR model. Section 5 covers the results of how the bias influences the outcome of the MMR model. Section 6 discusses related work. Finally, in Section 7 we conclude the paper, underlying the advantages of the MMR model, and further work that can be done.

2 THE GALAM MAJORITY RULE MODEL

Model States: The Galam model is composed of nodes existing in one of two states, *susceptible* and *infected*. Each state represents a definitive opinion with one in support of the opinion and the other against it.

Model Parameters: This model has three key parameters which are Fraction Infected, Q-group, and Iterations.

Fraction Infected: This parameter represents the percentage of nodes that start as *infected*. Its value is between zero and one, where zero means the whole population is *susceptible* and one means the entire population is *infected*.

Q-group: It represents a group of people of size Q, which is a value from one to the maximum number of

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nodes, where one means there is one person in the Q-group and the maximum number of nodes means the Q-group is equal to the total population. Regardless of size, all nodes in the Q-group are fully connected. After each iteration, the model adds up the total number of *susceptible* and *infected* nodes within the Q-group. Whichever state holds the most nodes becomes the majority, updating the Q-group state to the majority. If there is a tie in the Q-group, then the Q-group defaults to *infected*.

Iterations: It determines the number of times a Q-group is selected.

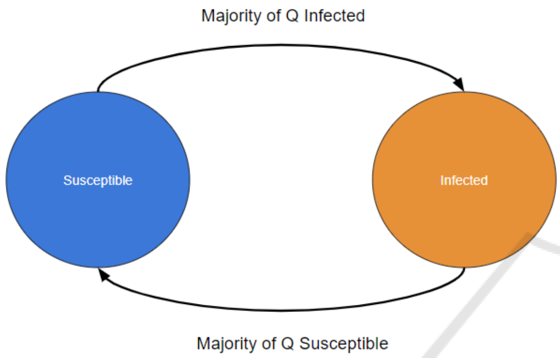


Figure 1: Diagram of the possible states and their relationship for the Galam Majority Rule model. Nodes can change from susceptible to infected and from infected back to susceptible. The status of the nodes depends on the majority status of the selected nodes for the Q-group.

2.1 Drawbacks with the Galam Majority Rule Model

The Galam Majority Rule model is meant to model the change in people's opinions by using two states. They found that the consensus for the nodes generally tends towards whichever state had the majority in the beginning (Krapivsky, 1). In real life, opinions that start as a minority initially can become the majority in the end. This model does not take into consideration social biases that might influence the decision making of the population. The algorithm used by NDLib to compute the majority is biased towards the *infected status* in every instance where there is a tie, which results in a skewed experiment when the size of the Q-group is even (Rossetti, 2017).

3 MULTIPLE MAJORITY RULE MODEL (MMR)

Model States: Depending on the configuration, the model can have up to three states. *Undecided*, people who have yet to develop an opinion. *Adopters* and *Rejectors*, which are those states that reflect the people who have a predetermined opinion on a given topic.

Model Parameters: This model has five key parameters which are Fraction Adopter, Fraction Rejector, Q-group, Iterations, and Bias.

Fraction Adopter: This parameter represents the percentage of nodes that start as *adopters*. Its value is between zero and one.

Fraction Rejector: This parameter represents the percentage of nodes that start as *rejectors*. Its value is between zero and one.

Q-group: Same functionality as in the Galam Majority Rule Model but If there is a tie in the Q-group, then the Q-group uses the bias parameter to determine the majority.

Iterations: It determines the number of times a Q-group is selected.

Bias: Given the case where a Q-group contains two equal majorities, the bias value represents the probability that the Q-group will change their opinion to the *adopter* state. This value should only be set inclusively between zero and one. When the bias value is zero, the probability that the *adopters* are selected as the majority in the Q-group is 0%. If the bias value is one, the probability that the *rejectors* are selected as the majority is 0%.

If the sum of Fraction Adopter and Fraction Rejector is one, the MMR model behaves as a two-state system. However, if the sum of these two parameters does not sum to 1, the difference between the sum and one represents the *undecided* population, which models a three-state system.

