CC-separation Measure Applied in Business Group Decision Making

Jonata C. Wieczynski[1][a], Giancarlo Lucca[2,3][b], Eduardo N. Borges[2][c], Graçaliz P. Dimuro[2,5][d], Rodolfo Lourenzutti[4][e] and Humberto Bustince[5][f]

[1]*Programa de Pós-Graduação em Computação, Universidade Federal do Rio Grande, Av. Itália Km 8, Rio Grande, Brazil*

[2]*Programa de Pós-Graduação em Modelagem Computacional, Universidade Federal do Rio Grande, Rio Grande, Brazil*

[3]*Centro de Ciências Computacionais, Universidade Federal do Rio Grande, Rio Grande, Brazil*

[4]*Department of Statistics, University of British Columbia, Vancouver, Canada*

[5]*Departamento de Estadística, Informática y Matemáticas, Universidad Publica de Navarra, Pamplona, Spain*

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Abstract: In business, one of the most important management functions is decision making. The Group Modular Choquet Random TOPSIS (GMC-RTOPSIS) is a Multi-Criteria Decision Making (MCDM) method that can work with multiple heterogeneous data types. This method uses the Choquet integral to deal with the interaction between different criteria. The Choquet integral has been generalized and applied in various fields of study, such as imaging processing, brain-computer interface, and classification problems. By generalizing the so-called extended Choquet integral by copulas, the concept of CC-integrals has been introduced, presenting satisfactory results when used to aggregate the information in Fuzzy Rule-Based Classification Systems. Taking this into consideration, in this paper, we applied 11 different CC-integrals in the GMC-RTOPSIS. The results demonstrated that this approach has the advantage of allowing more flexibility and certainty in the choosing process by giving a higher separation between the first and second-ranked alternatives.

1 INTRODUCTION

Business managers rely on the right decisions to keep their business competitive. Many times a decision has to be made by multiple analysts and considering various criteria. This is a time consuming and expensive task. Although, most of the time, it can be solved by an algorithm or mathematical model, like route, supplier chain, and location problems (Deveci et al., 2017; Alazzawi and Żak, 2020; Shyur and Shih, 2006), releasing the pressure of the decision from the managers, and allow them to work on other processes of the company/industry.

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Huang and Yoon, 1981) is one of the multi-criteria decision making (MCDM) methods that ranks the best possible solution among a set of alternatives. This approach is based on pre-defined criteria, using the alternative's distance to the best and worst possible solutions for the problems, Positive and Negative Ideal Solutions (PIS and NIS), respectively.

In 2017, the Group Modular Choquet Random TOPSIS (GMC-RTOPSIS) (Lourenzutti et al., 2017) was introduced. The method generalized the original TOPSIS allowing it to deal with multiple and heterogeneous data types. The approach models the interaction among the criteria by using the discrete Choquet integral (Choquet, 1954). The Choquet integral allows a function to be integrated by using non-additive fuzzy measures (Choquet, 1954; Candeloro et al., 2019), which means that it can consider the interaction among the elements that are being integrated (Murofushi and Sugeno, 1989; Dimuro et al., 2020). The GMC-RTOPSIS learns the fuzzy measure associated with the criteria with a Particle Swarm Optimization (PSO) algorithm (Wang et al., 2011) .

The C_T-integrals (Lucca et al., 2016) is a generalization of the Choquet integral that replaces the product operation by triangular norm (t-norm) functions (Klement et al., 2011). The C_T-integrals are a family

[a] https://orcid.org/0000-0002-8293-0126
[b] https://orcid.org/0000-0002-3776-0260
[c] https://orcid.org/0000-0003-1595-7676
[d] https://orcid.org/0000-0001-6986-9888
[e] https://orcid.org/0000-0003-2434-4302
[f] https://orcid.org/0000-0002-1279-6195

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of integrals that are pre-aggregation functions (Lucca et al., 2016). Additionally, C_T-integrals are averaging functions, i.e., the result is always between the minimum and maximum of the input.

The T-separation measure (Wieczynski et al., 2020) was introduced and applied in the GMC-RTOPSIS instead of the Choquet integral. In this study, the authors considered five different T-separation measures to tackle Case Study 2 from (Lourenzutti et al., 2017). The problem consists of choosing a new supplier for a company by asking various decision-makers to give their opinions with different criteria. The problem is posed with a variety of data types, such as probability distributions, fuzzy numbers, and interval numbers. The paper also proposed to use the t-norm that better discriminates the first ranked alternative to the second one by calculating the difference of the rankings. The approach presented good results when using the Łukasiewicz t-norm ($T_{Ł}$), giving a better separation between the ranked alternatives than the standard Choquet integral.

After introducing the C_T-integrals, Lucca et al. have proposed the *CC*-integrals (Lucca et al., 2017a). CC-integrals are a generalization of the Choquet integral in its expanded form, satisfying some properties, such as averaging, idempotency, and aggregation (Grabisch et al., 2009). The authors applied the CC-integral in classification problems, showing that the function based on the minimum is the one that produced the highest performance of the classifier. The CC-integrals have been studied in the literature by Dimuro et al., where the properties of CMin integrals (Lucca et al., 2017b; Dimuro et al., 2018; Mesiar and Stupňanová, 2019) were analyzed.

In this paper, we define the CC-separation measure and use it in the GMC-RTOPSIS instead of the Choquet integral. We also apply our approach in an application as an example, the same used in (Lourenzutti et al., 2017; Wieczynski et al., 2020). Finally, we analyze its results and compare them to other works.

