# Robust Emergency Medical System Design as a Multi-objective Goal Programming Problem

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Abstract: The main goal of this paper is to introduce and compare three different mathematical modelling approaches to robust emergency medical service system design. The idea of system robustness follows from the necessity of making the system resistant to various detrimental events, which may unexpectedly occur in the associated transportation network and thus negatively affect service. Such viewpoint is important mainly in different kinds of public service systems including rescue services, in which time necessary for service delivery plays a very important role. While the standard method of robust system designing takes into account only the worst possible situation considering the set of detrimental scenarios, suggested modeling approaches compute a separate objective function value for each scenario and then a special constraint is added to the original mathematical model. This way, an epsilon-constraint principle to the problem solution is applied. In this paper, numerical experiments to study the performance characteristics of suggested solving methods accompany the theoretical explanation of all presented models.

## 1 INTRODUCTION

This research paper deals with a special class of discrete network location problems, which are solved under uncertainty following from randomly and often unexpectedly occurring failures in the transportation network (Correia and Saldanha da Gama, 2015, Pan et al., 2014, Scaparra and Church, 2015). The main focus is on the application of suggested optimization approaches to the emergency medical system (EMS) designing. The rescue system performance efficiency is directly influenced by locations of service centers, which send the emergency vehicles to satisfy the requests raised at system users' locations. Obviously, the number of service providing centers is limited due to economic and technological restrictions. It is not possible to locate a separate service center to each served geographical area or to each system users' community.

The most commonly used objective function in the mathematical models for EMS designing takes into account the service accessibility of an average user. This way, the emergency service system design problem can be described as the weighted *p*-median problem broadly studied by many researchers (Avella et al., 2007, Current et al., 2002, Ingolfsson et al., 2008, Jánošíková, 2007, Snyder and Daskin, 2005) mainly from the points of developing effective exact and approximate solving techniques. It is worth mentioning, for example, the radial formulation of the problem, successfulness of which is based on the fact that there is only finite set of radii, which need to be taken into account (Elloumi et al., 2004, García et al., 2011, Janáček, 2008). Such model reformulation makes the problem easier, smaller and thus better solvable.

Simultaneously, several approximate approaches have been developed to get a good solution of the problem in acceptably short time (Doerner et al. 2005, Gendreau and Potvin, 2010).

It is often assumed that service center has enough capacity to serve all assigned users and thus, each system user can be serviced from the nearest located service center. Otherwise, the concept of so-called generalized disutility can be applied to incorporate stochastic behavior of real system into the mathematical model (Kvet and Janáček, 2018). This model extension enables to consider more service centers, which can provide the associated service to the same user. In the research reported in this paper, only the nearest located service centers for each system user are considered.



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When the emergency service system is designed, the designer must take into account that the transfer time from a service center to the affected user might be negatively impacted by various random events caused by weather or traffic. Furthermore, possible failure of a part of critical infrastructure should be taken into account because of congestion, disruptions or blockages. In other words, the system resistance to such critical events should be included into the decision-making process.

Most of available approaches to increasing the system robustness (Correia and Saldanha da Gama, 2015, Kvet and Janáček, 2017b, Pan et al., 2014, Scaparra and Church, 2015) are based on making the system resistant to possible failure scenarios, which can appear in the associated transportation network as a consequence of random failures due to congestion, disruptions or blockages. Thus, a finite set of failure scenarios is considered and each individual scenario is characterized by particular time distances between the users' locations and possible center locations.