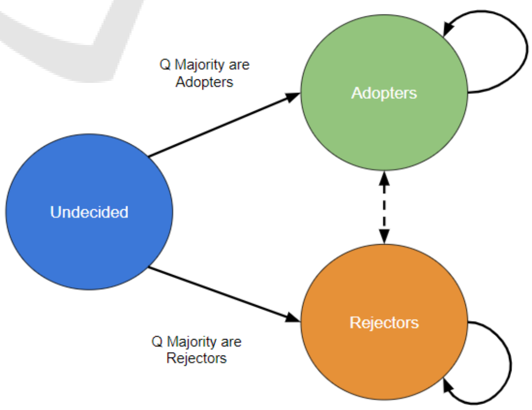


Figure 2: Diagram of the possible states and their relationship for the MMR model. If a node is undecided, it can only go to the adopter or rejector state. If a node is either an adopter or a rejector, it can be swayed to the opposite state or stay at its current state depending on the Q-group majority.

4 ANALYZING DIFFERENCE BETWEEN GALAM AND MMR MODEL

In this section, we tested the behavior of both models using different cases to compare the reliability of their results. The first subsection (A) discusses how each model behaves when a tie occurs. In subsection (B), we test the MMR model as a two-state system against the Galam Majority Rule model. Lastly, in subsection (C), we analyze the behavior of the MMR model as a three-state system.

4.1 The Case of a Tie

The Galam Majority Rule model produces a skewed result when the Q-group size is even. For Q-groups that have an even number, there is the possibility of a draw between both populations. A tie can only occur if the Q-group is even. In the event of a tie, the Galam Majority Rule model defaults the *infected state* to the majority. This clearly shows that the model design is skewed against the *susceptible state*. Table 1 & Figure 3 below show the probability of the *susceptible state* becoming the majority using different even numbers for the size of the Q-group initializing both populations at 50%.

Given a two-state system, like the Galam Majority Rule model, using a Q-group size of 2, there are 3 possible outcomes in the Q-group. Two *susceptible* nodes, one *susceptible* and one *infected*, or two *infected* nodes. From this information, we can deduce that the probability of the *susceptible* becoming the majority in an iteration is one-third. The results in Table 1 show the probability of the *susceptible* population becoming the majority in a total of 400 iterations. The number of iterations selected was the average number of iterations needed to reach a clear majority. Hence, this is the reason we used 400 iterations to calculate the probability of a *susceptible* majority.

Table 1: Probability of a Susceptible Majority with an even q-Group.

Q-group size	Probability of Susceptible Majority
2	< 0.00001
4	0.00002123787
8	0.011234105
16	0.109768941
32	0.25510399
64	0.36009636
128	0.418360442
256	0.449851167
512	0.4641378
1024	0.472096004

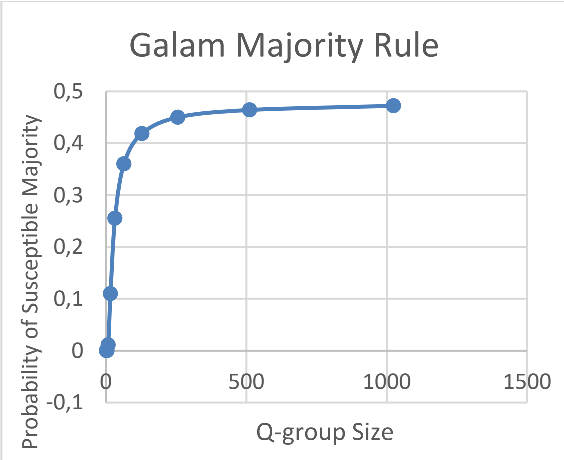


Figure 3: Plot showing how changing the size of the Q-group using even terms affects the probability that the susceptible state will become the majority.