The paper is organized as follows: Section 2 introduces the basic concepts about the fuzzy set theory and TOPSIS decision making. In Section 3 we define the CC-separation measure. In Section 4 we show our methodology, the problem used to test the CC-separation measure, and the results achieved by using it. Lastly, the conclusion is in Section 5.

2 PRELIMINARY CONCEPTS

In this section, we recall the preliminary concepts necessary to develop the paper.

2.1 Fuzzy Set Theory

A Fuzzy Set (Zadeh, 1965) is defined on a universe X by a membership function $\mu_a : X \to [0,1]$, denoted by

$$a = \{\langle x,\ \mu_a(x)\rangle \mid x \in X\}.$$

We call a trapezoidal fuzzy number (**TFN**) the fuzzy set denoted by $a = (a_1, a_2, a_3, a_4)$, where $a_1 \le a_2 \le a_3 \le a_4$, if the membership function μ_a is defined on $\mathbb{R}$ as:

$$\mu_a(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \le x < a_2 \\ 1, & \text{if } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{if } a_3 < x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

A measure of the distance between two TFNs $a = (a_1,\ a_2,\ a_3,\ a_4)$ and $b = (b_1,\ b_2,\ b_3,\ b_4)$ is defined as:

$$d(a,b) = \sqrt{\frac{1}{4}\sum_{i=1}^{4}(a_i - b_i)^2}.$$

The defuzzified value of a TFN $a = (a_1, a_2, a_3, a_4)$ is given by:

$$m(a) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

An intuitionistic fuzzy set (**IFS**) A is defined on a universe X by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\nu_A : X \to [0,1]$ such that $\mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$, that is:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}.$$

Let $\tilde{\mu}_A$ and $\tilde{\nu}_A$ be the maximum membership degree and the minimum non-membership degree, respectively, of an IFS A.

An IFS A is an intuitionistic trapezoidal fuzzy number (**ITFN**), denoted by

$$A = \langle (a_1, a_2, a_3, a_4), \tilde{\mu}_A, \tilde{\nu}_A \rangle$$

where $a_1 \le a_2 \le a_3 \le a_4$, if μ_A and ν_A are given, for all $x \in \mathbb{R}$, by

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}\tilde{\mu}_A, & \text{if } a_1 \le x < a_2 \\ \tilde{\mu}_A, & \text{if } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}\tilde{\mu}_A, & \text{if } a_3 < x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} \frac{1-\tilde{\nu}_A}{a_1-a_2}(x-a_1)+1, & \text{if } a_1 \le x < a_2 \\ \tilde{\nu}_A, & \text{if } a_2 \le x \le a_3 \\ \frac{1-\tilde{\nu}_A}{a_4-a_3}(x-a_4)+1, & \text{if } a_3 < x \le a_4 \\ 1, & \text{otherwise.} \end{cases}$$

The distance between two ITFNs $A = \langle (a_1,\, a_2,\, a_3,\, a_4)\,,\, \tilde{\mu}_A,\, \tilde{\nu}_A\rangle$ and $B = \langle (b_1,\, b_2,\, b_3,\, b_4)\,,\, \tilde{\mu}_B,\, \tilde{\nu}_B\rangle$ is:

$$d(A,B) = \frac{1}{2}\left[d_{\tilde{\mu}}(A,B) + d_{\tilde{\nu}}(A,B)\right]$$

where

$$d_\kappa(A,B) = \left\{ \frac{1}{4}\left[(a_1-b_1)^2 + (1+(\kappa_A-\kappa_B)^2) \right.\right. \\ (1+(a_2-b_2)^2+(a_3-b_3)^2) \\ \left.\left. -1+(a_4-b_4)^2 \right] \right\}^{1/2}$$

for $\kappa_A = \tilde{\mu}_A$ and $\kappa_B = \tilde{\mu}_B$ when $\kappa = \mu$; and for $\kappa_A = \tilde{\nu}_A$ and $\kappa_B = \tilde{\nu}_B$ when $\kappa = \nu$.

Aggregation functions (**AF**) (Grabisch et al., 2009) are used to unify inputs into a single value representing them all and are defined as a function that maps $n > 1$ arguments onto the unit interval, that is, a function $f : [0,1]^n \to [0,1]$ such that the boundaries, $f(\mathbf{0}) = 0$ and $f(\mathbf{1}) = 1$, with $\mathbf{0},\mathbf{1} \in [0,1]^n$, and the monotonicity properties, $\boldsymbol{x} \leq \boldsymbol{y} \implies f(\boldsymbol{x}) \leq f(\boldsymbol{y})$, $\forall \boldsymbol{x},\boldsymbol{y} \in [0,1]^n$, hold.

A triangular norm (t-norm) is an aggregation function $T : [0,1]^2 \to [0,1]$ that satisfies, for any $x,y,z \in [0,1]$: the commutative and associative properties and the boundary condition.

An overlap function (Bustince et al., 2010) $O : [0,1]^2 \to [0,1]$ is a function that satisfies the following conditions:

- O is commutative;
- $O(x,y) = 0 \iff xy = 0$;
- $O(x,y) = 1 \iff xy = 1$;
- O is increasing;
- O is continuous.

A bivariate function $Co : [0,1]^2 \to [0,1]$ is called a copula (Nelsen, 2007) if, for all $x,x',y,y' \in [0,1]$ with $x \leq x'$ and $y \leq y'$, the following conditions hold:

- $Co(x,y) + Co(x',y') \geq Co(x,y') + Co(x',y)$;
- $Co(x,0) = Co(0,x) = 0$;
- $Co(x,1) = Co(1,x) = x$.