The most commonly used objective function in the above-mentioned weighted *p*-median problem consists in minimizing the time accessibility of the service for an average user, i.e. a min-sum objective function is minimized subject to associated solution feasibility constraints. On the other hand, the most frequently used objective function of the robust design focuses on minimizing the maximal objective function of the individual instances corresponding with particular scenarios. It means that the worst possible impact of individual scenarios is minimized. It follows that the original min-sum objective function used in the weighted *p*-median problem is replaced by the min-max criterion. The min-max model uses the link-up constraints to limit the individual scenario min-sum objectives by their upper bound corresponding to the objective function of the resulting min-max model. In addition, incorporating the scenarios into the model causes the model soize to increase proportionally to the cardinality of the scenario set. Both the model structure and the increase in model size represent a burden to the computational process of most available IP-solvers. Thus, complementary approximate approaches to the robustness constitute a big challenge to operational researchers and professionals in the field of Applied Informatics (Janáček and Kvet, 2016, Janáček and Kvet, 2017, Kvet and Janáček, 2017a, Kvet and Janáček, 2017b).

This paper focuses on the main disadvantage of the min-max approach to robust EMS design; only the worst impact of individual scenarios is minimized. The set of scenarios may contain a bad scenario with very low probability of occurring, yet this scenario may seriously affect the optimal robust system design. This paper discusses three approaches to the problem based on multi-objective optimization (Antunes and Henriques, 2016). These techniques consider each scenario to form a separate objective function and they apply epsilon-constraint principle to the problem solution.

The remainder of this paper is organized as follows: Section 2 is devoted to the description of the original min-max robust design of emergency systems, in which all scenarios are taken into account. Section 3 explains two coefficients for robustness evaluation. The core of this contribution is reported in Section 4, in which all suggested multi-objective approaches are introduced and explained. The fifth Section discusses numerical experiments and yields a brief comparative analysis of the resulting designs. The results and findings are summarized in Section 6.

## 2 STANDARD APPROACH TO ROBUST EMS DESIGN

The standard approach to emergency medical service system design usually leads to formulation of a min-sum problem (Current et al., 2002, Ingolfsson et al., 2008, Jánošíková, 2007), in which the average system accessibility for users (average response time) is minimized. The robust system design is formulated as a min-max model bringing some difficulties into the computational process (Kvet and Janáček, 2017b).

To formulate the mathematical model for robust EMS design, we introduce the following notations..

Let symbol $J$ denote the set of users' locations and let symbol $I$ denote the set of possible service center locations. Furthermore, let $b_j$ denote the number of users sharing the location $j$. To solve the problem, $p$ locations from $I$ must be chosen so that the maximal scenario objective function value is to be minimized. The objective function value of an individual scenario is defined as a sum of users' distances from the location of the service center providing them with service multiplied by $b_j$. To incorporate system robustness into the mathematical model, a set $U$ of possible failure scenarios is needed to be introduced. This set contains also one specific scenario called *basic scenario*, which represents standard conditions in the associated transportation network. For the purpose of conciseness, let $U_0$ denote the set of scenarios without the basic one, i.e. $U_0 = U - \{basic\ scenario\}$. The integer distance between locations $i$ and $j$ under a specific scenario $u \in U$ is denoted by $d_{iju}$.

Even if the radial model is originally suggested for integer distance or time values only, the used principle enables us to adjust the model also for real values without any big problems.

The decisions, which determine the structure of the rescue service system, are modeled by decision variables $y_i \in \{0,1\}$ modeling the decision on service center location at the place $i \in I$ by the value of 1 if a service center is located at $i$ and by the value of 0 otherwise. In the robust problem formulation, the variable $h$ denotes the upper bound of the objective functions over the set $U$ of scenarios. To formulate the radial model, the integer range [0, $v$] of all possible distances of the matrices $\{d_{iju}\}$ is partitioned into zones according to (García et al., 2011, Janáček, 2008). The value of $v$ is computed according to the expression (1).

