Approaches to Parameter Identification for Hybrid Multilinear Time Invariant Systems

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Keywords: Multilinear Systems, Parameter Identification, Tensor Decomposition, Industrial Building Modelling.

Abstract: Industrial buildings often have interacting continuous- and discrete-valued signals. Hybrid multilinear time invariant (MTI) models have been shown to be able to describe this hybrid dynamics appropriately for many cases. White box modelling methods from first principles have been used in this application domain before. The parameters of these models can be efficiently represented by higher order tensors. This paper introduces as alternatives black and grey box approaches for the parameter identification of MTI models from data. The methods are tested with the help of simulation data produced from a multilinear model of an industrial hall. It is assumed that all state variables are measured with additative noise and the input and disturbances are exactly measured, too. Two black box methods obtain either the full parameter tensor or a rank-r decomposition of it. Numerical examples using the industrial building model show the principle applicability of these approaches for real data.

1 INTRODUCTION

Systems in many application areas as buildings engineering show discrete-valued as well as continuous-valued signals, e.g. if a continuous state like a temperature depends on the switching ON/OFF of a binary input like a gas heater. This example can be sufficiently good described by multilinear time invariant (MTI) models, as well as many more from other application areas, (Lichtenberg, 2011).

There are three traditional approaches in order to obtain a state space model depending on the available information about the system. If no sufficient prior knowledge about the system is available, then the black box identification approach is chosen (Chavan and Talange, 2018), (Tayamon, Zambrano, Wigren and Carlsson, 2011), (Royer, Thil, Talbert and Polit, 2014). On the contrary, in some cases through first principles like laws of physics, sufficient information about the system can be obtained. In these cases, white box techniques are used. However, if the information through first principles is incomplete, in the sense that there are unknown parameters in the system, the grey box identification approach is used (Bacher and Madsen, 2011).

In (Batselier, Ko, Phan, Cichoki and Wong, 2018), the system identification problem of multilinear state space models has been considered. In this work, the coefficients of the state space matrices have been estimated by representing them as tensor train matrices. The computational complexity is reduced by representation as tensor train matrices. The contribution of this paper are different methods for parameter identification of an MTI state space model. In order to solve the identification problem, both the grey and black box methods are considered. The grey box problem involves finding the unknown physical parameters of the system. In this case the structure of the model is known beforehand.

On the other hand, the black box problem involves no prior knowledge about the system. The identification problem is to find parameters of a multilinear state space model, given a set of input state data and

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Sridharan, A., Lichtenberg, G., Salvador, A. and Salgado, C.
Approaches to Parameter Identification for Hybrid Multilinear Time Invariant Systems.
DOI: 10.5220/0009887502550262
In *Proceedings of the 10th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2020)*, pages 255-262
ISBN: 978-989-758-444-2

a specific system order. The first black box method shows how the parameter identification problem can be transformed to a linear least squares problem and thus solved efficiently. The parameters of the MTI state space model increase exponentially with respect to the order of the system and the number of inputs. As the parameters of MTI state space models can naturally be represented as tensors, decomposition methods can be used to compute low rank approximations, which need only a remarkable less number of parameters, especially for the case of canonical polyadic (CP) decompositions, (Kruppa, 2017). The second approach to black box parameter identification given in this paper is a direct estimation method for the CP decomposition factors of the parameter tensor from data. Throughout the paper, discrete time models are used and the inputs as well as state signals are assumed to be sampled with a fixed sampling time. The effects of measurement noise are not dealt with by a formal approach. However, simulation results are presented by adding noise as well.

The paper is organized as follows. Firstly, a brief introduction to tensor calculus and MTI systems is given. Thereafter, the identification problem is setup and is dealt with by the methods described above. Finally, a numerical example simulated using MATLAB Simulink is presented, followed by conclusion and future scope.

2 TENSOR BASICS

The state space model of an MTI system can be formulated in a tensor framework. Thereby, in this section, some basic concepts about tensors and tensor decompositions are presented (Kolda and Bader, 2009).

Definition 1 (Tensor)**.** *A multidimensional n-way array*

$$\mathsf{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_n} \tag{1}$$

where $(I_1, I_2, ..., I_n) \in \mathbb{N}^n$ is called a real tensor of order n.