The Choquet integral is defined based on a fuzzy measure (Sugeno, 1974), that is, a function m from the power set of N to the unit interval, $m : 2^N \to [0,1]$, that for all $X,Y \subset N$ holds the conditions:

(1) $m(\emptyset) = 0$ and $m(N) = 1$;

(2) if $X \subset Y$, then $m(X) \leq m(Y)$.

From this, Choquet defined the integral as: Let m be a fuzzy measure. The Choquet integral (Choquet, 1954) of $\boldsymbol{x} \in [0,1]^n$ with respect to m is defined as:

$$\mathfrak{C}_m : [0,1]^n \to [0,1] \\ \boldsymbol{x} \to \sum_{i=1}^{n} \left(x_{(i)} - x_{(i-1)}\right)\, m(A_{(i)})$$

where (i) is a permutation on 2^N such that $x_{(i-1)} \leq x_{(i)}$ for all $i = 1,\ldots,n$, with $x_{(0)} = 0$ and $A_{(i)} = \{(1),\ldots,(i)\}$.

Notice that one can use the distributive law to expand the Choquet integral into:

$$\mathfrak{C}_m = \sum_{i=1}^{n} \left(x_{(i)} m(A_{(i)}) - x_{(i-1)} m(A_{(i)})\right) \qquad (1)$$

Recently, the Choquet integral was generalized by copula functions. By substituting the product operator by copulas in the expanded form of the Choquet integral (Eq. 1), CC-Integrals (Lucca et al., 2017a) were introduced.

Let m be a fuzzy measure and Co be a bivariate copula. The Choquet-like integral based on copula with respect to m is defined as a function $\mathfrak{C}_m^{Co} : [0,1]^n \to [0,1]$, for all $\boldsymbol{x} \in [0,1]^n$, by

$$\mathfrak{C}_m^{Co} = \sum_{i=1}^{n} Co\left(x_{(i)},\, m(A_{(i)})\right) - Co\left(x_{(i-1)},\, m(A_{(i)})\right) \qquad (2)$$

where (i), $x_{(i)}$ and $A_{(i)}$ is defined as the Choquet integral.

It is important to note that the Choquet integral, the C_T-integrals, and the CC-integrals are averaging functions, i.e., the results from them are always bounded by the minimum and maximum of their input.

2.2 Decision Making

The GMC-RTOPSIS (Lourenzutti et al., 2017) is a decision making algorithm that improved the classic TOPSIS (Huang and Yoon, 1981) by allowing groups of decision-makers, modularity in the input, multiple input types and, by using the Choquet integral, the ability to measure the interaction among different criteria.

Figure 1 shows an overview of the decision making process with the Choquet integral. Here three different decision-makers give their ratings for three products based on three criteria. These ratings are then processed and inserted in the Choquet integral, where the interaction between the criteria is calculated. After, the results are ranked according to their highest classiness coefficient value.

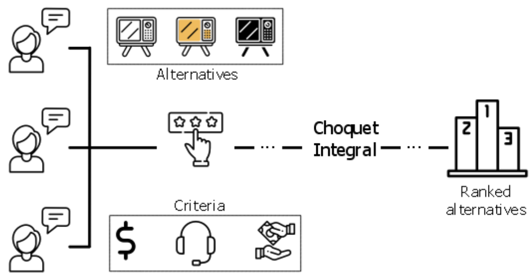

Figure 1: Image description of the decision making process using the Choquet integral. Source: the authors.

To describe the GMC-RTOPSIS method let q represent the q-th decision maker in a collection of $Q \in \mathbb{N} = \{1,2,3,\ldots\}$ ones. Let $\boldsymbol{A} = \{A_1,\ldots,A_m\}$ be the set of alternatives for the problem and $\boldsymbol{C}_q = \{C_1,\ldots,C_{n_q}\}$ represent the criteria set for decision maker q. With $\boldsymbol{C} = \{\boldsymbol{C}_1,\ldots,\boldsymbol{C}_Q\} = \{C_1,\ldots,C_n\}$, where $n = \sum_{q=1}^{Q} n_q$, representing the criteria set of all the decision makers. From these notations we can represent each of the q-th decision maker by the matrix below (Eq. (3)), called decision matrix DM:

$$DM^q = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_{n_q} \end{array} \\ \begin{pmatrix} s_{11}^q(\boldsymbol{Y}^q) & s_{12}^q(\boldsymbol{Y}^q) & \cdots & s_{1n_q}^q(\boldsymbol{Y}^q) \\ s_{21}^q(\boldsymbol{Y}^q) & s_{22}^q(\boldsymbol{Y}^q) & \cdots & s_{2n_q}^q(\boldsymbol{Y}^q) \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1}^q(\boldsymbol{Y}^q) & s_{m2}^q(\boldsymbol{Y}^q) & \cdots & s_{mn_q}^q(\boldsymbol{Y}^q) \end{pmatrix} \end{array} \quad (3)$$

Each matrix cell $s_{ij}^q(\boldsymbol{Y}^q)$, with $1 \leq i \leq m,\ 1 \leq j \leq n_q$, is called the rating of the criterion j for alternative i. Also, notice that the rating is a function of $\boldsymbol{Y} = (\boldsymbol{Y}_{rand},\ \boldsymbol{Y}_{det})$, which are factors that model random and deterministic events. Random events are modeled by stochastic processes, and deterministic are events which are not random, like time, location or a parameter of a random event. A fixed value x of the deterministic vector is called a state, and the set of all states is represented by $\mathcal{X}$.

