ACO Algorithms to Solve an Electromagnetic Discrete Optimization Problem

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Abstract: The paper proposes and studies the efficiency of the ant colony optimization (ACO) algorithms for solving an inverse problem in non-destructive electromagnetic testing (NDET). The inverse problem, which consists in finding the shape and parameters of cracks in conducting plates starting from the signal of an eddy current testing (ECT) probe, is formulated as a discrete optimization problem. Two of the most widely known ant algorithms are adapted and applied to solve the optimization problem. The influence over the optimization algorithms performances of some problem specific local search strategies is also analyzed.

1 INTRODUCTION

Eddy Current Testing (ECT) is one of the most used electromagnetic methods commonly employed in the non-destructive evaluation of conductive materials (Yusa et al., 2016). The ECT principle is based on the interaction between induced eddy currents and an examined conductive structure, interaction due to the electromagnetic induction phenomena. The method is applied in various application fields for thickness measurements, material corrosion evaluation, proximity measurements, and so on. However, at the present time the most widely spread area of application is the diagnosis and detection of discontinuities in conductive materials. Real cracks (such as stress corrosion cracks) usually appear in steam generator tubes used in pressurized water reactor of nuclear power plants (Yusa, 2017).

The Non-destructive Electromagnetic Testing (NDET) inverse problem deals with the identification of crack parameters using the ECT measured signal (Yusa et al., 2016) (Yusa, 2017). The optimization problem associated with the inverse problem aims to minimize the difference between the simulated signal corresponding to a potential solution and the measured (real) signal.

Since deterministic methods can not be applied because of multiple local minimum, heuristics based methods, like genetic algorithms, tabu search, particle swarm optimization, and so on, have emerged as the standard techniques for solving these non-convex and ill conditioned difficult inverse problems.

The present paper proposes and deals with studying and comparing the efficiency of ant algorithms to solve the optimization problem associated with the inverse NDET problem.

The first ant algorithm, the Ant System (AS), was proposed by (Dorigo et al., 1996) and was targeted towards hard non-determinist polynomial (NP) combinatorial optimization problems such as the Travelling Salesman Problem (TSP) (Stutzle, Hoos, 1997) (Dorigo et al., 1999) (Ridge, Kudenko, 2007), Quadratic Assignment Problem (QAP) (Stutzle, Hoos, 1997) (Dorigo et al., 1999) or Multiple Knapsack Problem (MKP) (Fidanova, 2007) (Ke et al., 2013). The algorithm simulates the behaviour of ants in real ant colonies when searching for food. The ants are social insects which communicate information about food sources using a substance called pheromone, substance secreted along their search path.

During time, to improve the performances of the initial algorithm a significant number of solutions have been proposed, the most notorious ant algorithms being the Max-Min Ant System (MMAS) (Stutzle, Hoos, 1997) and the Ant Colony Optimization (ACO) (Dorigo et al., 1999). In the same time several algorithms derived from ant algorithms for combinatorial optimization have been

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proposed for continuous optimization problems (ACOR) (Socha, Dorigo., 2008).

To solve the inverse NDET problem two approaches will be studied: the first one is based on an ant algorithm for continuous optimization (ACOR) and the second is based on an algorithm designated to combinatorial optimization (MMAS).

To speed up the optimization process and to avoid getting trapped in local minimum points some problem specific local search strategies are used to enhance the ant algorithms when solving the inverse NDET problem. The influence over the ant based algorithms performances of the local search frequency is also studied.

2 THE NDET PROBLEM

2.1 Tested Configuration

The problem used for testing is a slightly different version of JSAEM (Japan Society for Applied Electromagnetics) benchmark #2 similar with the one in (Janousek et al., 2017). The non-magnetic conductor (Ω_0) surface is scanned using pancake coil with a self-induction. The non-magnetic plate (40 x 40 x 1.25 mm³) has the conductivity σ =106 S/m and contains one crack located in the region Ω_1 (10 x 1 x 1.25 mm³) divided uniformly in a grid of cells (13 x 5 x 10) (Figure 1). The cracks are cubes described by 6 integer parameters, $\mathbf{c} = [ix_1, ix_2, iy_1, iy_2, iz, s]$, the indices of the first / last cells along x [length] and y[width], number of cell along z [depth iz], and conductivity $\sigma_c = s \% \sigma$. The crack is considered as having a uniform conductivity, zero or a percentage from the plate conductivity.



