S*ReLU: Learning Piecewise Linear Activation Functions via Particle Swarm Optimization

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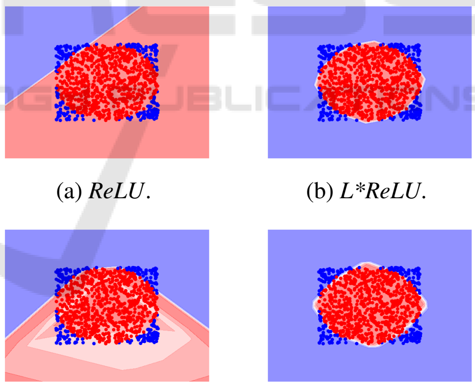
Keywords: Visual Categorization, Activation Functions, Particle Swarm Optimization.

Abstract: Recently, it was shown that using a properly parametrized *Leaky ReLU* (*LReLU*) as activation function yields significantly better results for a variety of image classification tasks. However, such methods are not feasible in practice. Either the only parameter (i.e., the slope of the negative part) needs to be set manually (*L*ReLU*), or the approach is vulnerable due to the gradient-based optimization and, thus, highly dependent on a proper initialization (*PReLU*). In this paper, we exploit the benefits of piecewise linear functions, avoiding these problems. To this end, we propose a fully automatic approach to estimate the slope parameter for *LReLU* from the data. We realize this via Stochastic Optimization, namely Particle Swarm Optimization (PSO): *S*ReLU*. In this way, we can show that, compared to widely-used activation functions (including *PReLU*), we can obtain better results on seven different benchmark datasets, however, also drastically reducing the computational effort.

1 INTRODUCTION

In the recent years, there has been a large scientific interest in manually designing (Klambauer et al., 2017; Clevert et al., 2016; Elfwing et al., 2018; Glorot et al., 2011; Gulcehre et al., 2016)) and learning (Ramachandran et al., 2018; Basirat and Roth, 2019; Hayou et al., 2018; Li et al., 2018) new activation functions (AFs). Even though these works could show that using more sophisticated non-linearities is beneficial in terms of convergency and training stability, for many tasks—including image classification—those are not reflected by the final performance. The reported improvements, if any, are often only very small. Thus, for many practical applications, *ReLU* is still used as non-linearity. This, however, is only true if the data is well separable. In contrast, if we have to deal with hard-to-distinguish classes, and even subtle differences are of relevance, using more sophisticated activation functions gives significantly better results (Basirat and Roth, 2020). In particular, using a properly initialized *Leaky ReLU* (*L*ReLU*) enforces the class separability and, finally, gives significantly better results.



(a) *ReLU*. (b) *L*ReLU*. (c) *PReLU*. (d) *S*ReLU* (proposed).

Figure 1: Nonlinear classification problem solved using different activation functions: (a) *ReLU*, (b) *L*ReLU*, (c) *PReLU*, and (d) *S*ReLU* (proposed).

To illustrate this, in Fig. 1, we show the decision boundaries for a simple nonlinear two-class classification problem, defined as separating the red points within a circle from the blue points outside the circle. Therefore, we apply a shallow fully connected network with three layers (2,4,2), using different activation functions. From Fig. 1(a) we can see that using *ReLU* even this simple problem cannot be solved. In contrast, Fig. 1(b) shows that *L*ReLU* using a prop-

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Basirat, M. and Roth, P.
S*ReLU: Learning Piecewise Linear Activation Functions via Particle Swarm Optimization.
DOI: 10.5220/0010338506450652
In *Proceedings of the 16th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications (VISIGRAPP 2021) - Volume 5: VISAPP*, pages 645-652
ISBN: 978-989-758-488-6

erly set negative slope allows for perfectly separating the two classes. However, in this case, the optimal parameter needs to be searched by a time-consuming grid-search, which is not feasible in practice.

To reduce the effort and the computational costs, an automatic approach preserving the positive properties of linear piecewise activation functions would be desirable. For instance, *Parametric ReLU* (*PReLU*) (He et al., 2015) learns the slope parameter from the training data. However, the approach is based on gradient-based optimization. Thus, the optimization likely gets stuck into a local optimum, and the success of the approach strongly depends on the initialization. This is illustrated in Fig. 1(c), showing that, in the expected case, the classification problem cannot be solved properly.