$$v = \max\left\{d_{iju} : i \in I, j \in J, u \in U\right\} - 1 \qquad (1)$$

The radial formulation of the problem is based on the idea of making a system of zones. The zone $s$ corresponds to the interval ($s$, $s$+1]. To complete the radial model, auxiliary zero-one variables $x_{jus}$ for $j \in J$, $u \in U$ and $s$ = 0 … $v$ need to be introduced. The variable $x_{jus}$ takes the value of 1, if the distance of the user at $j \in J$ under the scenario $u \in U$ from the nearest located center is greater than $s$ and it takes the value of 0 otherwise. Based on these assumptions, the expression $x_{ju0} + x_{ju1} + x_{ju2} + \ldots + x_{juv}$ constitutes the distance $d_{ju*}$ from user location $j$ to the nearest located service center under scenario $u \in U$. Similarly to the set covering problem, let us introduce a zero-one constant $a_{iju}^s$ under scenario $u \in U$ for each $i \in I$, $j \in J$, $s \in [0..v]$. The constant $a_{iju}^s$ is equal to 1, if the disutility $d_{iju}$ between the user location $j$ and the possible center location $i$ is less than or equal to $s$, otherwise $a_{iju}^s$ is equal to 0. Then the model of the robust system design problem can be formulated as follows.

$$Minimize \qquad h \qquad (2)$$

$$Subject\ to: \quad x_{jsu} + \sum_{i \in I} a_{iju}^s y_i \geq 1$$
$$for\ j \in J, s = 0, 1, \ldots, v,\ u \in U \qquad (3)$$

$$\sum_{i \in I} y_i = p \qquad (4)$$

$$\sum_{j \in J} b_j \sum_{s=0}^{v} x_{jsu} \leq h \quad for\ u \in U \qquad (5)$$

$$y_i \in \{0, 1\} \quad for\ i \in I \qquad (6)$$

$$x_{jsu} \in \{0, 1\} \quad for\ j \in J, s = 0, 1, \ldots, v,\ u \in U \qquad (7)$$

$$h \geq 0 \qquad (8)$$

The objective function (2) gives an upper bound of all objective function values corresponding to the scenarios. The constraints (3) ensure that the variables $x_{jus}$ are allowed to take the value of 0, if there is at least one center located in radius $s$ from the user location $j$ and constraint (4) limits the number of located service centers by $p$. The link-up constraints (5) ensure that each perceived disutility (time or distance) is less than or equal to the upper bound $h$. The obligatory constraints (6), (7) and (8) are included to ensure the domain of the decision variables $y_i$, $x_{jus}$ and $h$.

# 3 SERVICE SYSTEM ROBUSTNESS EVALUATION

The main goal of robust service system design is to make the system resistant to randomly occurring failures on the associated transportation network. To evaluate the gauges of robustness, we introduce the following additional notations. As before, let $U$ denote the set of all considered failure scenarios, which contains also the *basic scenario*. Let **y** denote the vector of location variables $y_i$; $i \in I$. Let $\mathbf{y}^b$ correspond to the basic system design, i.e. the solution of a simple weighted $p$-median problem, in which only the *basic scenario* is taken into account. Let $f^b(\mathbf{y})$ denote the associated objective function value. Similarly, let $\mathbf{y}^r$ denote the solution of the model (2)-(8), which brings the robust system design. Finally, the objective function (2) will be denoted by $f^r(\mathbf{y})$. The *price of robustness* (*POR*) expresses the relative increment (additional cost) of the basic scenario objective function, when $\mathbf{y}^r$ is applied instead of the optimal solution $\mathbf{y}^b$ obtained for the basic scenario. Its value is defined by (9).

$$POR = 100 * \frac{f^b\left(\mathbf{y}^r\right) - f^b\left(\mathbf{y}^b\right)}{f^b\left(\mathbf{y}^r\right)} \qquad (9)$$

The *price of robustness* expresses the percentage increase in cost in the basic scenario when the robust system design is chosen, but it does not express what we gain by applying the robust solution. Therefore, we introduce also a coefficient called *gain of robustness* (*GOR*) expressed by (10).

$$GOR = 100 * \frac{f^r\left(\mathbf{y}^b\right) - f^r\left(\mathbf{y}^r\right)}{f^r\left(\mathbf{y}^r\right)} \tag{10}$$

This coefficient evaluates the profit following from applying the robust solution instead of the standard one in the worst case ignoring detrimental scenarios.