Table 1 shows that when the size of the Q-group is an even number such as 2, the *susceptible* population has a significantly less than 1% chance of becoming the majority. Given an even number, the plot shows that, as the size of the Q-group approaches infinity, the probability of the *susceptible state* winning approaches 50%. This means that the Galam Majority Rule model results in a very skewed experiment when the Q-group consists of a small even number of nodes. However, the experiment becomes less skewed as you increase the size of the Q-group. For example, when the size of the Q-group is 1024 the *susceptible* population has a 47% chance of becoming the prevalent majority.

Although the experiment becomes more balanced as the Q-group increases in size, the simulation also becomes unrealistic. It is assumed that every person in a Q-group interacts with everyone else in the same Q-group simultaneously. Using a large Q-group size such as 500 would mean each node is connected to the other 499 nodes in the Q-group. To assume that every node share 499 common connections is unrealistic.

$$\lim_{Q\to\infty} 1 - \frac{\left(\frac{Q}{2}\right)+1}{Q+1} = \frac{1}{2} \quad (1)$$

Q is the value assigned to the size of the Q-group. Equation (1) is used to calculate the probability of the *susceptible state* becoming the majority in a single iteration. Using limits, as the size of the Q-group reaches infinity, this probability of *susceptible* majority approaches one-half.

We used (1) to calculate the probability of the *susceptible state* becoming the majority in a single iteration. To compute the probability of the *susceptible state* becoming the majority after the total number of iterations, we calculated the cumulative binomial probability, where the probability of success in a single iteration was the value coming from (1).

Now let us measure the probability of the *susceptible* state becoming the majority after the total number of iterations instead of a single iteration.

$$\sum_{k=201}^{400} \binom{400}{k} 0.4^k 0.6^{400-k} \tag{2}$$

k is the number of successful iterations for *susceptible* majority. To find out the probability of the *susceptible* state becoming the majority after the total number of iterations, we used the cumulative binomial probability function (2). The probability of success on a single trial is 0.40. The number of trials equals the number of iterations, which we set to 400.

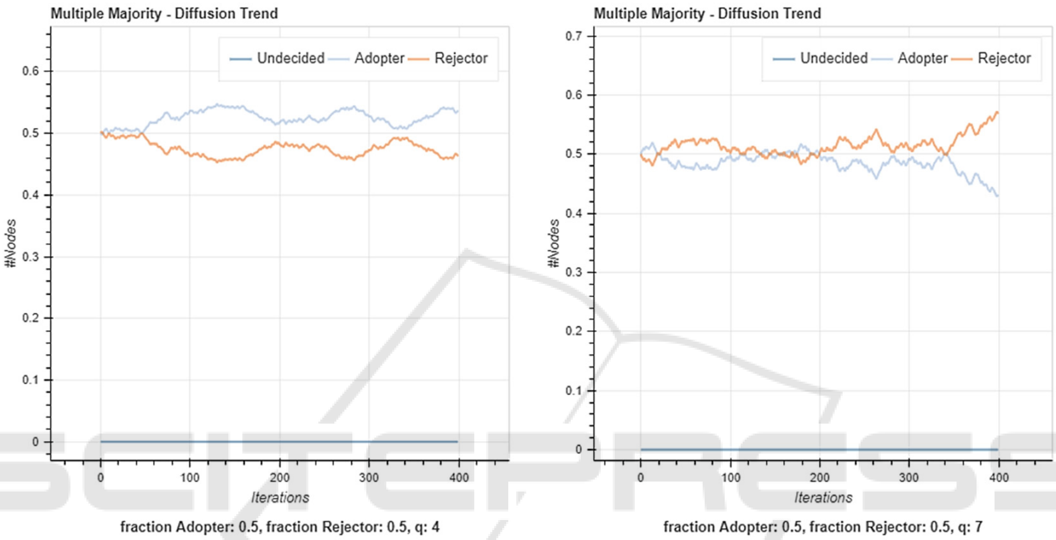


Figure 4: MMR model with a two-state system where both populations are set to 50%. 1000 nodes were selected for this experiment along with 400 iterations, a neutral bias, and a Q-group size of 4 and 7 from left to right.