To summarize, it is neither desirable to estimate the necessary parameter manually (*L*ReLU*), nor to rely on a gradient-based approach, which highly depends on a proper initialization (*PReLU*). To avoid both shortcomings, we propose to automatically learn the slope parameter from the data, adopting ideas from Stochastic Optimization (Spall, 2003). In contrast to gradient-based approaches, we avoid getting stuck into local optima, and can ensure a globally optimal solution. Thus, as can be seen in Fig. 1(d), we can estimate the decision boundary very well, even though no prior knowledge was used. In favor of other stochastic optimization methods, we finally selected Particle Swarm Optimization (PSO) (Eberhart and Kennedy, 1995) due to its simplicity and efficiency.

The experimental results on seven different datasets show that, in this way, the optimal slope can be estimated, even though no prior knowledge was used, and that even the accuracy level of *L*ReLU* can be matched. In addition, we compare our results with those obtained by well-known and widely-used activation functions as well as to *PReLU*, which also estimates the slopes automatically from the data.

Thus, the main contributions of this paper can be summarized as follows:

- We introduce a Particle Swarm Optimization (PSO) based approach to automatically estimate the optimal slope of the negative part of *Leaky ReLU* (*LReLU*). Thus, also building on the ideas of *L*ReLU*, we would like to refer to it as *S*ReLU*.
- We demonstrate that our fully automatic approach finally matches the accuracy level of *L*ReLU*, which uses the manually-set optimal parameter and, thus, yields the best possible solution (estimated via a thorough parameter search).

2 RELATED WORK

In the following, we, first, give a summary of fixed (Sec. 2.1) and learned (Sec. 2.2) activation functions. Then, we revise *Leaky ReLU* in more detail (Sec. 2.3). Finally, we give a short introduction to metaheuristic stochastic optimization (Sec. 2.4).

2.1 Fixed Activation Functions

Inspired by simple thresholding functions, in the first place, squashing functions such as *Sigmoid* and *Tanh* (Hornik, 1991) were of interest. In particular, as they are continuous, bounded, and monotonically increasing, they fulfill the required properties of the universal approximation theorem (Hornik, 1991), making them a valid choice when learning continuous non-linear functions. However, such functions often suffer from the vanishing gradient problem (Hochreiter, 1998), which is, in particular, a problem when networks are getting deeper.

To overcome this problem, *Rectified Linear Units* (*ReLU*) (Nair and Hinton, 2010) have been introduced. As the derivative of any positive input is one, the gradient cannot vanish, whereas all negative values are mapped to zero. On the one hand, this makes *ReLU* easily and efficiently applicable; on the other hand, we are facing two main problems: (a) There is no information flow for negative values, which is known as dying *ReLU*. (b) The statistical mean of the activation values is larger than zero, leading to a bias shift in successive layers.

Both shortcomings can be alleviated by using *Exponential Linear Unit* (*ELU*) (Clevert et al., 2016). By pushing the mean activation value towards zero, *ELU* is more robust to noise and eliminates the bias shift in the succeeding layers. This idea was later extended by introducing a self-normalizing network: *Scaled Exponential Linear Unit* (*SELU*) (Klambauer et al., 2017). In this way, convergence towards a normal distribution with zero mean and unit variance can be ensured.

2.2 Learned Activation Functions

More efficient activation functions can be obtained by learning the parameters from the data. In this way, *Parametric ELU* (Trottier et al., 2017) overcomes the vanishing gradient problem and allows for controlling the bias shift. More complex functions (also non-convex ones) can be learned using *Multiple Parametric Exponential Linear Units* (Li et al., 2018). In this way, better convergence properties can be ensured, finally, leading to a better classification performance.

Similar results can also be achieved by adopting ideas from reinforcement learning (Ramachandran et al., 2018) and genetic programming (Basirat and Roth, 2019) by exploring complex search spaces, allowing to construct new, more complex AFs. In this way, *Swish* (Ramachandran et al., 2018), a combination of a squashing and a linear function, was found, which has been demonstrated to work very well for many problems. As an extension, *Parametric Swish* (*PSwish*) additionally allows to train a scaling parameter. Similar functions, also yielding slightly better results in the same application domains, were found by (Basirat and Roth, 2019). Recently, a theoretic justification for these results has been given in (Hayou et al., 2018), demonstrating that *Swish*-like functions propagate information better than *ReLU*.