# 4 MULTI-OBJECTIVE APPROACHES

The main disadvantage of the standard approach to robust service system design described by the model (2)-(8) consists in minimizing only the worst possible impact of detrimental scenarios on the resulting system performance measured by average service accessibility for system users (average response time). It must be noted that the partial objective functions corresponding to individual scenarios may take different values and it is assumed that not only the highest one should be considered. Therefore, three different multi-objective-based approaches will be introduced in the following subsections.

## 4.1 Function GetGoalMinMax

The first approach is based on minimization of the objective function for the *basic scenario* under the condition that the objective functions corresponding to the detrimental scenarios do not increase too much. To achieve this goal, the following denotation must be introduced and it will be used in the remaining parts of this paper. If the index *u* is set to the value of zero, it means that the *basic scenario* is concerned. In other words, the matrix $\{d_{ij0}\}$ corresponds to the *basic scenario*.

If all the scenario objective functions are to be taken into account in the form of separate constraints, the goal value $G(u)$ for each scenario $u\in U_0$ should be computed. The expression (11) shows the weighted *p*-median problem solved for each failure scenario. Remember, that the symbol $U_0$ denotes the set of detrimental scenarios without the *basic scenario*.

$$G(u) = \min\left\{\sum_{j\in J} b_j \min\left\{\begin{matrix} d_{iju} : i \in I_1, \\ I_1 \subseteq I, |I_1| = p \end{matrix}\right\}\right\} \tag{11}$$

Based on these preliminaries, a non-negative parameter $\varepsilon$ can be introduced to limit the maximal increase of the objective function $G(u)$ for scenario $u\in U_0$. The parameter $\varepsilon$ can either take a given exact value or it can be expressed as some percentage of the objective function $G(u)$. Then, the model for robust EMS design can be formulated by the expressions (12)-(17).

$$\text{Minimize} \quad \sum_{j\in J} b_j \sum_{s=0}^{v} x_{js0} \tag{12}$$

$$\text{Subject to:} \quad x_{jsu} + \sum_{i\in I} a_{iju}^s y_i \geq 1 \quad \text{for } j\in J, s = 0,1,\dots,v, \ u \in U \tag{13}$$

$$\sum_{i\in I} y_i = p \tag{14}$$

$$\sum_{j\in J} b_j \sum_{s=0}^{v} x_{jsu} \leq G(u) + \varepsilon \quad \text{for } u\in U_0 \tag{15}$$

$$y_i \in \{0,1\} \quad \text{for } i \in I \tag{16}$$

$$x_{jsu} \in \{0,1\} \quad \text{for } j\in J, s = 0,1,\dots,v, \ u\in U \tag{17}$$

Since the mathematical model (12)-(17) has very similar structure as the original model (2)-(8), it is not necessary to explain each constraint separately. There are only two differences to be noted.

The first one is the objective function, which now corresponds to the service accessibility of all users under the *basic scenario*.

The second difference consists in the link-up constraints (15), in which the objective functions of all scenarios are limited by their goal values instead of their upper bound.

## 4.2 Function AdjGetGoalMinMax

The second suggested approach to robust EMS design follows from the previous *GetGoalMinMax* function described by the model (12)-(17) and goal values $G(u)$ for all scenarios from the set $U_0$.

The adjustment consists in replacing the link-up constraints (15) of the former model by their adjusted version (18), in which only the maximal goal is taken into account.

$$\sum_{j\in J} b_j \sum_{s=0}^{v} x_{jsu} \le MG + \varepsilon \quad for\ u \in U_0 \tag{18}$$

The maximal goal value *MG* can be obtained by the following expression (19). The individual goals $G(u)$ are defined by (11).

$$MG = \max\left\{G(u) : u \in U_0\right\} \tag{19}$$

This way, the *AdjGetGoalMinMax* strategy can be described by minimizing the objective function (12) under the constraints (13), (14), (16), (17) and (18).