Although such a structure can store complex values as well, in the current work, only real values are dealt with. An element

$$x(\mathbf{i}) = x(i_1, i_2, ..., i_n) \in \mathbb{R}$$

of the tensor X can be selected by the index vector $\mathbf{i} = (i_1, i_2, ..., i_n)$.

For tensors, numerous arithmetic operations exist and two of them are used in this paper: the contracted product of two tensors A, B is denoted by $\langle \mathsf{A} | \mathsf{B} \rangle$ whereas the outer product is denoted by $\mathsf{A} \circ \mathsf{B}$. In depth information on these products can be found in (Kolda and Bader, 2009).

Tensors can have very high storage demands, since the number of elements of a tensor depends exponentially on the number of dimensions of the tensor. In order to address this problem, tensor decomposition methods have been developed over the last decades, (Cichocki, Zdunek, Phan and Amari, 2009). Decomposition methods range from canonical polyadic (CP), Tucker, tensor trains to hierarchical decompositions. Because of their superior storage effort savings and suitability to MTI models, CP decompositions are used in this paper. The simplest of it is given in the next definition.

Definition 2 (Rank One Tensor)**.** *An n dimensional tensor*

$$\mathsf{X} = x_1 \circ x_2 \circ \cdots \circ x_n \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_n} \tag{2}$$

has rank one if it can be computed by the outer product of n vectors $x_i \in \mathbb{R}^{I_i} \, \forall i = 1, ..., n$.

All full tensors are representable by a sum of rank one tensors, which is given in the next definition.

Definition 3 (CP Tensor)**.** *A canonical polyadic representation*

$$\mathsf{K} = [\mathbf{X_1}, \mathbf{X_2}, ..., \mathbf{X_N}] \cdot \lambda \tag{3}$$

$$= \sum_{l=1}^{r} \lambda(l) x_1(:,l) \circ \cdots \circ x_N(:,l) \tag{4}$$

of a real tensor of dimension $I_1 \times I_2 \times \cdots \times I_n$ is given by a sum of rank 1 tensors.

The elements are computed by the sums of the outer products of the column vectors of factor matrices $X_i \in \mathbb{R}^{I_i \times r}$. The introduction of a weighting vector $\lambda \in \mathbb{R}^r$ allows to normalize the column vectors of the factor matrices. The minimum number of rank 1 tensors required to reproduce the original tensor K is called the CP rank of the tensor K.

Example 1. Figure 1 visualizes a 3rd order CP tensor

$$\mathsf{K} = [\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3}] \cdot \lambda$$

$$= \sum_{l=1}^{r} \lambda(l) \mathbf{X}_1(:,l) \circ \mathbf{X}_2(:,l) \circ \mathbf{X}_3(:,l)$$

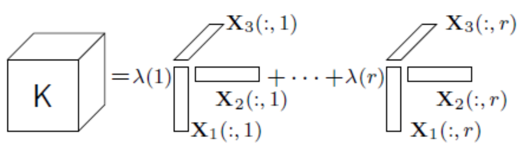

Figure 1: CP tensor (Kruppa and Lichtenberg, 2017).

3 MULTILINEAR MODELS

MTI systems are described by multilinear functions depending on states and inputs. Multilinearity means that the function is linear if all but one variables are held constant. Thus, they are polynomials of order n but showing a maximum order of one for each variable, (Pangalos, Eichler and Lichtenberg, 2015).

Definition 4 (Multilinear Function). *The function*

$$f(x) = \alpha^T \mathbf{m}(\mathbf{x}) \tag{5}$$

where

$$\alpha = \begin{bmatrix}\alpha_1 & \cdots & \alpha_{2^n}\end{bmatrix}^T \in \mathbb{R}^{2^n}$$

is a coefficient row vector and

$$\mathbf{m}(\mathbf{x}) = \begin{bmatrix}1\\x_n\end{bmatrix} \otimes \cdots \otimes \begin{bmatrix}1\\x_1\end{bmatrix} \in \mathbb{R}^{2^n}$$

is a column vector of monomial is called multilinear.

Here, $\otimes$ is the well known Kronecker product.