Even though competitive results have been shown for many problems, the beneficial theoretical properties did not allow to (significantly) outperform neural networks using *ReLU*. Thus, for most applications, *ReLU* is used as nonlinearity, and other activation functions are not considered at all.

2.3 Leaky ReLU

Nevertheless, it was shown that using *Leaky ReLU* (*LReLU*) (Maas et al., 2013), just introducing a small slope ($\alpha = 0.01$) for the negative part of *ReLU*, gives better results for many tasks. Even though we can overcome the dying-*ReLU*-problem, we are still suffering from the bias shift. In this way, *Randomized Leaky Rectified Linear Unit* (*RReLU*) (Xu et al., 2015) sets the slope for the negative part randomly.

Recently, it has been shown that not only using a non-zero negative part is essential, but that also the slope of the negative part is of importance. In particular, the goal of *L*ReLU* (Basirat and Roth, 2020) was to demonstrate two points: First, piecewise linear activation functions can deal with more complex data very well, i.e., if the data is not well separable. This is, in particular, the case when classes are very similar and small subtle differences need to be modeled. Therefore, also the absence of features needs to be covered. To this end, an activation function needs to be uniform-continuous and strictly increasing, which can be ensured by a probably initialized *LReLU*. Second, the optimal slope is highly data-dependent and related to the Lipschitz constant (c.f., (Sokolić et al., 2017; Anil et al., 2019; Tsuzuku et al., 2018)). Thus, for different tasks also different parameters for the negative slope are needed. Even though these are interesting insights, the slope for *L*ReLU* needs to be set manually, which is infeasible in practice.

Parametric ReLU (*PReLU*) (He et al., 2015) overcomes this problem by learning the slopes for the negative part based on the training data. Moreover, there are two main differences to *L*ReLU*: (a) a separate activation function is learned for each layer, and (b) the activation functions are not kept fixed but are updated during neural network training. Even though this sounds reasonable, the main drawback of *PReLU* is—building on a gradient-based optimization—that the solver often gets stuck into a local optimum, making the approach very sensitive to a proper initialization. To allow to generate complex functions, *SReLU* (Jin et al., 2016) is defined via three piecewise linear functions finally forming a crude S shape.

Overall, neither manually setting the slope parameter nor a high sensitivity to the initialization are desirable in practice. Thus, in the following, we introduce an approach for learning piecewise linear functions from the data, overcoming the problems discussed above by building on ideas from metaheuristic stochastic optimization.

2.4 Metaheuristic Stochastic Optimizations

During the last decades, various metaheuristic stochastic optimization approaches (Osman and Laporte, 1996) such as Evolutionary Algorithms (EA) (e.g., (Koza et al., 1999; Schwefel, 1987; Goldberg, 1989; Holland, 1992)) or Particle Swarm Optimization (PSO) (Eberhart and Kennedy, 1995) have been introduced. The key idea of such approaches is that a population of candidate solutions competes and/or collaborates to find the optimal solution through evaluating the fitness of each candidate solution. Then, each candidate solution is tweaked over several iterations in the direction of the fittest solution. Indeed, Evolutionary Algorithms (EA), including Genetic Algorithms (GA) (Goldberg, 1989; Holland, 1992), Genetic Programming (GP) (Koza et al., 1999), and Evolution Strategies (ES) (Schwefel, 1987), are inspired from biology. In contrast, PSO is a swarm intelligence algorithm inspired by the social behaviors of a swarm. Therefore, candidate solutions are referred to as a swarm of particles, not as a population of individuals. In contrast to EAs, PSO does not re-sample particles to generate new ones. PSO has been widely used for hyper-parameter tuning and designing the topology of deep neural networks (Sinha et al., 2018; Lorenzo et al., 2017; Wang et al., 2018). Due to its simplicity and the ability also to deal with rather complex problems, for our problem, we finally decided to build on PSO in favor of other metaheuristic stochastic optimization approaches.

3 LEARNING PIECEWISE LINEAR ACTIVATION FUNCTIONS

In the following, in Sec. 3.1, we first review the main concepts and motivation of piecewise linear activation functions, in particular of *Rectified Linear Unit* (*ReLU*) and *Leaky Rectified Linear Unit* (*LReLU*). Then, in Sec. 3.2, we introduce our new, more robust automatic approach to estimate the optimal slope for the negative part of *LReLU*.