In possession of all decision matrices from all decision-makers Q, the algorithm can be applied. The process is quite similar to the original TOPSIS, presented in 1981. It uses the same definition of Positive Ideal Solution (**PIS**) and Negative Ideal Solution (**NIS**) that are, respectively, the one that is closer to the best possible solution and the one that is distant from the best possible solution, see Eq. (4). The most significant difference is that each criterion may use a different distance measure since each may have its own type. So, the distances of each criterion are calculated separately and aggregated afterward in the separation measure step of the algorithm (see Figure 2).

In order to ease the comprehension of our approach, we present in Figure 2 the steps of the GMC-RTOPSIS, where:

Step 0. Select a state $x \in \mathcal{X}$ not yet processed;

Step 1. Normalize all matrices;

Step 2. Select the PIS, denoted by $s_j^+(\boldsymbol{Y})$, and the NIS, denoted by $s_j^-(\boldsymbol{Y})$, considering, for each $j \in \{1,\ldots,n\}$, respectively:

$$s_j^+(\boldsymbol{Y}) = \begin{cases} \max\limits_{1\leq i\leq m} s_{ij}, & \text{if it is a benefit criterion,} \\ \min\limits_{1\leq i\leq m}, s_{ij} & \text{if it is a cost/loss criterion,} \end{cases} \quad (4)$$

$$s_j^-(\boldsymbol{Y}) = \begin{cases} \min\limits_{1\leq i\leq m} s_{ij}, & \text{if it is a benefit criterion,} \\ \max\limits_{1\leq i\leq m} s_{ij}, & \text{if it is a cost/loss criterion;} \end{cases}$$

Step 3. Calculate the distance measure for each criterion C_j, with $j \in \{1,\ldots,n\}$, to the PIS and NIS solutions, that is,

$$d_{ij}^+ = d(s_j^+(\boldsymbol{Y}), s_{ij}(\boldsymbol{Y})),$$
$$d_{ij}^- = d(s_j^-(\boldsymbol{Y}), s_{ij}(\boldsymbol{Y})),$$

where $i \in \{1,\ldots,m\}$ and d is a distance measure associated with the criteria data type;

Step 4. Calculate the separation measure, for each $i \in \{1,\ldots,m\}$, using the Choquet integral as follows:

$$S_i^+(\boldsymbol{Y}) = \sqrt{\sum_{j=1}^{n}\left(\left(d_{i(j)}^+\right)^2 - \left(d_{i(j-1)}^+\right)^2\right) m_{\boldsymbol{Y}}(\boldsymbol{C}_{(j)}^+)}$$

$$S_i^-(\boldsymbol{Y}) = \sqrt{\sum_{j=1}^{n}\left(\left(d_{i(j)}^-\right)^2 - \left(d_{i(j-1)}^-\right)^2\right) m_{\boldsymbol{Y}}(\boldsymbol{C}_{(j)}^-)}$$

where $d_{i(1)}^+ \leq \ldots \leq d_{i(n)}^+$, $d_{i(1)}^- \leq \ldots \leq d_{i(n)}^-$, for each $j \in \{1,\ldots,n\}$, $C_{(j)}^+$ is the criterion correspondent to $d_{i(j)}^+$, $C_{(j)}^-$ is the criterion correspondent to $d_{i(j)}^-$, $\boldsymbol{C}_{(j)}^+ = \{C_{(j)}^+, C_{(j+1)}^+, \ldots, C_{(n)}^+\}$, $\boldsymbol{C}_{(j)}^- = \{C_{(j)}^-, C_{(j+1)}^-, \ldots, C_{(n)}^-\}$, $\boldsymbol{C}_{(n+1)}^+ = \boldsymbol{C}_{(n+1)}^- = \emptyset$, $d_{i(0)}^+ = d_{i(0)}^- = 0$ and m_Y is the learned fuzzy measure by a particle swarm optimization algorithm (Wang et al., 2011).

Here, the separation measure is the square root of the Choquet integral of squared distances, and this means that it is the square root of a d-Choquet integral (Bustince et al., 2020). Also, for each state, we may have a different fuzzy measure, which means that the fuzzy measure is dependent on $\boldsymbol{Y}_{det}$

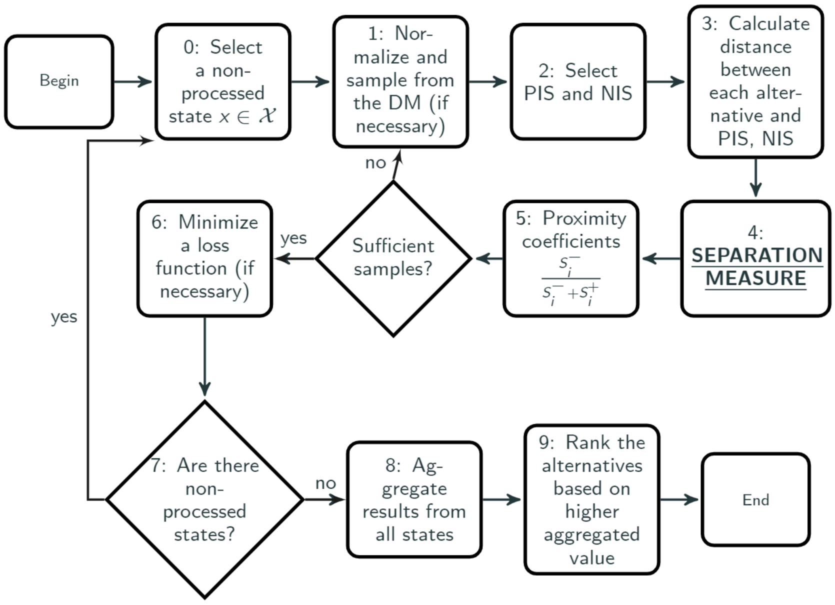

Figure 2: Diagram of the GMC-RTOPSIS process. The separation measure step is where the CC-separation measure is used. Source: The authors.