Figure 1: Conductor plate with a crack.

2.2 ECT Signal Simulation

For the simulation of the ECT signals a fast FEM-BEM solver is used (Chen et al., 1999) (Rebican et al., 2006). The simulated ECT signals use a database generated in advance for cracks with different widths (Yusa et al., 2003), (Chen et al., 2006). To calculate the ECT signal for a crack with the FEM-BEM a linear equations system of small dimension needs to be solved, corresponding to the finite elements composing the crack. This leads to a significant computational time decrease.

The optimization problem associated to the NDET inverse problem has the following objective function:

$$\varepsilon(c) = \sqrt{\sum_{i=1}^{N_{sc}} (\Delta Z_i(c) - \Delta Z_i^{true}(c))^2}$$
(1)

where **c** is the vector containing the crack parameters, N_{sc} is the scanning points number, and $\Delta Z_i / \Delta Z_i^{true}$ are the simulated / measured coil impedance variations in the *i*-th scanning point.

3 ACO ALGORITHMS 3.1 MMAS

The MMAS is an ant colony optimization algorithm proposed by Stutzle and Hoos which proved its efficiency on combinatorial optimization problem such as TSP and QAP (Dorigo et al., 1999). As ACO optimization algorithms, MMAS is based on the natural phenomenon of ants forage for food. During their search path ants create tours (graphs) on which they deposit a substance called pheromone. An ant movement along the edges of the graph is a probabilistic decision based on the pheromone information.

In practice MMAS is implemented as an iterative stochastic algorithm with the next stages: pheromone initialisation, tour (solution) construction and evaluation, local search, pheromone evaporation, pheromone deposit on the best global route, pheromone limitation, and if necessary pheromone reinitialization. The pseudocode of the algorithm is as follows:

```
; initialize pheromones table
do
   foreach ant of the colony
        ; construct solution
        ;evaluate solution
   end for
   ;local search
   ;evaporate pheromones
    ;deposit pheromone for best ant
    ;apply correction to pheromones
     foreach pheromone value
           if pheromone < \taumin
            pheromone = \taumin
           if pheromone > \tau_{max}
           pheromone = \taumax
   end for
     ; reset pheromones if necessary
   noIterations ++
while
    (noIterations < maxNoIterations)</pre>
   and
     (optimal solution not found)
```

Pheromone Initialization. Each pheromone from the pheromones table (graph) is assigned an initial value which equals the maximum allowed value (τ_{max}). This value is usually set to 1 / (ρ *Fmin*) where ρ is the evaporation rate and *Fmin* is the smallest value of the objective function to be minimized.

Solution Construction. The ants construct an initial solution starting from a random node. Starting from a node an ant movement can be exploitative or explorative. The decision is made using a random number and an exploration threshold, which is a parameter value of the algorithm. If the decision is exploitation then the ant computes the probabilities for choosing the next possible nodes in the graph and choses the node with the highest probability. The probability to choose a node j when starting from a node i is:

$$p_{ij} = ([\tau_{ij}]^{\alpha} [\eta_{ij}]^{\beta}) / (\sum_{k} [\tau_{ik}]^{\alpha} [\eta_{ik}]^{\beta}), \qquad (2)$$

where τ_{ij} is the pheromone for the edge between the nodes *i* and *j*, *k* is a node which can be selected from the node *i*, and η is a heuristic information representing the attractiveness of the move (in case of TSP the length of the *ij* edge).

If the decision is exploration than the computed probabilities are used as weights to choose the next node using a probabilistic method such as wheel selection. **Local Search.** Local search is used to improve the solution quality with neighbourhood strategies. Two different approaches can be applied: a problem independent heuristic (as tabu search), and secondly some problem specific local search strategy.

Pheromone Evaporation. Each pheromone corresponding to an edge of the graph is decreased with the following formula:

$$\tau_{ij} = (1 - \rho) \tau_{ij}, \tag{3}$$

where ρ is the evaporation rate.