3.1 Piecewise Linear Activation Functions

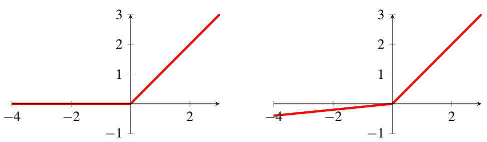
Piecewise functions, in general, are defined via different functions for different parts of the domain. In our case, we are interested in piecewise linear function $f(x)$ defined for the negative, $f(x \leq 0)$, and the positive, $f(x > 0)$, domain of $\mathbb{R}$. The most prominent function of this family is *Rectified Linear Unit* (*ReLU*) defined as

$$f(x) = \max(x, 0). \tag{1}$$

In practice, for most applications, including image classification, *ReLU* is used due to its simplicity and efficiency. Moreover, for a wide range of applications, good results can be obtained. However, due to the constantly zero negative part, we are suffering from the *dying ReLU* problem and a bias shift in subsequent layers. To this end, it has been shown that introducing a small negative slope (i.e., $\alpha = 0.01$) gives better results for many applications:

$$f(x) = \max(x, 0) + \min(\alpha x, 0), \tag{2}$$

where $\alpha \geq 0$ defines the slope of the linear function for the negative part. The function $f(x)$ defined in Eq. (2) is referred to as *Leaky ReLU* (*LReLU*). As can be seen from Fig. 2, the main difference compared to *ReLU* is that not all negative values are mapped to zero but are mapped using a linear function with a small slope α.



(a) $f(x) = ReLU$. (b) $f(x) = LReLU$.

Figure 2: Piecewise linear functions: (a) *ReLU* and (b) *Leaky ReLU (LReLU)*, consisting of two linear parts: $f(x \leq 0)$ and $f(x > 0)$.

More recently, in (Basirat and Roth, 2020) it has been demonstrated that not only using a non-zero negative part is of importance, but that also the selected slope is of relevance. Moreover, it has been revealed that the optimal slope is data-dependent. In other words, for different tasks a different parametrization is necessary. In general, as long as the data is well-separable in the latent space, the choice of the activation function is not critical. This is also illustrated in Table 3, where it can be seen that for different choices of activation functions very similar results are obtained for the *CIFAR*-datasets.

If this is not ensured, it has been shown (Basirat and Roth, 2020; Clevert et al., 2016) that also modeling the absence of features is of importance, which can be achieved via using strictly monotonically increasing non-linear functions that are uniform-continuous. In practice, this means that the negative values should neither be mapped to zero nor saturating, and that similar inputs should produce similar outputs. Both properties are fulfilled by *LReLU*, where, additionally, also the desirable properties of *ReLU* are inherited.

Even though this is an important and interesting insight, it is not convenient to set the necessary parameter manually. To overcome this problem, *PReLU* (He et al., 2015) can be used, which automatically estimates the optimal slopes (i.e., separately for each layer) from the data. However, as also can be seen from Tables 3 and 4, the success of the approach is highly dependent on an initialization close to the optimal solution. Building on a gradient-decent-based optimization scheme, the approach gets often stuck into a local minimum.

Since neither manually setting the parameter α nor depending on a critical initialization are meaningful for practical applications, in Sec. 3.2, we introduce an automatic approach for optimally selecting the negative slope of *LReLU* not depending on a proper initialization, building on ideas of Particle Swarm Optimization (Eberhart and Kennedy, 1995).

3.2 Learning Negative Slopes via Particle Swarm Optimization

To overcome the problems of gradient-based optimizers, we propose to build on ideas from metaheuristic stochastic algorithms. In particular, we apply *Particle Swarm Optimization* (PSO) (Eberhart and Kennedy, 1995) to compute the optimal negative slope for *LReLU*.

Given a swarm $S = \{x_1, \ldots, x_n\}$, $n \geq 1$, of candidate solutions x_i, the key idea of PSO is to move the individual candidate solutions in the search space towards an optimal solution, while sharing their experiences. In general, a candidate solution $x_i = \langle x_i^1, \cdots, x_i^d \rangle$, also referred to as "particle", is a d-tuple, representing a location in a d-dimensional search space. Being interested in computing the slope of the negative part of a piecewise linear function, we set $d = 1$, i.e., a particle[1] is a scalar value $x \in \mathbb{R}^+$.