Definition 5 (MTI Matrix Model). *A discrete-time multilinear model is represented by a next state equation*

$$\mathbf{x}(k+1) = \mathbf{F} \cdot \mathbf{m}(\mathbf{x}(k), \mathbf{u}(k)) \tag{6}$$

where $\mathbf{m}(\mathbf{x}(k), \mathbf{u}(k))$ *is given by*

$$\begin{bmatrix}1\\u_m(k)\end{bmatrix} \otimes \cdots \otimes \begin{bmatrix}1\\u_1(k)\end{bmatrix} \otimes \begin{bmatrix}1\\x_n(k)\end{bmatrix} \otimes \cdots \otimes \begin{bmatrix}1\\x_1(k)\end{bmatrix}$$

i.e. the monomial vector containing all multiplicative combinations of states and inputs at time k.

The transition matrix $\mathbf{F} \in \mathbb{R}^{n \times 2^{n+m}}$ holds the parameters of the model as the following example shows.

Example 2. A MTI state space model of order two

$$x_1(k+1) = f_{11} + f_{12}x_1(k) + f_{13}x_2(k) + f_{14}x_1(k)x_2(k)$$
$$x_2(k+1) = f_{21} + f_{22}x_1(k) + f_{23}x_2(k) + f_{24}x_1(k)x_2(k)$$

has a transition matrix and monomial vector given by

$$\mathbf{F} = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \end{pmatrix},$$

$$\mathbf{m}(x_1(k), x_2(k)) = \begin{pmatrix} 1 \\ x_1(k) \\ x_2(k) \\ x_2(k)x_1(k) \end{pmatrix}.$$

Next, the multilinear structure of the functions can be exploited to represent the next state equation in a tensor format.

Definition 6 (MTI Tensor Model). *The next state equation (6) can equivalently be written by*

$$\mathbf{x}(k+1) = \langle \mathsf{F} \,|\, \mathsf{M}(\mathbf{x}(k), \mathbf{u}(k)) \rangle \tag{7}$$

where $\mathsf{F} \in \mathbb{R}^{\times^{(n+m)}2 \times n}$ *contains the parameters of the model arranged as a tensor and* $\mathsf{M}(\mathbf{x}, \mathbf{u}) \in \mathbb{R}^{\times^{(n+m)}2}$ *is a monomial tensor. The notation* $\times^{(n+m)}2$ *stands for* $\underbrace{2 \times 2 \times ... \times 2}_{(n+m)\,times}$.

Example 3. An autonomous MTI state space model of order two in tensor form is given by

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \langle \mathsf{F} \,|\, \mathsf{M}(x_1(k), x_2(k)) \rangle .$$

For this example, the contracted product of the transition tensor $\mathsf{F} \in \mathbb{R}^{2\times2\times2}$ and the monomial tensor $\mathsf{M} \in \mathbb{R}^{2\times2}$ leads to the same next state equations as in the matrix case of Example 2.

The monomial tensor is already rank one and thus, minimal. The parameter tensor can be decomposed as CP tensor

$$\mathsf{F} = [\mathbf{F}_{u_m}, .., \mathbf{F}_{u_1}, \mathbf{F}_{x_n}, .., \mathbf{F}_{x_1}, \mathbf{F}_\Phi] \cdot \lambda_F \tag{8}$$

which is e.g. discussed in (Pangalos, 2016).

With the parameter tensor (8), the next state equation (7) can be simplified

$$\mathbf{x}(k+1) = \mathbf{F}_\Phi \Bigg(\lambda_F \circledast \mathbf{F}_{u_m}^T \begin{pmatrix}1\\u_m(k)\end{pmatrix} \circledast \cdots \circledast \circledast \mathbf{F}_{u_1}^T \begin{pmatrix}1\\u_1(k)\end{pmatrix} \circledast \mathbf{F}_{x_n}^T \begin{pmatrix}1\\x_n(k)\end{pmatrix} \circledast \cdots \circledast \mathbf{F}_{x_1}^T \begin{pmatrix}1\\x_1(k)\end{pmatrix} \Bigg) \tag{9}$$

The element wise multiplication, also known as the Hadamard product is denoted as $\circledast$ operation and matrix vector multiplication has precedence here.