Step 5. For each $i \in \{1,\ldots,m\}$, calculate the relative closeness coefficient to the ideal solution with:

$$CC_i(\boldsymbol{Y}) = \frac{S_i^-(\boldsymbol{Y})}{S_i^-(\boldsymbol{Y}) + S_i^+(\boldsymbol{Y})};$$

Step 6. By using probability distributions in the DM, it is introduced a bootstrapped probability distribution in the CC_i values, so as a point representation for this distribution we minimize a predefined risk function:

$$\begin{aligned} cc_i &= \arg\min_c R(c) \\ &= \arg\min_c \int_{\mathbb{R}} L(c,\, CC_i(\boldsymbol{Y}))\, dF(CC_i(\boldsymbol{Y})); \end{aligned} \quad (5)$$

Step 7. If there is at least one non-processed state x, return to Step 0;

Step 8. Aggregate the cc_i values from all the states with $\widehat{cc_i} = f_{x\in X}(cc_i(x))$, where f is an aggregation function.

Step 9. Finally, rank the alternatives from the highest to the lowest $\widehat{cc_i}$ values.

3 GENERALIZATION OF THE GMC-RTOPSIS BY USING CC-INTEGRALS

Using the Choquet integral in the separation measure, the GMC-RTOPSIS method allows for interaction among different criteria. This is the step where this study incorporates the CC-integrals in place of the Choquet integral.

We introduce the CC-separation measure by:

Definition 3.1 (CC-separation measure)**.** *Let Co be a bivariate copula and m a fuzzy measure. A CC-separation measure $S^* : [0,1]^2 \to [0,1]$ is defined, for all $i \in \{1,\ldots,m\}$, by the functions:*

$$S_i^+(\boldsymbol{Y}) = \left[\sum_{j=1}^{n} Co\left(\left(d_{i(j)}^+\right)^2,\, m_{\boldsymbol{Y}}\left(\boldsymbol{C}_{(j)}^+\right)\right) - Co\left(\left(d_{i(j-1)}^+\right)^2,\, m_{\boldsymbol{Y}}\left(\boldsymbol{C}_{(j)}^+\right)\right)\right]^{1/2}$$

$$S_i^-(\boldsymbol{Y}) = \left[\sum_{j=1}^{n} Co\left(\left(d_{i(j)}^-\right)^2,\, m_{\boldsymbol{Y}}\left(\boldsymbol{C}_{(j)}^-\right)\right) - Co\left(\left(d_{i(j-1)}^-\right)^2,\, m_{\boldsymbol{Y}}\left(\boldsymbol{C}_{(j)}^-\right)\right)\right]^{1/2}$$

where $d_{i(j)}^+$, $d_{i(j)}^-$, $\boldsymbol{C}_{(j)}^+$, $\boldsymbol{C}_{(j)}^-$ and $m_{\boldsymbol{Y}}$ are defined as in Step 4 of the GMC-RTOPSIS algorithm. Note that the separation measure is the squared root of the CC-integral, which is an aggregation function as shown in (Lucca et al., 2017a).

4 EXPERIMENTAL FRAMEWORK

In this section, we present the application of the CC-separations in the GMC-RTOPSIS. To do so, we start describing the methodology adopted in the study; after that, the example in which we apply our approach is described, and lastly, the obtained results are presented and discussed.

4.1 Methodology

In this study, we will apply the proposed CC-separation measure to the Case Study 2 introduced in (Lourenzutti et al., 2017) and used in (Wieczynski et al., 2020) to ease the comparison between the different CC-integrals.

To perform the simulation, we used 10,000 samples from the DM. We also applied a particle swarm optimization to learn the fuzzy measure using 30 particles and 100 interactions. The PSO is used since the original method had good outcomes with the method.

In this study we used the copula functions from Table 1. We highlight that we used 2 different values for the α parameter, which are $\alpha = 0.1$, selected from the literature (Lucca et al., 2017a), and $\alpha = 0.6$, which was the best fit for the problem presented in this paper based on the difference between the first and second ranked alternatives, when compared to other α values.

For the risk function, given in Eq. (5), it was used the squared loss:

$$L(cc,\, CC_i) = (cc - CC_i)^2.$$

This result in the mean function being the point estimator for the process.

Also, we use the aggregation function bellow in the Step 8 of the algorithm:

$$WAM_i = w(S_1)\cdot cc_i(S_1) + w(S_2)\cdot cc_i(S_2).$$

For the analysis of the results from the different copula functions, we use the difference between the alternative ranked first to the second ranked with:

$$\Delta_{R1,R2} = \max(\hat{c}_1) - \max(\hat{c}_2)$$

where $\hat{c}_1 = \{cc_i \mid i \in \{1,\dots,m\}\}$ and $\hat{c}_2 = \hat{c}_1 - \{\max(\hat{c}_1)\}$.

Lastly, since we are altering only the Choquet function, the algorithm maintains its original complexity.

4.2 The Considered Problem

A company needs a new supplier for a provision and is evaluating four different suppliers, namely A_1, A_2, A_3 and A_4. The company called three of its managers to analyze the suppliers and give their ratings based on their criteria.