In this way, also each particle x is assigned a velocity $v(x)$, steering the current update:

$$x \leftarrow x + \varepsilon v(x), \tag{3}$$

where ε defines the jump size of the particles. To compute the current estimate of $v(x)$, we need to define three optimal values, estimated via a so-called fitness function, for each particle x: (a) x^*, the best solution for particle x (personal best), (b) x^+, the best solution among all neighbors of x (local best), and (c) $x^!$, the best solution of all particles $x \in S$ (global best).

Given the current location of particle x, the update for $v(x)$ is computed by

$$v(x) \leftarrow wv(x) + a(x^* - x) + b(x^+ - x) + c(x^! - x), \tag{4}$$

where $w \in [0,1]$ is called inertia weight and controls to which extend a particle's velocity is steered by its history of locations; $a \in [0, \beta]$ is called the cognitive parameter and controls exploitation; $b \in [0, \gamma]$ and $c \in [0, \delta]$ are called the social parameters and control the local and the global exploration of the feasible search space, respectively. Please note, the parameters a, b, and c are randomly chosen from their respective ranges in each iteration. Starting from a random initialization for x and $v(x)$, in each iteration x and $v(x)$ are updated according to Eqs. (3) and (4) until a predetermined stopping criterion is met. Even though starting from a random initialization, finally, the optimal slope for the negative part of the piecewise linear activation function can be estimated.

Algorithm 1 summarizes the overall procedure. Starting from a swarm of random particles x assigned a random velocity $v(x)$ (lines 2–6), we randomly tag a particle $x^r \in S$ as the ideal solution ϕ (line 7). Then, the following steps are iterated until the optimal solution is obtained or a stopping criterion is met (lines 8–15): (a) evaluate the fitness of all particles x and update the optimal values; (b) update the velocity $v(x)$ based on the own experience (personal best), the experience discovered from its neighbors (local best), and the best experience of all particles in the swarm (global best); (c) update particle x.

In general, there are two main PSO strategies, namely *Global PSO* (*GPSO*) and *Local PSO* (*LPSO*). The difference between them lies in the neighborhood structure of the particles. In GPSO, the neighborhood of a particle, for which information is shared, is defined by all particles in the entire swarm. On the contrary, in LPSO, the neighborhood of a particle is defined by a fixed number of particles.

Algorithm 1: Particle Swarm Optimization.

```
Require:
    n ← desired swarm size
    w ← proportion of velocity to be retained
    β ← proportion of personal best to be retained
    γ ← proportion of the neighbors' best to be retained
    δ ← proportion of global best to be retained
    ε ← jump size of a particle
Ensure:
    φ ← the fittest particle so far
 1: procedure PARTICLESWARMOPTIMIZATION
 2:   S ← ∅
 3:   for n times do
 4:     x ← RANDOM-PARTICLE(d)
 5:     v(x) ← RANDOM-VELOCITY(d)
 6:     S ← S ∪ {x}
 7:   φ ← x^r
 8:   repeat
 9:     for x ∈ S do
10:       if FITNESS(x) < FITNESS(φ) then
11:         φ ← x
12:       UPDATE v(x): Eq. (4)
13:       UPDATE x: Eq. (3)
14:   until φ is optimal solution or stopping criterion is met
15:   return φ
```

In this way, for GPSO, the parameter γ is set to 0, enforcing $b = 0$; thus the term $(x_i^+ - x_i)$ is omitted. More specifically, given the location of particle x, the update of $v(x_i)$ is reduced to

$$v(x_i) \leftarrow wv(x_i) + a(x_i^* - x_i) + c(x_i^! - x_i). \tag{5}$$

In contrast, for LPSO, the parameter δ is set to 0, enforcing $c = 0$; thus the term $(x_i^* - x_i)$ is omitted. More specifically, given the location of particle x, the update of $v(x_i)$ is reduced to

$$v(x_i) \leftarrow wv(x_i) + a(x_i^* - x_i) + b(x_i^+ - x_i). \tag{6}$$

4 EXPERIMENTAL RESULTS

To demonstrate our approach, we run it for seven different image classification benchmarks, two coarse-grained visual categorization (CGVC) and five fine-grained visual categorization (FGVC) problems. To this end, in the first step, we run Local and Global PSO to estimate the optimal slopes for *LReLU* for each dataset, which are then used to train a neural network in the second step. The benchmarks and details about the experimental setups are given in Sec. 4.1 and Sec. 4.2, respectively.