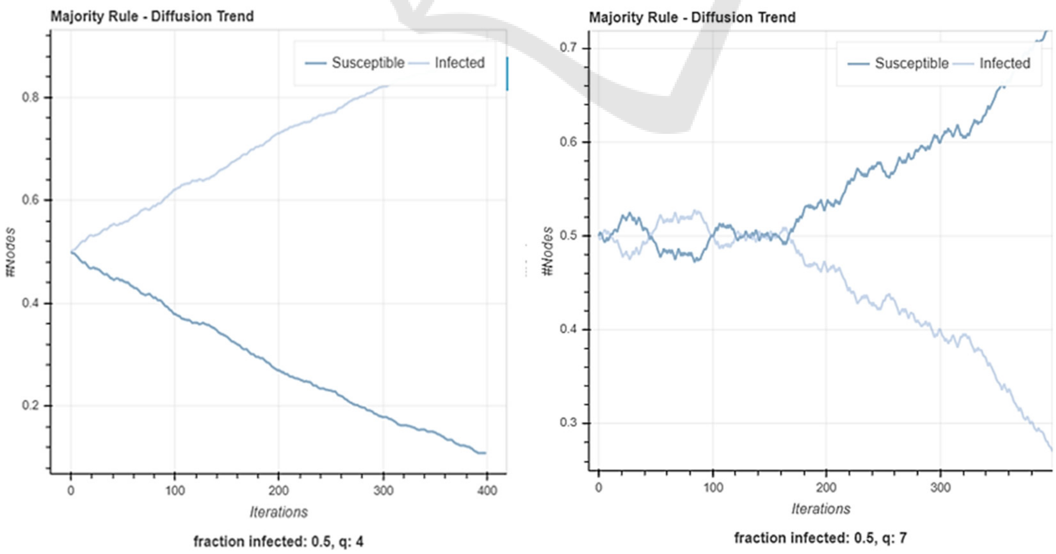


Figure 5: Galam Majority Rule model configured with the same initial parameters as shown in Figure 4 without the bias parameter.

Finally, the last field is the number of successes, which is the minimum number of successful iterations for a population to take over the majority, which in this case is 201. After you input all the values in their respective fields you will find the probability of success on a single trial. The sum of all the equations with the different values of k will give you the probability of the *susceptible* population becoming the majority after 400 iterations. This probability could be expressed by $P(400 \leq X \geq 201)$ where X is the number of iterations required to be the final majority.

In the case of the MMR model if a tie occurs, regardless if Q is even or odd, the outcome of the majority is chosen depending on what the bias value is. The user can choose to have a biased or unbiased experiment favoring either *adopters* or *rejectors*, or neither. If the user chooses to make a balanced simulation, the bias value must be set to 0.5, which will result in the majority being chosen stochastically whenever a tie occurs (Chaouiya, 2013).

$$\frac{Q/2}{Q} = \frac{1}{2} \tag{3}$$

Equation 3 is used to calculate the probability of the *adopter* or *rejector* population becoming the majority using a two-state MMR model. The formula assumes the model uses a neutral bias and both populations start at 50%.

Since (3) can be simplified to one-half, we can conclude that, given a neutral bias, both populations have exactly a 50% chance of becoming the majority in a two-state system. It must be noted that you can use a Q-group of any size, even or odd and the probability will not change.

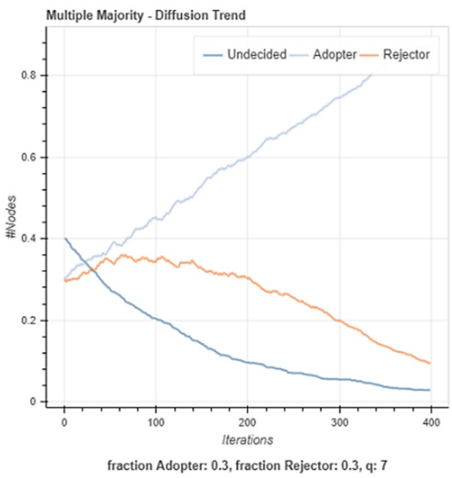
4.2 The Case of Two States

The following charts showcase the MMR model functioning as a two-state system in comparison with the Galam Majority model in its respective order. Figure 4 illustrates that when using the MMR model, changing the size of the Q-group from an even or odd number does not dictate that a specific population will become the majority.