Trail Update. Pheromone is deposited on all edges connecting the components of the solution for the best ant. There are the following approaches: the best overall solution, or the best solution at the current iteration and the best overall solution. The update formula is:

$$\tau_{ij} = 1/F + \tau_{ij},\tag{4}$$

where *F* is the objective function value for the best solution (in the case of TSP the length of the tour).

Pheromone Correction. In the case of MMAS, to avoid the algorithm stagnation the pheromones are limited to an interval $[\tau_{min} \tau_{max}]$. The minimum and maximum values of the pheromones are usually chosen as:

$$\tau_{min} = 1/2n, \ \tau_{max} = 1 / (\rho \ Fmin),$$
 (5)

where Fmin is smallesr objective function value (smallest length of tour for TSP) and n is the problem size (number of cities for TSP).

Reinitialise Pheromones. Pheromone table can be reinitialised if the algorithm stagnates and does not improve the overall best after an imposed number of iteration. The pheromone values are set to their initial values τ_{max} .

3.2 ACOR

Proposed in (Socha, Dorigo., 2008) the ACOR algorithm is an extension of ant based optimization algorithms for continuous optimization problems.

ACOR is a population based algorithm which stores the pheromones table as a solutions archive (6). The solutions are ordered using their fitness values in ascending order ($f(s_k) < f(s_{k+1})$), where $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function to be minimized. Each

solution has an associated weight ω corresponding to its fitness value ($\omega_k > \omega_{k+1}$).



Solution Construction. The construction of a new solution starts from a solution l from the archive. The *l*th solution can be chosen using a wheel selection mechanism. The selection probability for the lth solution is:

$$p_l = \omega_l / \sum_k \omega_k. \tag{7}$$

After choosing the start solution, an ant constructs a new solution in n steps. At each step i the ant calculates a value for the corresponding optimization variable using only information about the *i*th dimension.

The new solutions are constructed using the solution archive by calculating the parameters of the Gaussian kernels G^k (the number of the Gaussian kernels is equal with the number of variables of the optimization problem *n*). More details about calculating the parameters of the Gaussian kernels can be found in (Socha, Dorigo., 2008).

After constructing a set of solutions the algorithm evaluates them, add them to the solution archive, sorts the solutions archive according to the fitness values, calculates weights and Gaussian kernels, and in the end removes the worst solutions by keeping the solutions archive size to a specified number.

3.3 Ant Algorithms Approach for the NDET Inverse Problem

The ant algorithms used for the inversion process have to be customized for this type of NDET problems. The ACO for continuous domains, such as ACOR, store their pheromone table as a solution archive and can simply be adapted to the discrete optimization problem by rounding the coordinates values before the evaluation of the objective function.

The ant algorithms for combinatorial

optimization, such as MMAS and ACO, need specific design modification to be used for the NDET inverse problem.

The first design issue is to map the inverse problem on a graph. This paper proposes the use of a layered graph. Each layer in a graph corresponds to a variable of the optimization function (a parameter of the crack) and its vertices (the nodes) are given by the number of possible values of the discrete variable. The edges (the arcs) between the nodes of different layers have assigned pheromone levels and represent a possibility of choice: for example, an edge between a node i from a layer x and a node j from another layer y means that when constructing a candidate solution after the parameter y might receive a value j.

The second design issue is related to the *tour* construction (candidate solution). At each step, in order to move from a vertex to another an ant has to compute a probability distribution (2). If in the case of TSP the attractiveness was represented by the distance between the two cities in our case the proposed solution is to be the best value of the objective function which was previously obtained with that combination of parameter values.

The maximum and minimum values for the MMAS pheromone levels will include the best value of the objective function obtained at the current step (instead of the length of the tour) and the number of vertices in the graph. To avoid extreme cases the objective function will be normalized and have a minimum non-zero value.

The last issue is the local search methodology. The proposed local search methods are NDET problem specific, and they aim to avoid local minima and increase the speed of convergence of the inversion algorithm.

4 LOCAL SEARCH STRATEGY

Initially proposed in (Duca et al., 2014) and used in PSO conjunction with (Particle Swarm Optimization) (Kennedy, Eberhart, 1995) based algorithms, and also successfully applied in (Duca et al., 2014, 2) in conjunction with advanced PSO algorithms (Sun et al., 2004) (Clerc, 2012) (Altinoz et al., 2015), the local search methods are applied after a number of iterations performed by the optimization procedure. The local search strategy generates 16 potential solutions starting from the solution with the best fitness. A test point is generated changing one parameter of the starting point using expansion, contraction or displacement.