The first manager is a budget manager. He considered the price per batch (in thousands) as $C_1^{(1)}$, warranty (in days) as $C_2^{(1)}$ and payment conditions (in days) as $C_3^{(1)}$. Also, it was considered that the demand for the product is higher in December. He modeled it by using a binary variable τ, that is $\tau = 0$ when the month is between January and November, and $\tau = 1$ when it is December. Finally, he assigned a weight for each of his criterion with a weighting vector: $\boldsymbol{w}^{(1)} = (0.5,\, 0.25,\, 0.25)$.

The second manager, a product manager, considered the price as $C_1^{(2)}$, delivery time (in hours) as $C_2^{(2)}$, production capacity $C_3^{(2)}$, product quality $C_4^{(2)}$ and the time to respond to a support request (in hours) as $C_5^{(2)}$. Additionally, to account for the reliability in the production process and what a failure in the process could cause to the supplier's production capacity, he let P_i be a random variable such that $P_i = 0$ occurs when there are no failures in the production process of the supplier A_i, and $P_i = 1$ when there are failures. Also, in December, the production is accelerated, so the chance of failure is higher, so he modeled a stochastic process with the help of the function:

$$f_i(x,y) = x\big(1 + y(P_i + \tau)^2\big).$$

Lastly, the production capacity was modeled by using ITFNs:

$$s_{13}^2 = \big((0.8^{1+P_1}, 0.9^{1+P_1}, 1.0^{1+P_1}, 1.0^{1+P_1}),\, 1.0,\, 0.0\big)$$
$$s_{23}^2 = \big((0.8^{1+4P_2}, 0.9^{1+4P_2}, 1.0^{1+4P_2}, 1.0^{1+4P_2}),\, 0.7,\, 0.1\big)$$
$$s_{33}^2 = \big((0.6^{1+2P_3}, 0.7^{1+2P_3}, 0.8^{1+2P_3}, 1.0^{1+2P_3}),\, 0.8,\, 0.0\big)$$
$$s_{43}^2 = \big((0.5^{1+3P_4}, 0.6^{1+3P_4}, 0.8^{1+3P_4}, 0.9^{1+3P_4}),\, 0.8,\, 0.1\big).$$

This manager selected the same weight for all criteria, i.e $\boldsymbol{w}^{(2)} = (0.2,\, 0.2,\, 0.2,\, 0.2,\, 0.2)$.

The commercial manager was the third. He considered the product lifespan (in years) as $C_1^{(3)}$, social and environmental responsibility as $C_2^{(3)}$, the quantity of quality certifications as $C_3^{(3)}$and the price as $C_4^{(3)}$. The weighting vector provided by this manager is $\boldsymbol{w}^{(3)} = (0.25,\, 0.12,\, 0.23,\, 0.4)$.

The P_i distribution was determined by historical data of each supplier and it is given as follows:

- For $\tau = 0$:

$$p(P_1 = 0|S_1) = 0.98,$$
$$p(P_2 = 0|S_1) = 0.96,$$
$$p(P_3 = 0|S_1) = 0.97,$$
$$p(P_4 = 0|S_1) = 0.95.$$

Table 1: Examples of Copulas.

(I) T-norms	
Definition	Name/Description
$T_M(x,y) = \min\{x,y\}$	Minimum
$T_P(x,y) = xy$	Algebraic Product
$T_L(x,y) = \max\{0, x+y-1\}$	Łukasiewicz
$T_{NM}(x,y) = \begin{cases} \min\{x,y\} & \text{if } x+y>1 \\ 0 & \text{otherwise} \end{cases}$	Nilpotent Minimum
$T_{HP}(x,y) = \begin{cases} 0 & \text{if } x=y=0 \\ \frac{xy}{x+y-xy} & \text{otherwise} \end{cases}$	Hamacher Product
(II) Non-associative overlap functions	
Definition	Reference/Description
$O_B(x,y) = \min\{x\sqrt{y}, y\sqrt{x}\}$	Cuadras-Augé family of copulas (Nelsen, 2007)
$O_{mM}(x,y) = \min\{x,y\}\max\{x^2,y^2\}$	(Dimuro and Bedregal, 2014; Pereira Dimuro et al., 2016)
$O_\alpha(x,y) = xy(1+\alpha(1-x)(1-y))$, where $\alpha \in [-1,0[\, \cup\,]0,1]$	(Alsina et al., 2006; Lucca et al., 2015)
(III) Non-associative copulas, which are neither t-norms nor overlap functions	
Definition	Reference/Description
$C_F(x,y) = xy + x^2y(1-x)(1-y)$	(Klement et al., 2011)
$C_L(x,y) = \max\{\min\{x, \frac{y}{2}\}, x+y-1\}$	(Alsina et al., 2006)
$C_{Div}(x,y) = \frac{xy+\min\{x,y\}}{2}$	(Alsina et al., 2006)

- For $\tau = 1$:

$$p(P_1 = 0|S_2) = 0.96,$$
$$p(P_2 = 0|S_2) = 0.92,$$
$$p(P_3 = 0|S_2) = 0.96,$$
$$p(P_4 = 0|S_2) = 0.90.$$

Considering all the *DM*s, we have the following underlying factors: a random component $\boldsymbol{Y}_{rand} = (P_1, P_2, P_3, P_4)$ and a deterministic component $Y_{det} = \tau$ that has two states: S_1 when $\tau = 0$ and S_2 when $\tau = 1$. The underlying factors can be represented by $\boldsymbol{Y} = (\boldsymbol{Y}_{rand}, \boldsymbol{Y}_{det})$. The managers agreed that the state S_2 was more important, since the production is higher, so they gave it a higher weight for it in the aggregation step (Step 8 of the method) by setting $w(S_1) = 0.4$ and $w(S_2) = 0.6$.