The left plot in Figure 5 demonstrates that when you use an even number as the size of the Q-group, the results are skewed towards the *infected state*. The plot on the right shows that once the Q-group size has been modified to an odd number, the Galam Majority Rule model produces a balanced simulation where neither population is advantaged.

4.3 The Case of Three States

The Galam Majority Rule model cannot simulate a three-state system. The MMR model introduces the *undecided* state which represents a group of nodes without a defined stance. It was found that the addition of the *undecided* nodes increases the rate at which nodes become opinionated. This does not necessarily mean that a majority will be computed sooner. The higher the population of *undecided*, the more variance there is in the results. This means that ignoring those nodes which are *undecided* might make a big difference when computing a majority with regards to time. In most cases, if you conduct an experiment where the population is divided into 50% of *adopters* and 50% of *rejectors*, the rate at which a majority is computed is slower than that of an experiment conducted with 30% of *adopters*, 30% of *rejectors* and 40% of *undecided*.



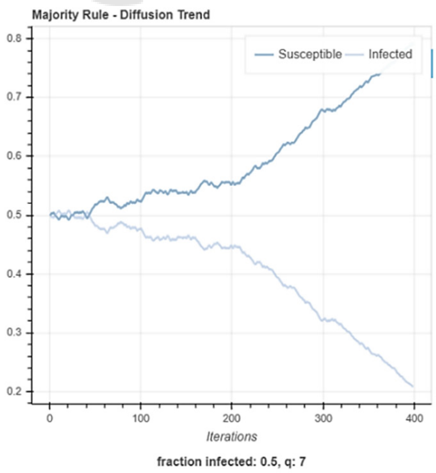


Figure 6: Plots showing the impact of using the MMR model as a three-state system on the left in respect to a two-state system Galam Majority Rule model simulation on the right. These plots also use a population of 1000 nodes.

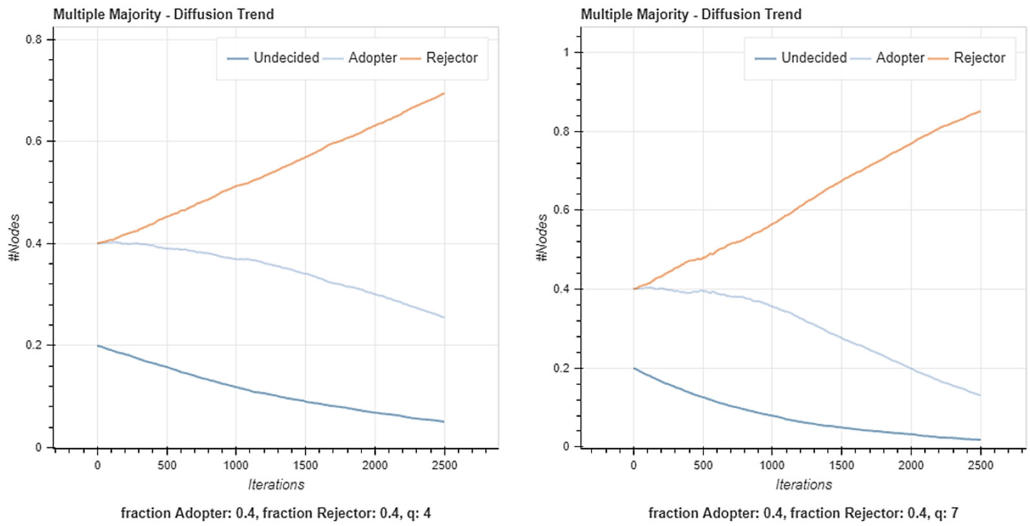


Figure 7: Plots showing a 0% bias for adopters and 100% bias for rejectors.