Example 4. The previous example in decomposed tensor representation has the next state equation

$$\mathbf{x}(k+1) = \mathbf{F}_\Phi \Bigg(\lambda_F \circledast \mathbf{F}_{x_2}^T \begin{pmatrix}1\\x_2(k)\end{pmatrix} \circledast \circledast \mathbf{F}_{x_1}^T \begin{pmatrix}1\\x_1(k)\end{pmatrix} \Bigg) \tag{10}$$

the weighting vector

$$\lambda_F = \begin{pmatrix} f_{11} & \ldots & f_{14} & f_{21} & \ldots & f_{24} \end{pmatrix}^T$$

and the factor matrices

$$\mathbf{F}_\Phi = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{F}_{x_1} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{F}_{x_2} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

4 IDENTIFICATION PROBLEM

In this section, the identification problem is formulated for both grey and black box methods. Within black box identification, the problem is tackled by two different approaches.

4.1 Grey Box Identification

In this sub-section, it is assumed that a model from the laws of physics is already available, albeit with unknown parameters. Considering the following model with n states, m inputs and l disturbances in discrete time domain

$$\begin{pmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \end{pmatrix} = \begin{pmatrix} f_{11} + f_{12}x_1(k) + .. + f_{12^{n+m+l}} \prod_{i=1}^{n} x_i \prod_{i=1}^{m} u_i \prod_{i=1}^{l} d_i \\ \vdots \\ f_{n1} + f_{n2}x_1(k) + .. + f_{n2^{n+m+l}} \prod_{i=1}^{n} x_i \prod_{i=1}^{m} u_i \prod_{i=1}^{l} d_i \end{pmatrix} \tag{11}$$

The discrete time MTI state space model for the system in (11) is of order n with state vector $\mathbf{x} \in \mathbb{R}^n$, having an input vector $\mathbf{u} \in \mathbb{R}^m$ inputs and disturbances $\mathbf{d} \in \mathbb{R}^l$.

$$\mathbf{x}(k+1) = \mathbf{Fm}\left(\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k)\right) \tag{12}$$

where $\mathbf{F} \in \mathbb{R}^{n \times 2^{n+m+l}}$ is the matrix containing the parameters $(f_{11}, .., f_{n2^{n+m+l}})$ of the MTI state space model. $\mathbf{m}(\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k))$ is the monomial vector as already introduced in section 3.

Assuming that the states of the MTI state space model in (12) can be measured, then the parameter estimation problem can be formulated as, given an initial state $\mathbf{x}(0)$, a set of inputs $\mathbf{u}(k)$, set of disturbance inputs $\mathbf{d}(k)$ and a set of real measured data $\tilde{\mathbf{x}}(k)$ where $k = 0, 1, .., T$, find the parameter matrix $\mathbf{F}$ containing the parameters $(f_{11}, .., f_{n2^{n+m+l}})$ in the sense that the sum J of squared errors $\mathbf{e}$ is minimum.

$$\min_{\mathbf{F}} J \tag{13}$$

$$J = \frac{1}{2}(\mathbf{e}^T \mathbf{Q} \mathbf{e}) \tag{14}$$

where $\mathbf{Q} \in \mathbb{R}^{nT \times nT}$ is a diagonal, positive semi definite weighting matrix for normalization of the measured signals.

$$\mathbf{Q} = diag(q_1, q_2, .., q_{nT})$$

$$\mathbf{e} = \begin{bmatrix} \mathbf{x}(1) - \tilde{\mathbf{x}}(1) \\ \vdots \\ \mathbf{x}(T) - \tilde{\mathbf{x}}(T) \end{bmatrix} \tag{15}$$