[1]For sake of readability, in the following discussion, we also leave out the index i.

We split our evaluations into two different experiments. First, in Sec. 4.3, we compare the slopes estimated via our PSO-approach to the optimal slopes estimated for *L*ReLU* (Basirat and Roth, 2020). Second, in Sec. 4.4, we compare the finally obtained classification results to *L*ReLU*, defining the "upper bound", to well-known and widely used activation functions, and to *PReLU* (He et al., 2015). Finally, in Sec. 4.5, we discuss the computational effort of *S*ReLU* compared to *L*ReLU*.

4.1 Benchmark Datasets

To demonstrate the generality of our approach, we demonstrate it for benchmarks of different characteristics: (a) Coarse-grained Visual Categorization (CGVC): CIFAR-10 and CIFAR-100 (Krizhevsky, 2009); and (b) Fine-grained Visual Categorization (FGVC): Caltech-UCSD Birds-200-2011 (Wah et al., 2011), Car Stanford (Krause et al., 2013), Dog Stanford (Khosla et al., 2011), Aircrafts (Maji et al., 2013), and iFood[2].

In contrast to CGVC, where the classes are well separable, in FGCV the single classes are more similar. Demonstrating CGVC and FGVC results side by side should also show that the choice of the activation function has a different impact on the two problems. However, we can also see that PSO performs well on both tasks.

4.2 Experimental Setup

To allow for a fair comparison on these rather heterogeneous datasets, we used the same experimental setup for all experiments:

For neural networks training, we used an architecture consisting of eight convolutional plus two fully connected layers with 400 and 900 units, respectively. For training, we applied an Adam optimizer with batch normalization, using a batch size of 70 and a maximum pooling size of seven. To avoid random effects and to keep the results traceable, we abstained from using dropout regularization. Using a random initialization, we ran all experiments five times, where the mean results are reported, respectively.

For the PSO-step (Miranda, 2018), we initialized the particles in the expected range, i.e., $x \in [0,1]$, however, during optimization the search space was given by $[0,\infty)$. The parameters ε, w, β, γ, and δ were set to 1, 0.5, 0.5, 0.3, and 0.3, respectively. We used a swarm-size of 10 and run the optimization using the loss on an independent test set as the fitness function for 15 epochs. The number of iterations in PSO was set to 5, and for Local PSO 4 neighbors have been considered. Please note that the estimated slopes are kept fixed and are not changed during neural network training. That is, in particular, a significant difference to *PReLU*, where the slope is adapted during the neural network training.

4.3 PSO vs. Optimal Slope

In the first experiment, we compare the slopes estimated using Global PSO (GPSO) and Local PSO (LPSO) to the optimal slopes identified in (Basirat and Roth, 2020). The corresponding results for CGVC and FGVC are summarized in Tables 1 and 2, respectively.

It can be seen that for the simpler CGVC both approaches, GPSO and LPSO, yield similar results close to the optimal slope. In contrast, for the FGVC benchmarks, GPSO yields consistently better results, i.e., the estimated slopes are closer to the optimal ones. Thus, from the practical (data-agnostic) point of view, we would recommend using GPSO, which would give better results in the expected case. The ambiguity* for Aircraft can be explained by the fact that in (Basirat and Roth, 2020) for this dataset two optimality peaks (i.e., $\alpha = 0.2$ and $\alpha = 0.35$) have been reported. Overall, these results clearly show that our fully automatic approach, in particular the GPSO variant, allows us to recover the optimal slope parameter even without having any prior knowledge.

Table 1: Comparison of the best slope of *L*ReLU* and slopes found via GPSO and LPSO for the CGVC benchmarks.

Dataset	**Best slope (*L*ReLU*)**	**Slope (GPSO)**	**Slope (LPSO)**
CIFAR-10	0.10	0.12	0.11
CIFAR-100	0.10	0.11	0.11

Table 2: Comparison of the best slope of *L*ReLU* and slopes found via GPSO and LPSO for the FGVC benchmarks.