As shown in Figure 6, after 200 iterations using the MMR model, 30% of the population changed their initial stance. While using the Galam Majority Rule model resulted in ~10% of the population changing their initial status on the same mark. As expected, using the three-state system MMR model results in a faster simulation even though the two-state system reaches a consensus sooner because of the lead start in population.

5 THE IMPACT OF BIAS IN THE MMR MODEL

In this section, we discuss how various values for the bias can affect the outcome of a given simulation and how the size of the Q-group has an effect on the bias and its effectiveness.

5.1 Analyzing Bias Results

The results below show as expected, the two lines that represent both states display polarizing results at 0% and 100% bias. These lines get closer as they reach the neutral bias value which is 0.5. From this point, shifting the bias to one side or the other will be the reason a state will have an advantage over the other.

With the ability to be able to set a bias before running the model, we can visualize how the value of the bias affects the outcome of the simulation. For the data collected, we looked at different breakpoints of bias values to visually see the impact it had. For Figures 7-9, all the charts were initially configured to have 7500 nodes, 2500 iterations, *adopter*, and *rejector* populations at 40% each and *undecided* population at the remaining 20%. Except for Figure 10.2, every left plot has a Q-group size of 4 and the right plot has a Q-group size of 7. At the extreme ends where the bias for either *rejector* or *adopter* is 0% or 100%, the charts show clearly that the majority opinion with the favorable bias will end up with an overwhelming majority. As the bias gets to the 25/75% split, this gap between the majority and minority opinions begins to close. When there is a neutral bias, the majority opinion only slightly hovers above the 50% mark and not straying far from it.

As seen from these results, the dispersion between the two states as we modify the values of the bias from 0 to 1. The closer it approaches either 0 or 1 the more disparate the lines become. As we can see 0.5 is the midpoint between the minimum and the maximum bias and it is the point where neither state has an advantage over the other. As we can see from this diagram, both sides of the plot are symmetric, meaning that setting a bias of 0.25 or a bias of 0.75 will make both simulations behave the same, just changing which state is in the majority.

6 RELATED WORK

The Galam Majority Rule model has been studied and researched extensively. It has been shown that the Galam Majority Rule model generally results in the final opinion being equal to the initial majority (Krapivsky, 2003). This paper explores the spread of opinion using the Galam Majority Rule model. It found that for most systems the final opinion always

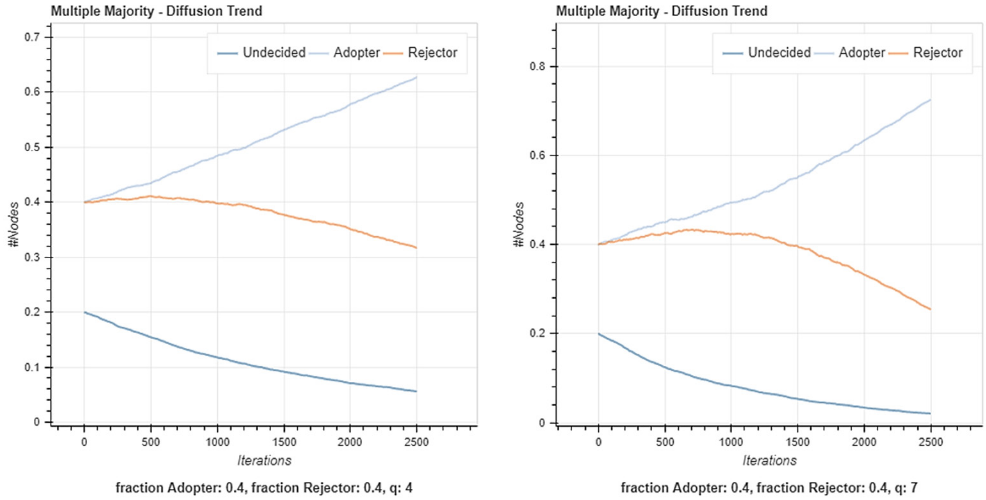


Figure 8: Plots showing a 25% bias for adopters and 75% bias for rejectors.