### 4.3 Function GetGoalMinH

The last modeling strategy *GetGoalMinH* is based on a different principle. Here, the robust service system design is obtained in such a way that the goal objective function value $G(0)$ for the *basic scenario* is computed first. Then, the value of parameter $\varepsilon$ must be given to limit the maximal possible increase of mentioned goal value $G(0)$. The objective function used in this approach minimizes possible increase $h$ of the maximal goal *MG* over the set of scenarios. The associated mathematical model can be formulated in the following way.

$$Minimize \quad h \tag{20}$$

$$Subject\ to: \quad x_{jsu} + \sum_{i\in I} a_{iju}^{s} y_i \ge 1 \quad for\ j \in J, s = 0,1,\dots,v,\ u \in U \tag{21}$$

$$\sum_{i\in I} y_i = p \tag{22}$$

$$\sum_{j\in J} b_j \sum_{s=0}^{v} x_{jsu} \le MG + h \quad for\ u \in U_0 \tag{23}$$

$$\sum_{j\in J} b_j \sum_{s=0}^{v} x_{js0} \le G(0) + \varepsilon \tag{24}$$

$$y_i \in \{0, 1\} \quad for\ i \in I \tag{25}$$

$$x_{jsu} \in \{0, 1\} \quad for\ j \in J, s = 0,1,\dots,v,\ u \in U \tag{26}$$

$$h \ge 0 \tag{27}$$

The minimized objective function (20) expresses the increase of the maximal goal value *MG* over the objective functions corresponding to individual scenarios. The constraints (21) ensure that the variables $x_{jus}$ are allowed to take the value of 0, if there is at least one center located in radius $s$ from the user location $j$ and constraint (22) limits the number of located centers by $p$. The link-up constraints (23) ensure that each perceived disutility (time or distance) is less than or equal to the maximal goal value *MG* increased by $h$. The constraint (24) does not allow to exceed given value of the objective function for the *basic scenario* $G(0)$ by more than $\varepsilon$. Finally, the obligatory constraints (25), (26) and (27) are included to ensure the domain of the decision variables.

## 5 CASE STUDY

The main goal of the computational study reported in this section was to study:

- robustness coefficients,
- computational time demands.

The first aspect consists in robustness coefficients *POR* and *GOR* introduced in Section 3. Since each approach minimizes different objective function, for each resulting vector $\mathbf{y}$ of location variables $y_i$ the robust objective functions $f^b$ and $f^r$ were computed in order to evaluate *POR* and *GOR*. The values of $f^b$ and $f^r$ are defined by (28) and (29) respectively.

$$f^b(\mathbf{y}) = \sum_{j\in J} b_j \min\left\{d_{ij0} : y_i = 1\right\} \tag{28}$$

$$f^r(\mathbf{y}) = \max\left\{\sum_{j\in J} b_j \min\left\{d_{iju} : y_i = 1\right\} : u \in U\right\} \tag{29}$$

The second studied characteristic of all suggested modeling approaches consists in computational time.

All numerical experiments were performed using the optimization software FICO Xpress 7.3. They were run on a PC equipped with the Intel® Core™ i7 5500U processor with 2.4 GHz and 16 GB RAM.

The benchmarks were derived from the real emergency health care system, which was originally implemented in eight regions of Slovak Republic. For each region (Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA)), all cities and villages with corresponding population $b_j$ were taken into account. The coefficients $b_j$ were rounded to hundreds. In the benchmarks, the set of communities represents both the set $J$ of users' locations and the set

*I* of possible center locations as well. The cardinalities of these sets are reported in the tables together with the number *p* of located centers. The network time - distances from a user to the nearest located center were derived from the real transportation network. Due to the lack of scenario benchmarks for the experiments, the problem instances used in the computational study were created in the way used in (Janáček and Kvet, 2016). There were selected one quarter of matrix rows so that these rows corresponded to the biggest cities concerning the number of system users. Then same of them were chosen randomly and the associated time distance values were multiplied by the randomly chosen constant from the numbers 2, 3 and 4. The rows, which were not chosen by this random process, stay unchanged. This way, 10 different scenarios were generated for each self-governing region. These benchmarks and generated scenarios were used also in the research reported in (Janáček and Kvet, 2016, Janáček and Kvet, 2017, Kvet and Janáček, 2017a, Kvet and Janáček, 2017b).