Dataset	**Best slope (*L*ReLU*)**	**Slope (GPSO)**	**Slope (LPSO)**
Birds	0.35	0.36	0.01
Cars	0.25	0.30	0.41
Dogs	0.05	0.06	0.01
iFood	0.35	0.35	0.43
Aircraft*	0.35	0.17	0.22

[2]https://sites.google.com/view/fgvc5/competitions/fgvcx/ifood (last accessed: October 31, 2020)

4.4 Comparison to the State-of-the-Art

Next, we compare the classification accuracy of *S*ReLU (GPSO and LPSO)* to (a) *L*ReLU* (defining the "upper bound"), (b) well-known and widely used AFs and known to yield good results in practice, namely *ReLU* (Nair and Hinton, 2010), *ELU* (Clevert et al., 2016), *SELU* (Klambauer et al., 2017), and *Swish* (Ramachandran et al., 2018), and (c) to *Parametric ReLU (PReLU)* (the most related approach.) The thus obtained mean accuracies for CGVC and FGVC are shown in Tables 3 and 4, respectively.

Table 3 shows that for CGVC all activation functions perform more or less on par. In addition, we see that both GPSO and LPSO match the optimal results of *L*ReLU*, where—as expected from the estimated slopes—LPSO gives slightly better results. In contrast, for FGVC, as can be seen in Table 4, LPSO provides—as was expected from the estimated slopes—worse results compared to GPSO. However, again *S*ReLU* matches the optimal results of *L*ReLU* very well. The other activation functions give worse results, as was also shown in (Basirat and Roth, 2020). In particular, this applies for *PReLU*, which also estimates the slope parameter automatically. However, the approach tends to get stuck into a local minimum, whereas *S*ReLU* ensures a globally optimal solution in any case.

4.5 Computational Effort

In Secs. 4.3 and 4.4, we demonstrated that, compared to other widely-used activation functions, *S*ReLU* gives better results, even matching those of *L*ReLU*, where the optimal parameter was estimated via a brute-force grid search. The main advantage of *S*ReLU* is that this parameter is estimated automatically, drastically reducing the computational effort. In fact, for *L*ReLU* the search space was defined by 16 slopes equidistantly sampled in the range $[0,1]$. To reduce the impact of random effects, each experiment, running 90 epochs, was repeated 5 times. In contrast, for *S*ReLU*, using a swarm size of 10 and 5 PSO iterations, we need to run only 15 epochs. In this way, the computational effort can be reduced by a factor of 10, still preserving the same classification accuracy.

5 CONCLUSION

For many classification problems, *Leaky ReLU (LReLU)* provides better results compared to other widely-used activation functions. However, the necessary parameter, i.e., the slope of the negative part, has to be estimated empirically from the data, or the approaches are very sensitive to proper initialization. In this paper, we propose an automatic approach, not suffering from these problems. In particular, we learn the negative slope of *LReLU* via Particle Swarm Optimization (PSO): *S*ReLU*. This metaheuristic stochastic optimization method ensures a globally optimal solution, where we applied two different approaches: *LPSO* and *GPSO*. The experimental results revealed that we match the (optimal) accuracy of *L*ReLU*, but at a significantly reduced computational effort. Since, in the expected case, *GPSO* gives better results than *LPSO*, we would recommend using *GPSO*.

ACKNOWLEDGEMENTS

This work is (in part) supported by the German Federal Ministry of Education and Research (BMBF) in the framework of the international future AI lab AI4EO (Grant number: 01DD20001).

Table 3: Mean accuracies for different activation functions for CGVC, where the proposed approaches are in color.

Dataset	**ELU**	**Swish**	**ReLU**	**SeLU**	**PReLU**	**L*ReLU**	**GPSO**	**LPSO**
CIFAR-10	90.81%	91.23%	90.83%	89.72%	87.83%	90.98%	91.02%	91.06%
CIFAR-100	63.64%	64.36%	65.32%	63.29%	60.28%	66.44%	65.90%	66.19%

Table 4: Mean accuracies for different activation functions for FGVC, where the proposed approaches are in color.

Dataset	**ELU**	**Swish**	**ReLU**	**SeLU**	**PReLU**	**L*ReLU**	**GPSO**	**LPSO**
Birds	39.12%	38.89%	38.18%	36.86%	37.69%	44.30%	43.81%	36.93%
Cars	41.86%	44.72%	33.08%	39.17 %	40.07%	47.20%	47.88%	45.83%
Dogs	35.67%	37.04%	35.17%	33.65%	35.14%	37.63%	37.58%	35.65%
iFood	38.12%	*41.14%*	37.67%	34.67%	39.27%	42.81%	42.06%	40.86%
Aircraft	38.97%	39.63%	38.49%	30.54%	37.03%	40.42%	39.35%	39.89%

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