Figure 2: Negative displacement on OX for the crack c = [6, 13, 1, 3, 4, 3].

The contraction / expansion can be performed along the length (OX), width (OY) or depth (OZ), but also conductivity. The contraction / expansion operations generate 12 testing points, because the operations can be applied in two different ways for the length and width, changing ix1/iy1 or ix2/iy2. The displacement can be performed for OX or for OY axis but not for OZ (the crack always starts from the plate surface). The displacement operation generates four new testing points. Figure 2 shows a negative displacement on OX performed on a crack described by the parameters [6, 13, 1, 3, 4, 3].

5 RESULTS

In this paper, the inverse NDET problem is solved using six different schemes: three MMAS schemes (MMAS, MMAS with high frequency local search MMAS-LS-hf, MMAS with low frequency local search MMAS-LS-lf) and three ACOR schemes (ACOR, ACOR with high frequency local search ACOR-LS-hf, ACOR with low frequency local search ACOR-LS-lf). For the schemes with high frequency the local search is applied on the best solution after each iteration of the algorithm, while for the schemes with low frequency the local search is applied after 80 evaluations of the objective function (equivalent with 16 iterations for the ACOR and 40 iterations for MMAS).

To compare the efficiency of the employed schemes six inner defects (ID, the crack is on the same side with the coil) are considered: four with zero conductivity (ID1-ID4) and two with non-zero conductivity (ID5-ID6, crack conductivity is 3% and 2% of the plate conductivity). The values of the crack parameters are given in Table 1. For example, ID4 has the length of 5.39mm (7cells x 0.77mm), the width of 0.4mm (2cells x 0.2mm), the depth of

40% from the plate thickness (iz=4), and zero conductivity (s=0).

Table 1: Cracks used for testing.

	Crack parameters							
Crack	ix_1	ix2	<i>iy</i> 1	iy2	iz	S		
ID1	4	10	2	4	5	0		
ID2	5	9	1	5	3	0		
ID3	3	11	2	3	2	0		
ID4	4	10	3	4	4	0		
ID5	4	10	2	4	5	3		
ID6	3	11	2	3	2	2		

To make a relevant statistical study, 30 numerical simulations (tests) were performed for each crack reconstruction. After a previous tuning the most suitable ACOR parameters were: archive size 40, number of ants 5, locality of the search process 0.01, convergence speed 0.85. The MMAS parameters were chosen as suggested in (Ridge, Kudenko, 2007): number of ants 2, alpha pheromone term 4, distance heuristic term beta 3, exploration / exploitation threshold 0.75, pheromone update frequency for best so far 1, random chosen start variable for solution construction, limits of trail pheromone 0.01 and 2, number of iterations without improvement (which resets the pheromone table) 10.

The optimization algorithms were stopped when the exact solution was found (the objective function is zero) or the algorithm completed a maximum number of 1000 objective function (OF) evaluations.

Table 2 (see Appendix) presents the numerical results of the reconstructions as the minimum, the maximum, the average value and the standard deviation of the objective function for the best solution for each of the 30 tests, and the number of tests in which the exact parameters of the crack were found (the exact fit).

The performances obtained with the ACOR based algorithms outperform the ones with the MMAS for all the six tested cracks. The local search strategy improves the converge speed for both algorithms. The inversion schemes when the local search is applied with lower frequency performed significantly better than the schemes with high frequency, providing better average values and higher number of exact findings. The exceptions are in the case of ID6 and partially ID3 (for ACOR algorithms) and ID3 (for the MMAS algorithms).

The improvements and superiority of the algorithms with local search can also be seen from mean-best evolution during the optimization process (Figures 3-8). Besides the fact that statistical mean values are smaller, the LS-lf algorithms are more

stable having a smoother evolution for the cracks ID 1/2/4/5, while the LS-hf algorithms perform better for the cracks ID 3/6.



Figure 3: Mean-best OF value variation for test ID1.



Figure 4: Mean-best OF value variation for test ID2.