The *DM*s of all managers are presented in Table 2, where the linguistic variables (W, P, I, G and E) are defined as in Table 3.

The company, considering the opinion of manager 2 more important, assigned a weighting vector for the managers represented by $\boldsymbol{w} = (0.3,\ 0.4,\ 0.3)$. Furthermore, they wanted to include some interaction between the criteria, so a variation of 30% was allowed for each fuzzy measure in relation to the coefficient in the additive fuzzy measure. This measure is calculated computationally by means of the PSO algorithm (Wang et al., 2011; Lourenzutti et al., 2017).

4.3 Obtained Results

The aggregated ranked results are presented in Table 4 (for all the results see APPENDIX Table 5). The table shows for each copula function *Co*, the rank of alternatives from columns 2 to 5, with its aggregated values inside parenthesis. The last column shows the difference between alternatives ranked first and ranked second.

We can see that for the t-norms the values are pro-

Table 2: Decision matrices for the managers.
(a) Budget manager

Alternatives	$C_1^{(1)}$	$C_2^{(1)}$	$C_3^{(1)}$ $\tau = 0$	$C_3^{(1)}$ $\tau = 1$
A_1	$260.00(1+0.15\tau)$	90	G	G
A_2	$250.00(1+0.25\tau)$	90	P	W
A_3	$350.00(1+0.20\tau)$	180	G	I
A_4	$550.00(1+0.10\tau)$	365	I	W

(b) Production manager

Alternatives	$C_1^{(2)}$	$C_2^{(2)}$	$C_3^{(2)}$	$C_4^{(2)}$	$C_5^{(2)}$
A_1	260.00	$\mathrm{U}(f_1(48,\ 0.10),\ f_1(96,\ 0.10))$	s_{13}^2	I	[24, 48]
A_2	250.00	$\mathrm{U}(f_2(72,\ 0.20),\ f_2(120,\ 0.20))$	s_{23}^2	P	[24, 48]
A_3	350.00	$\mathrm{U}(f_3(36,\ 0.15),\ f_3(72,\ 0.15))$	s_{33}^2	G	[12, 36]
A_4	550.00	$\mathrm{U}(f_4(48,\ 0.25),\ f_4(96,\ 0.25))$	s_{34}^2	E	[0, 24]

(c) Commercial manager

Alternatives	$C_1^{(3)}$	$C_2^{(3)}$	$C_3^{(3)}$	$C_4^{(3)}$
A_1	Exp(3.5)	W	1	260.00
A_2	Exp(3.0)	W	0	250.00
A_3	Exp(4.5)	P	3	350.00
A_4	Exp(5.0)	I	5	550.00

Table 3: Linguistic variables and their respective trapezoidal fuzzy numbers.

Linguistic variables	Trapezoidal fuzzy numbers
Worst (W)	(0, 0, 0.2, 0.3)
Poor (P)	(0.2, 0.3, 0.4, 0.5)
Intermediate (I)	(0.4, 0.5, 0.6, 0.7)
Good (G)	(0.6, 0.7, 0.8, 1)
Excellent (E)	(0.8, 0.9, 1, 1)

portional to the ones presented in the study that used C_T-integral instead of the Choquet integral (Wieczynski et al., 2020). As in that paper, here the T_L t-norm has the greatest difference, with $\Delta_{R1,\ R2} = 0.0700$. Although, only the T_{MN} t-norm performs well compared with other copulas such as O_α and C_F.

The copula O_α with $\alpha = 0.6$ achieved the second greatest difference among the tested ones, with $\Delta_{R1,\ R2} = 0.0502$. The next of this family tested was the one with $\alpha = 0.1$, as it is the main value used in the literature, it resulted in a quite lower difference value, with only $\Delta_{R1,\ R2} = 0.0294$. Among the copulas from the α family, we have C_F, with $\Delta_{R1,\ R2} = 0.0466$ and T_{MN} with $\Delta_{R1,R2} = 0.0454$.

Notice that when using C_{div}, C_L and T_M copulas the alternatives A_3 and A_4 change position. This is from the influence of the state 2 result, where these functions may have weighted higher criteria for alternative A_4. Furthermore, the relative small difference $\Delta_{R1,R2}$ make the top of the rank prone to invert positions.

Our last analysis considered the CC-separations that presented the lowest ranks. Precisely, it is observable in the obtained results that the copulas T_{HP}, T_P, O_B and T_M obtained a similar performance and the lowest separation.

5 CONCLUSIONS

The GMC-RTOPSIS is a decision method that chooses the alternative that is closer to an ideal solution. It is capable of dealing with multiple data types as inputs and, also, through the Choquet integral, considers the interaction among different criteria.

In this paper, we presented the CC-separation measure. A new measure to be used in the GMC-RTOPSIS method that utilizes the CC-integrals instead of the Choquet integral. The CC-integrals is a generalization of the Choquet integral that presented

Table 4: Rank of the alternatives with each of the C functions, ordered by difference between the value of the alternative ranked first and second.