For $i = 1..T$, the states $\mathbf{x}(i)$ are the one step ahead predictions from the model (12) using the measured states $\tilde{\mathbf{x}}(i-1)$, the inputs $\mathbf{u}(i-1)$ and the disturbances $\mathbf{d}(i-1)$ in the previous time step. The objective is to find the parameters set that minimizes the error between the measured state $\tilde{\mathbf{x}}(i)$ and predicted state $\mathbf{x}(i)$ from the MTI model in (12). The assumption is that the parameters are constants and not varying with time. The parameter matrix can be determined in a straight forward way by simply solving an inverse problem. Considering the MTI state space model in matrix form introduced in the equation (12). Given the data-set $(\tilde{\mathbf{x}}(k), \mathbf{u}(k), \mathbf{d}(k)$ for all $k = 0, 1, .., T)$, for a model order n, with m inputs and l disturbances

$$\tilde{\mathbf{X}} = \mathbf{F} \cdot \mathbf{M} \tag{16}$$

where $\tilde{\mathbf{X}} \in \mathbb{R}^{n \times T}$, $\mathbf{F} \in \mathbb{R}^{n \times 2^{n+m+l}}$ and $\mathbf{M} \in \mathbb{R}^{2^{n+m+l} \times T}$.

$$\tilde{\mathbf{X}}^T = \begin{pmatrix} \tilde{\mathbf{x}}(1) \\ \tilde{\mathbf{x}}(2) \\ \vdots \\ \tilde{\mathbf{x}}(T) \end{pmatrix} \tag{17}$$

$$\mathbf{M}^T = \begin{pmatrix} \mathbf{m}(\mathbf{x}(0), \mathbf{u}(0), \mathbf{d}(0)) \\ \mathbf{m}(\mathbf{x}(1), \mathbf{u}(1), \mathbf{d}(1)) \\ \vdots \\ \mathbf{m}(\mathbf{x}(T-1), \mathbf{u}(T-1), \mathbf{d}(T-1)) \end{pmatrix} \tag{18}$$

Assuming T is large enough such that the given system of equations is over-determined, the solution is:

$$\mathbf{F} = \tilde{\mathbf{X}} \cdot \mathbf{M}^{-1} \tag{19}$$

Solving the equation (19) will also minimize the sum of squared errors as mentioned in (13).

4.2 Black Box Identification

For this case, no prior knowledge about the system is available from the laws of physics. Thereby, the problem now is to find a state space model of order n, from a given a input-state data set, instead of finding the unknown physical parameters of the system.

We solve the problem in two ways. First, the parameter matrix $\mathbf{F}$ is calculated by solving an inverse problem. Thereafter, as shown in the section 3, MTI state space model can be written in tensor form as well and hence, the problem is solved by calculating the CP factors of the parameter tensor F.

4.2.1 Inverse Problem

The inverse problem approach is similar to the solution discussed for the grey box problem in the equation (19). It follows from the solution in (19) that, for a solution to exist, $\mathbf{M}\mathbf{M}^T$ must be non-singular and thereby, full rank. For this to have full rank, all the dynamic modes of the system must be excited. Since the only set of influence-able variables of the T sample monomial matrix are the inputs, they must be rich enough to excite all the modes of the system. This is the well known concept of persistence excitation (Ljung, 1999).

4.2.2 Estimation of Tensor Factors

In (7), it is shown how an MTI state space model can be written in tensor form and thereafter, in (8) the CP decomposed form of the tensor model is shown. The objective here is to estimate the CP factors of the parameter tensor directly from the measurements. Assuming a rank r CPD of the parameter tensor F such that

$$\mathsf{F} = [\mathbf{F}_{u_m}, .., \mathbf{F}_{u_1}, \mathbf{F}_{x_n}, .., \mathbf{F}_{x_1}, \mathbf{F}_{\Phi}] \cdot \lambda_F \qquad (20)$$

where all of $\mathbf{F}_{u_m}, .., \mathbf{F}_{u_1}, \mathbf{F}_{x_n}, .., \mathbf{F}_{x_1} \in \mathbb{R}^{2\times r}$, $\mathbf{F}_{\Phi} \in \mathbb{R}^{n\times r}$ and $\lambda_F \in \mathbb{R}^r$.