Figure 5: Mean-best OF value variation for test ID3.



Figure 6: Mean-best OF value variation for test ID4.



Figure 7: Mean-best OF value variation for test ID5.



Figure 8: Mean-best OF value variation for test ID6.

6 CONCLUSIONS

The paper studied the efficiency of ant based algorithms used for the reconstruction of cracks starting from the ECT signals supplied by a probe. Two type of ant algorithms have been adapted and analysed, ACOR for continuous domains and MMAS for discrete optimization problems. The paper also analysed the efficiency of the ant algorithms in conjunction with some problem specific local search methods aiming to enhance the inversion process.

The schemes based on ACOR provide better performances (higher number of exact findings and smaller average and standard values for the objective function) than the proposed MMAS schemes, for both conductive and non-conductive cracks.

The ant algorithms enhanced with local search strategies proved to be, by far, the best approach for solving the inverse problem. The schemes enhanced with local search significantly improve the performances of both type of algorithms, ACOR and MMAS, for cracks with zero or non-zero conductivity. In terms of frequency, a lower frequency use of the local search strategies seems to be preferable to a high frequency, which seems to lead to a premature convergence for most of the test cases.

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APPENDIX

Crack	c / Algorithm	Min-best OF value (× E-02)	Exact fit (OF=0)	Max - best OF value (× E-02)	Mean - best OF value $(\times E-02)$	Standard deviation (× E-02)
	ACOR	0	8 /30	10.87	5.35	3.57
ID1	ACOR-LS-hf	0	8 /30	10.87	5.11	3.37
	ACOR-LS-lf	0	9 /30	13.41	4.71	3.39
	MMAS	0	1 /30	45.31	24.68	10.95
	MMAS-LS-hf	6.4	0 /30	49.08	22.56	10.29
	MMAS-LS-lf	0	4 /30	49.08	19.51	11.43
ID2	ACOR	0	21 /30	6.22	1.40	2.36
	ACOR-LS-hf	0	20/30	15.40	2.62	4.45
	ACOR-LS-lf	0	22 /30	5.35	1.12	2.16
	MMAS	8.12	0 /30	25.62	17.96	5.00
	MMAS-LS-hf	14.98	0 /30	36.89	18.88	6.42
	MMAS-LS-lf	0	4 /30	19.43	9.64	6.28
ID3	ACOR	0	18 /30	7.23	2.25	2.99
	ACOR-LS-hf	0	19/30	7.22	1.68	2.39
	ACOR-LS-lf	0	20 /30	7.22	1.73	2.64
	MMAS	0	4 /30	11.43	7.36	3.28
	MMAS-LS-hf	0	14 /30	16.09	3.92	4.17
	MMAS-LS-lf	0	5 /30	9.64	5.50	2.78
ID4	ACOR	0	20 /30	10.40	2.65	4.01
	ACOR-LS-hf	0	21/30	16.16	3.19	5.09
	ACOR-LS-lf	0	29 /30	10.39	0.35	1.90
	MMAS	0	2 /30	25.02	15.84	5.83
	MMAS-LS-hf	0	2 /30	29.51	15.00	5.96
	MMAS-LS-lf	0	5 /30	19.57	10.56	5.93
ID5	ACOR	0	23 /30	10.64	1.63	3.09
	ACOR-LS-hf	0	20/30	19.25	3.41	5.54
	ACOR-LS-lf	0	28 /30	7.46	0.34	1.43
	MMAS	10.64	0 /30	43.22	21.62	6.80
	MMAS-LS-hf	7.46	0 /30	24.09	21.57	3.37
	MMAS-LS-lf	0	1 /30	24.09	16.49	6.55
ID6	ACOR	0	23 /30	5.08	1.08	2.06
	ACOR-LS-hf	0	28 /30	5.10	0.34	1.29
	ACOR-LS-lf	0	26/30	5.05	0.67	1.74
	MMAS	0	1 /30	13.08	8.02	3.34
	MMAS-LS-hf	0	5 /30	12.08	6.26	4.03
	MMAS-LS-lf	0	9 /30	5.10	3.34	2.35

Table 2: Objective function values and standard deviation for the NDET problem.(Bold RED is the best algorithm option for a crack, bold GREEN is the best MMAS option for a crack)