Co function	Ranked 1st	Ranked 2nd	Ranked 3rd	Ranked 4th	$\Delta_{R1,R2}$
$T_Ł$	$A_3(0.6462)$	$A_4(0.5762)$	$A_1(0.4616)$	$A_2(0.3782)$	0.0700
$O_{\alpha=0.6}$	$A_3(0.5897)$	$A_4(0.5395)$	$A_1(0.4716)$	$A_2(0.4282)$	0.0502
C_F	$A_3(0.5991)$	$A_4(0.5525)$	$A_1(0.4453)$	$A_2(0.4194)$	0.0466
T_{NM}	$A_3(0.5919)$	$A_4(0.5493)$	$A_1(0.4713)$	$A_2(0.3910)$	0.0425
$O_{\alpha=0.1}$	$A_3(0.5953)$	$A_4(0.5659)$	$A_1(0.4453)$	$A_2(0.3962)$	0.0294
O_{mM}	$A_3(0.5995)$	$A_4(0.5715)$	$A_1(0.4454)$	$A_2(0.3927)$	0.0280
C_{Div}	$A_4(0.5234)$	$A_3(0.5016)$	$A_1(0.4868)$	$A_2(0.4250)$	0.0218
C_L	$A_4(0.5273)$	$A_3(0.5097)$	$A_1(0.4914)$	$A_2(0.4361)$	0.0176
T_{HP}	$A_3(0.5351)$	$A_4(0.5221)$	$A_1(0.5049)$	$A_2(0.4308)$	0.0131
T_P	$A_3(0.5821)$	$A_4(0.5701)$	$A_1(0.4346)$	$A_2(0.3977)$	0.0120
O_B	$A_3(0.5511)$	$A_4(0.5395)$	$A_1(0.4713)$	$A_2(0.4133)$	0.0116
T_M	$A_4(0.5229)$	$A_3(0.5118)$	$A_1(0.4737)$	$A_2(0.4386)$	0.0110

good results when applied in classification problems.

By using an example from the literature, we tested the method with 11 different copula functions, with one of them using two distinct parameters. The results indicate that the Łukasiewicz t-norm is the best copula function to use in this example problem since it gives the greatest separation between the alternatives ranked first and second. Additionally, the Overlap alpha family, with $\alpha = 0.6$, the C_F and the T_{NM} also presented good separations.

By being able to verify the separation between the ranks, we can choose more confidently the alternative that better suits the problem. Therefore, by using multiple functions in the CC-separation measure, we can see how the problem behaves in different situations.

Finally, future work will consider learning the α parameter of the overlap alpha family, using distinct optimization methods.

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APPENDIX

Table 5: Mean and standard deviation of the alternatives for State 1 and State 2. The highest mean for each function and state is in **boldface** and the highest for the criterion has an asterisk*.

Alternatives	State 1 (S_1, $\tau = 0$)								State 2 (S_2, $\tau = 1$)							
	A_1		A_2		A_3		A_4		A_1		A_2		A_3		A_4	
Function	mean	std.dev	mean	std.dev	mean	std.dev	mean	std.dev	mean	std.dev	mean	std.dev	mean	std.dev	mean	std.dev
T_L	0.4702	0.0482	0.4360	0.0116	**0.6438***	0.0367	0.5588	0.0119	0.4558	0.0506	0.3397	0.0334	**0.6478***	0.0477	0.5878*	0.0250
$O_{\alpha=0.6}$	0.4852	0.0134	0.4791*	0.0232	**0.5742**	0.0731	0.5078	0.0108	0.4625	0.0266	0.3943	0.0165	**0.6001**	0.0593	0.5607	0.0214
C_F	0.4774	0.0181	0.4604	0.0173	**0.6074**	0.0858	0.5270	0.0135	0.4239	0.0194	0.3921	0.0208	**0.5935**	0.0710	0.5695	0.0174
T_{NM}	0.4643	0.0269	0.4276	0.0178	**0.5939**	0.0469	0.5654	0.0186	0.4759	0.0399	0.3666	0.0362	**0.5905**	0.0379	0.5386	0.0368
$O_{\alpha=0.1}$	0.4568	0.0127	0.4282	0.0117	**0.6013**	0.0791	0.5656	0.0098	0.4377	0.0275	0.3749	0.0223	**0.5913**	0.0611	0.5661	0.0232
O_{nM}	0.4519	0.0134	0.4218	0.0171	**0.6119**	0.0772	0.5668	0.0156	0.4411	0.0308	0.3733	0.0150	**0.5912**	0.0789	0.5746	0.0271
C_{Div}	0.5100	0.0101	0.4654	0.0187	0.5139	0.0202	**0.5221**	0.0191	0.4713	0.0293	0.3980	0.0088	0.4934	0.0242	**0.5242**	0.0307
C_L	0.5251*	0.0082	0.4670	0.0144	**0.5335**	0.0058	0.5242	0.0126	0.4690	0.0427	0.4155	0.0165	0.4938	0.0186	**0.5293**	0.0314
T_{HP}	0.4976	0.0083	0.4648	0.0141	**0.5328**	0.0169	0.5253	0.0125	0.5097*	0.0198	0.4081	0.0114	**0.5367**	0.0207	0.5199	0.0157
T_P	0.4567	0.0120	0.4270	0.0093	**0.5976**	0.0655	0.5674*	0.0098	0.4198	0.0396	0.3782	0.0190	0.5718	0.0605	**0.5719**	0.0360
O_B	0.4843	0.0148	0.4443	0.0138	**0.5584**	0.0422	0.5487	0.0101	0.4626	0.0360	0.3926	0.0156	**0.5462**	0.0388	0.5334	0.0306
T_M	0.4701	0.0478	0.4615	0.0217	**0.5326**	0.0018	0.5279	0.0231	0.4761	0.0270	0.4234*	0.0232	0.4980	0.0087	**0.5195**	0.0269