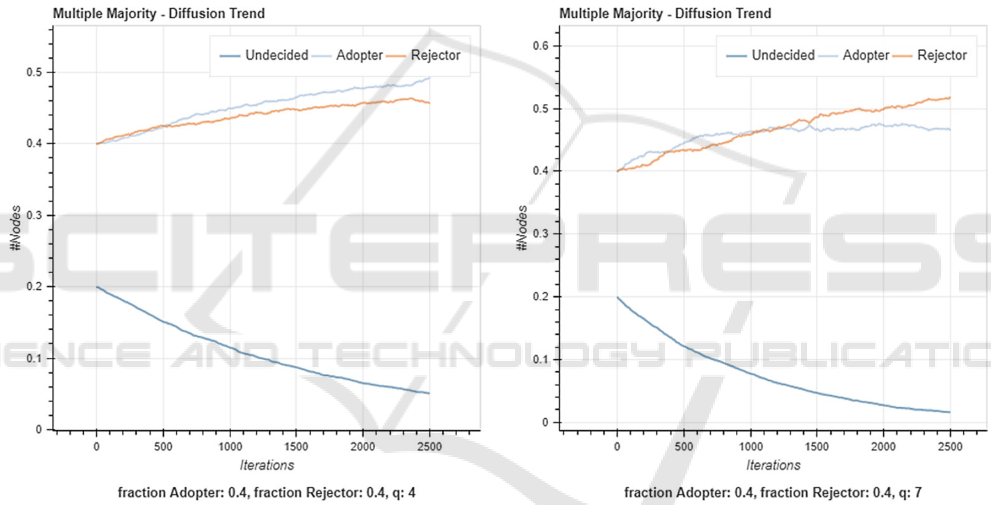


Figure 9: Plots showing a neutral bias (50%) for adopter and rejector.

equals that of the initial majority except when the dimension of the model is one. The Galam Majority Rule model when stubborn nodes are introduced into the network (Mukhopadhyay, 2020). It shows that when nodes are biased towards a preferred opinion, the bias can affect the consensus of the preferred opinion and can be achieved with a high probability. It also shows that when stubborn agents with fixed opinions are present, the resulting network will have metastability, fluctuating between each of the different states of the configuration. Our MMR model behaves similarly to the gene regulatory networks when they update their next gene when a tie occurs (Chaouiya, 2013).

7 CONCLUSION AND FUTURE WORK

The Galam Majority Rule model is a simple model that shows the spread of opinions throughout a population. However, it has an unmodifiable bias, which results in creating skewed results. This model does not allow for nodes without a definitive opinion. It was found that counting for undecided agents in a network makes the model more accurate with respect to time as well as giving the model more variance in the results. The bias has proved that it can be the determining factor of a population reaching a majority even if they started in the minority.

The Galam Majority Rule model has proven not to be consistent, as using an even number for the size of the Q-group results in a skewed experiment against the *susceptible* population. Alternatively, the MMR model has proven that while using a neutral bias neither population has an advantage. Furthermore, the MMR model can be used as a two-state system to replace the Galam Majority Rule model. Our model can also be used to include undecided agents and add a bias to account for how strong each of the opposing populations' opinions is.

For future work, the network graph the model uses on the back end consists of nodes and edges which are only formed between the nodes in the Q-group. We believe that our MMR model could be further improved if the nodes in the Q-group were not picked at random and there was an underlying network layer that simulated pre-existing connections like how people interact in real-life. Then Q-groups could be selected from nodes who are neighbors. This would more accurately represent the way information spreads in society. Further research could be conducted by considering the eccentricity and betweenness when selecting nodes to form Q-groups from the network graph.

Lastly, use of statistical analysis may be considered to find the average outcome of multiple model simulations, and machine learning, to find the value of the bias for different topics.

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