The one step ahead predicted states $\mathbf{x}(k)$ as shown in the error vector in (15) can be calculated from (9). Using the error vector in (15), the least squares optimization problem in (13) can be solved for the unknown factors of the parameter tensor. The optimization problem is:

$$\min_{\mathbf{F}_{u_m}, .., \mathbf{F}_{u_1}, \mathbf{F}_{x_n}, .., \mathbf{F}_{x_1}, \mathbf{F}_{\Phi}, \lambda_F} J \qquad (21)$$

The problem (21) turns out to be a nonlinear optimization problem. Solutions to such problems can be found by using the nonlinear optimization solvers in the simulation environment of MATLAB Simulink. An important variable in the given problem is the assumed rank r of the CPD. The more the multilinear terms present to calculate the state in the next time step, the denser the parameter tensor becomes and thereby, the CP rank of the parameter tensor also increases. Hence, while choosing the rank of the CPD one must consider that there is a trade off between the savings in storage effort and the accuracy of the decomposed tensor with respect to the original tensor.

5 APPLICATION

In this section, the simulation results of the MTI state space model identification are presented. The identification will be performed for the heating system of an industrial building.

5.1 Heating System

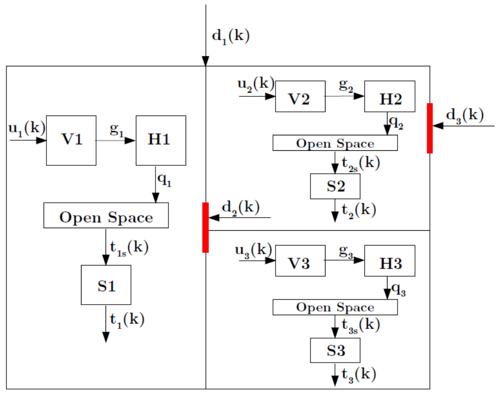

Figure 2: Heating system.

In figure 2, the schematic diagram of the heating system of three heat zones of an industrial building is clearly shown. Each heat zone has one heating device which is a gas heater denoted by Hi, i=1..3. Depending on the input $u_i(k) \in \mathbb{E}, i = 1..3$, where $\mathbb{E} \in \{0,1,2\}$ to the valves Vi, i=1..3, a volume of gas $g_i, i = 1..3$ goes into the gas heater. The gas heater emits heat flow $q_i, i = 1..3$, depending on the gas inlet. Naturally, the heat flow causes changes in the temperature $t_i(k) \in \mathbb{R}, i = 1..3$ of the zone. The absolute temperature of each the zones is measured by the respective sensors Si, i=1..3. Additionally, there are three external influences on the heating system as well. Two of the three influences ($d_2(k)$ and $d_3(k)$) are binary signals corresponding to the opening and closing of the doors and one of the doors ($d_2(k)$) is in between the zones and the other between the zone and the open area outside the industrial building, both the doors have been marked in red in the schematic. The $d_1(k)$ influence is the external temperature. The multilinearity in the model comes through the presence of these external disturbances. The equations describing the temperatures $t_1(k+1)$, $t_2(k+1)$ and $t_3(k+1)$ of the three zones in the next time step are

$$\begin{aligned} t_1(k+1) = t_1(k) &+ p_{11}u_1(k) \\ &- p_{12}(t_1(k) - d_1(k)) \\ &- p_{13}d_2(k)(t_1(k) - t_2(k)) \end{aligned} \qquad (22)$$

$$\begin{aligned} t_2(k+1) = t_2(k) &+ p_{21}u_2(k) \\ &- p_{22}(t_2(k) - d_1(k)) \\ &- p_{23}d_2(k)(t_2(k) - t_1(k)) \\ &- p_{24}d_3(k)(t_2(k) - d_1(k)) \end{aligned} \qquad (23)$$

$$\begin{aligned} t_3(k+1) = & \; t_3(k) + p_{31} u_3(k) \\ & - p_{32}(t_3(k) - d_1(k)) \\ & - p_{33} d_2(k)(t_3(k) - t_1(k)) \end{aligned} \tag{24}$$

The constants $p_{i1} \forall (i = 1..3)$ are the heat flow constants corresponding to the heating devices in the zone i. The $p_{i2} \forall (i = 1..3)$ constants are the heat flow constants between the respective zone and the outside. The constants $p_{i3} \forall (i = 1..3)$ are corresponding to the heat flow between zones when the door between them is open. Furthermore, the constant p_{24} is the heat flow constant between the second zone and the outside when the door is open.