The experiments were organized so that each of suggested multi-objective-based approaches was used to get the resulting robust EMS design for two different values of parameter $\varepsilon$. The obtained results are reported in the following six tables. The left part of all tables contains the sizes of used benchmarks. The right part contains the obtained results reported by four values:

- Let the symbol *ObjF* denote the particular model objective function.
- Computational time in seconds is reported in columns denoted by *CT*.
- Finally, the coefficients *POR* and *GOR* are reported in percentage.

The expressions (9) and (10) define their values. In this short computational study, the values of parameter $\varepsilon$ were set directly, i.e. they were not set to any percentage of the scenario goals.

Table 1: Results of the *GetGoalMinMax* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 1600.

| Region | \|I\| | p | ObjF | CT | POR | GOR |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | Problem infeasible | | | |
| BB | 515 | 52 | 17289 | 705.0 | 0.00 | 0.00 |
| KE | 460 | 46 | 20063 | 929.1 | 0.10 | 3.55 |
| NR | 350 | 35 | 22728 | 1299.7 | 0.34 | 2.24 |
| PO | 664 | 67 | 20025 | 1203.7 | 0.00 | 0.00 |
| TN | 276 | 28 | 15739 | 478.4 | 0.34 | 3.19 |
| TT | 249 | 25 | 18966 | 1422.9 | 0.49 | 3.55 |
| ZA | 315 | 32 | 21320 | 4593.6 | 1.55 | 8.53 |

Table 2: Results of the *GetGoalMinMax* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 3000.

| Region | \|I\| | p | ObjF | CT | POR | GOR |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | 21999 | 126.0 | 8.15 | 32.18 |
| BB | 515 | 52 | 17289 | 492.3 | 0.00 | 0.00 |
| KE | 460 | 46 | 20042 | 392.4 | 0.00 | 0.00 |
| NR | 350 | 35 | 22651 | 268.9 | 0.00 | 0.00 |
| PO | 664 | 67 | 20025 | 1111.3 | 0.00 | 0.00 |
| TN | 276 | 28 | 15686 | 75.9 | 0.00 | 0.00 |
| TT | 249 | 25 | 18873 | 133.7 | 0.00 | 0.00 |
| ZA | 315 | 32 | 21119 | 779.4 | 0.59 | 4.99 |

Table 3: Results of the *AdjGetGoalMinMax* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 1600.

| Region | \|I\| | p | ObjF | CT | POR | GOR |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | 22050 | 138.2 | 8.40 | 36.24 |
| BB | 515 | 52 | 17289 | 710.1 | 0.00 | 0.00 |
| KE | 460 | 46 | 20055 | 841.1 | 0.06 | 1.46 |
| NR | 350 | 35 | 22756 | 1132.7 | 0.46 | 3.76 |
| PO | 664 | 67 | 20025 | 1165.5 | 0.00 | 0.00 |
| TN | 276 | 28 | 15706 | 473.0 | 0.13 | 3.26 |
| TT | 249 | 25 | 18939 | 1011.5 | 0.35 | 1.72 |
| ZA | 315 | 32 | 21320 | 539.4 | 1.55 | 8.53 |

Table 4: Results of the *AdjGetGoalMinMax* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 3000.