For the given heating system, the identification problem will be solved by three approaches. Firstly, the grey box identification problem is tackled. The MTI state space model can be deduced from the equations in (22), (23) and (24). Hence, the grey box identification problem is to identify the unknown parameters $p_{i1}, p_{i2}, p_{i3} \forall (i = 1..3)$ and p_{24} from the given input-state data set. From the equations (22), (23), (24) we can write the model in the following form

$$\mathbf{t}(k+1) = \hat{\mathbf{F}} \hat{\mathbf{m}}(\mathbf{t}(k), \mathbf{u}(k), \mathbf{d}(k)) \tag{25}$$

where $\mathbf{t}(k) \in \mathbb{R}^3$ is the state vector comprising of the the temperatures of the respective zones

$$\mathbf{t}(k) = \begin{pmatrix} t_1(k) \\ t_2(k) \\ t_3(k) \end{pmatrix}$$

$\mathbf{u}(k) \in \mathbb{E}^3$ is the input vector

$$\mathbf{u}(k) = \begin{pmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{pmatrix}$$

and $\mathbf{d}(k) \in \mathbb{B}^2 \cup \mathbb{R}$ is the disturbance inputs vector

$$\mathbf{d}(k) = \begin{pmatrix} d_1(k) \\ d_2(k) \\ d_3(k) \end{pmatrix}$$

$\hat{\mathbf{F}} \in \mathbb{R}^{3 \times 12}$ contains the unknown parameters corresponding to the terms in the monomial vector $\hat{\mathbf{m}} \in \mathbb{R}^{12}$.

The second approach is the black box identification of the MTI state space model for the heating system. It is assumed that a model of order three is to be identified. Assuming that no information is available about the heating system, the task is to find the parameter matrix $\mathbf{F} \in \mathbb{R}^{3 \times 512}$. This objective can be achieved by solving an inverse problem as mentioned in the previous section.

The final approach is the estimation of a low rank approximation of the parameter tensor F, by finding the CP factors of the parameter tensor directly from the measurements (virtual data).

Due to unavoidable circumstances, real measurement data from the building is not available in this moment. Hence, to generate the temperature state data for solving the identification problem using all methods, a Simulink model is constructed with the help of information in equations (22), (23) and (24). The unknown parameters are set to certain values. Thereafter, the model constructed in Simulink is simulated to obtain the temperature response. The data is collected for one whole day and the sample time of the model is one minute. Thereby, in total $T = 1440$ samples are collected. The signals in the heating system (u_1, u_3, u_3, d_1, d_2, d_3 and measurement noise) are generated in the following way:

- u_1, u_2 and u_3: Random integers between 0 and 2.
- d_1 (external temperature): Temperatures for the day 30.03.2020 in Hamburg, Germany have been used. This day in particular is chosen, because substantial variations in temperature could be observed. The maximum and minimum recorded temperatures on this day were 8.25 and -3.46 degree celsius respectively. The JEVis platform (Palensky, 2003) was used to obtain the data, which in turn uses the data from Germany's National Meteorological Service to obtain the temperatures. The data is available only in 15 minute intervals in JEVis. Since the sample time of the model is one minute, it is assumed that the temperature is constant for every minute of the 15 minute time interval.
- d_2 (door between two zones): It is assumed that between 9:00 and 17:00 (core working hours) on the particular day, the door is opened and closed randomly by the workers. Hence, between the mentioned time interval, random integers between 0 and 1 are generated.
- d_3 (door between zone and outside open area): It is assumed that between 12:00 and 14:00 on the particular day, the door has to remain open for loading of the manufactured goods. Hence, between the mentioned time interval, $d_3 = 1$, otherwise $d_3 = 0$.
- Noise: The band limited white noise block is used to add measurement noise to the temperatures. A noise power of 100 is chosen.

5.2 Identification Results

The simulated responses in figures 3, 4, 5 and 6 show the results of all the approaches to identification discussed so far. The validation of the respective identification approaches are presented by displaying the plots of the temperatures generated through the Simulink model and the temperatures predicted by the identified model. Under the approach of estimating the low rank approximation of the parameter tensor in figures 5 and 6, two cases are considered. Case one with $r = 1$ and second case with $r = 10$.