| Region | \|I\| | p | ObjF | CT | POR | GOR |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | 21999 | 75.7 | 8.15 | 32.18 |
| BB | 515 | 52 | 17289 | 788.4 | 0.00 | 0.00 |
| KE | 460 | 46 | 20042 | 434.3 | 0.00 | 0.00 |
| NR | 350 | 35 | 22651 | 250.0 | 0.00 | 0.00 |
| PO | 664 | 67 | 20025 | 1284.7 | 0.00 | 0.00 |
| TN | 276 | 28 | 15686 | 188.8 | 0.00 | 0.00 |
| TT | 249 | 25 | 18873 | 134.2 | 0.00 | 0.00 |
| ZA | 315 | 32 | 21119 | 559.2 | 0.59 | 4.99 |

Table 5: Results of the *GetGoalMinH* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 500.

| Region | \|I\| | p | ObjF | CT | POR | GOR |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | 6871 | 83.7 | 1.70 | 10.98 |
| BB | 515 | 52 | 386 | 18380.0 | 2.61 | 2.73 |
| KE | 460 | 46 | 548 | 15768.5 | 2.16 | 6.29 |
| NR | 350 | 35 | 710 | 15775.4 | 2.14 | 6.84 |
| PO | 664 | 67 | 286 | 18419.1 | 2.00 | 5.48 |
| TN | 276 | 28 | 328 | 1522.6 | 3.16 | 9.85 |
| TT | 249 | 25 | 998 | 2226.6 | 2.08 | 4.61 |
| ZA | 315 | 32 | 601 | 2957.5 | 2.33 | 12.93 |

Table 6: Results of the *GetGoalMinH* approach applied on benchmarks derived from the self-governing regions of Slovakia. The parameter $\varepsilon$ was set to 1500.

| Region | $\|I\|$ | *p* | *ObjF* | *CT* | *POR* | *GOR* |
|---|---|---|---|---|---|---|
| BA | 87 | 14 | 3217 | 104.4 | 6.88 | 25.70 |
| BB | 515 | 52 | 386 | 18054.0 | 2.61 | 2.73 |
| KE | 460 | 46 | 548 | 23663.2 | 2.16 | 6.29 |
| NR | 350 | 35 | 657 | 13426.0 | 2.47 | 7.07 |
| PO | 664 | 67 | 286 | 19660.5 | 2.00 | 5.48 |
| TN | 276 | 28 | 284 | 2378.1 | 4.05 | 10.13 |
| TT | 249 | 25 | 788 | 2267.6 | 3.87 | 5.68 |
| ZA | 315 | 32 | 525 | 4196.0 | 3.75 | 13.31 |

Analyzing the results reported in Tables 1 - 6, the expectations have been confirmed. As can be observed, the quality of obtained resulting system designs measured by the values of coefficients *POR* and *GOR* depend on the parameter settings. As far as the service system robustness is concerned, presented approaches represent suitable contribution to the state-of-the-art methods for robust system designing.

Focusing on computational time requirements, the big difference between the first two approaches and the third one can be explained by the model structure. While the mathematical model used in the functions *GetGoalMinMax* and *AdjGetGoalMinMax* uses a min-sum optimization criterion, the model used in the *GetGoalMinH* approach takes the form of a min-max problem, which is generally harder to solve, leading to longer computation times.

# 6 CONCLUSIONS

This paper was focused on robust emergency medical service system design. The robustness follows the idea, which aims to make the system resistant to various randomly occurring detrimental events, which may negatively affect system performance and quality of the service provided. The main focus was on the set of detrimental scenarios, which allows forming an additional constraint to the model for each element of the scenario set. In this paper, three approaches were introduced and experimentally compared.

It can be observed that the computational time demands depend on the model structure. If we replace a min-sum objective by a min-max optimization criterion, then the model gets more complicated so it requires a longer computation time.. Besides that, quality of obtained results is very satisfactory.

The future research in this field could be aimed at other approximate techniques, which will enable to reach shorter computational time with acceptable solution accuracy. Another future research goal could be focused on mastering the presented problem with a larger set of detrimental scenarios.

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