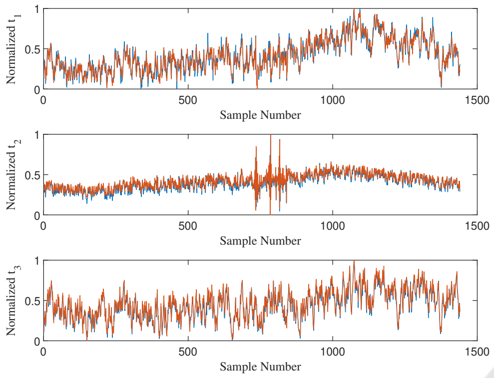

Figure 3: Grey box: measured (red) vs estimated (blue).

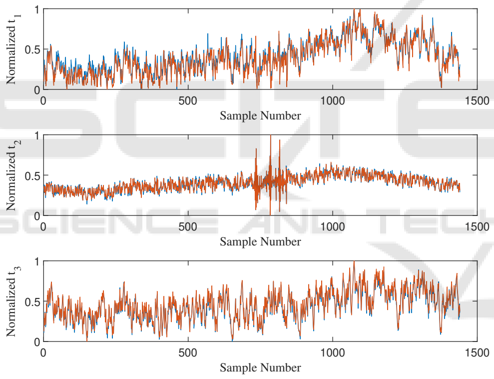

Figure 4: Black box: measured (red) vs estimated (blue).

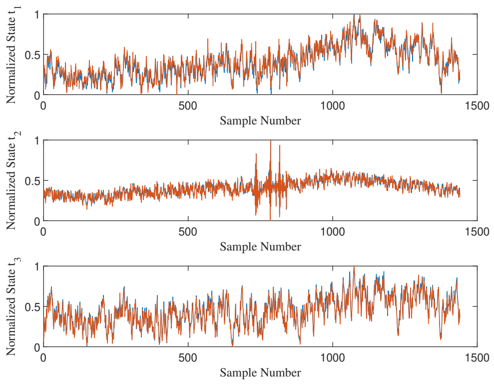

Figure 5: Black box rank 10 approximation: measured (red) vs estimated (blue).

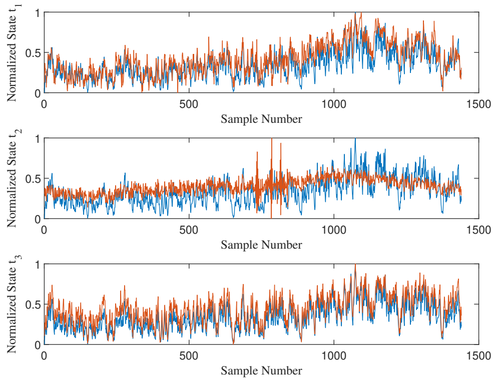

Figure 6: Black box rank 1 approximation: measured (red) vs estimated (blue).

From the figures 3, 4 and 5, it is clear that the temperatures predicted by the identified models and the temperatures generated through the Simulink model are in agreement. A similar accuracy in identification is not seen in figure 6. This means that the parameter tensor of the model to be identified has a CP rank substantially higher than 1. Therefore, a rank 1 CP parameter tensor does not adequately represent the model. The figures 5 and 6 also illustrate the trade off that is storage effort ($r = 1$ has 22 elements and $r = 10$ has 220 elements) and accuracy of the decomposed tensor with respect to the original parameter tensor.

6 CONCLUSIONS

The paper presents the system identification of an MTI state space model of a given order using both the grey and black box identification methods. Within the black box method, two different approaches are presented. The first approach focuses on solving an inverse problem. The result of this approach is a full parameter matrix of the state space model. The second approach focuses on finding a low rank approximation of the parameter tensor directly from the measurements.

A numerical example of identification of MTI state space model for a heating system of an industrial building was also presented where, state space models of order three were identified for the corresponding input-output data sets. Real measurement data was not available, therefore, virtual data was generated by simulating a known model of the heating system in MATLAB Simulink. The data was collected for a whole day, with signals as close to reality as possible.

The future work should be towards making the estimation process more robust to noises. Furthermore, identification could be carried out for larger systems with real measurement data.

ACKNOWLEDGEMENTS

This work was partly supported by the project SIOSTA of the Federal Ministry of Education and Research, Germany (Grant-No.: 01LY1812B).

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