A Fuzzy Cognitive Map Approach to Investigate the Sustainability of the Social Security System in Jordan

Keywords: Fuzzy Cognitive Maps, Time Series Prediction, Economic Modelling, Social Security System.

Abstract: Fuzzy Cognitive Maps are emerging as an important new tool in economic modelling. The aim of this study

is to investigates the use of fuzzy cognitive maps with their learning algorithms, based on genetic algorithms, for the purposes of prediction of economic sustainability. A Case study data are extracted from the Jordanian Social Security system for the last 120 months; The Real-Code genetic algorithm and structure optimization algorithm were chosen for their ability to select the most significant relationships between the concepts and to predict future development of the Jordanian social security revenues and expenses. The study shows that fuzzy cognitive maps models clearly predict the future of a complex financial system with incoming and outgoing flows. Therefore, this research confirms the benefits of fuzzy cognitive maps applications as a tool

for scholarly researchers, economists and policy makers.

1 INTRODUCTION

The Social Security is a package of social insurances, where each insurance defines and meets the citizens' needs in accordance with a legislation outlining the obligations and rights, and sets up a balance between them. Accordingly, Social Security is a general insurance symbiotic system aims to protect people socially and economically where the law defines its benefits and funding sources; the government, through institutions or bodies established under this system, achieves these benefits in case of any social risk such as old-age, disability, death, work injuries, and unemployment. Such benefits are financed by contributions paid by insured persons and employers. This system is interested in the achievement of social competency considerations.

Social security systems all over the world face financial problems. This phenomenon has a number of reasons like underfinancing, unclear and too lavish rules to benefit from them and insufficient control. Pension systems more specifically also suffer from a growing number of beneficiaries giving the increased life expectancy in many countries due to better medical conditions and improved life styles. As a

result, governments struggle with the development of social security systems, more specifically pension systems that are sustainable over a longer period.

The construct of an optimal pension system should be based on data available. However, there are limited research addressing the issue of sustainable social security and pension systems. The data available on any of these social security systems are vast, relatively complicated and cover many different variables like age, gender, family composition, average of salary, years of contribution, eventual reduction systems and so on. As a result, scientific analysis of the sustainability of a social security system, in particular a pension system needs very specific data analysis methods.

Fuzzy Cognitive Maps (FCMs) have been developed as a knowledge-based tool to model and analyze complex systems using causal relations (Kosko, 1986). From the structural perspective, an FCM can be defined as a fuzzy digraph that describes the underlying behavior of an intelligent system in terms of concepts (objects, states, variables or entities) and causal relations. Essentially, FCMs are a kind of recurrent neural networks that support backward connections that sometimes form cycles in

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the causal graph (Tsadiras, 2008). This implies that concepts in the information network can be understood as neural processing entities with inference capabilities

The objectives of this research is to investigate the sustainability of the Jordanian Social Security (JSS) system, as a case study, using FCMs with time series as a predictive method. This will be achieved by predicting the revenues and expense using real time series data extracted from the JSS system.

2 LITERATURE REVIEW AND RESEARCH MOTIVATION

Among earlier studies on the field of retirement is the study of Sala and Xavier (1996), the study considered the theory of social security. The author argued that programs of social security around the world connect public pensions to retirement. For example, individuals do not lose their pensions if they make a million dollars a year in the stock market, but they do face marginal tax rates of up to 100 % if they decide to work. In that context, the study constructed a positive theory that is compatible to resolve such issues. Furthermore, the study concluded that when the difference between the level of skill of the young and that of the old is big enough, total output in an economy where the old-aged do not work is higher. Thomas and Soares (1999), analysed the welfare implications of the social security system and compared overall equilibrium measures of welfare to the commonly used notion of actuarial fairness. The study revealed that social security has a considerable impact in the extent of the capital stock and the rate of return to private saving.

In many cases some retired people have to get back to the Labour field, that depends on many reasons among which the work on the informal work, which does not provide the pension according to the conditions of work (ILO, 2014). In support of this issue, the International Labour Organization (ILO) (2014) published a report "Social protection for older persons: Key policy trends and statistics". The report, included 178 (including Jordan) countries, revealed that in many countries with high percent of informal employment, only minority of people can access pensions, and many older people can count only on family support. The report also concluded that, nearly half (48 %) of all people over pensionable age do not obtain a pension. Many of those who obtains a pension, the level of income is not adequate. Thus, the majority of the world's older people have no income security, have no right to retire and have to work as long as they are able to and often badly paid and in insecure conditions.

In the context of Jordan, Alhawarin (2014) studied the issue of getting back to the Labour field. The author collected data from about 5000 households containing about 6000 individuals. The study revealed that nearly 85% of male retirees had retired early, and about 45% of them got back to the labour market and took jobs, which were characterized, to some extent by informality. Economically active early retired persons; however, seem to suffer from high unemployment rates, particularly those who retire from the private sector. The results of analysis showed that individuals retired from the Jordanian armed forces were more probably to retire early and to get back to labor market. Whereas the household wealth appears to affect the probability of early retirement, family size has a positive effect. Mohammed and Najim (2014) conducted a study aimed at identifying the influence of the services provided by the Public Institutions for JSS at the level of satisfaction on the work of the institution in the city of Amman. The study found that the level of services of public institutions and the level of satisfaction for Social Security. The outcomes also found a statistically significant effect of the services on the level of satisfaction with the work of the institution. In their study, Enoff and McKinnon (2011) aimed to promote sharing of knowledge and good practice on contribution collection and the compulsion of compliance. The data proposes that seven core factors unite often to form the basis of success in contribution collection and consent. Additionally the improvement of benefit adequacy and public standing of programs and the financial health, such success may support both national and international efforts to expand social protection coverage.

Early retirement is an important factor that affects the sustainability of social security systems. Fenge and Pestieau (2005) investigated theoretical and empirical evidence, which explains why early retirement became such a burden for systems of social security and suggested pension system reforms, which will reverse the trend. The study used evidence from the European Union (compared with other industrialized countries including Canada and the United States). The study showed that the effective retirement age is impacted by social security regulations (as a change in eligibility age) and discussed ways of measuring those embedded stimulus. Furthermore, pushing older workers to retire does not free jobs for young unemployed individuals. Finally, the study concluded that the gap

between productivity and salaries is a stimulus for employers to release themselves from older workers and claim that governments must not support this behavior by recompensing the elderly workers for the difference between salaries and severance payments in early retirement programs.

In their study, Imrohoroğlu and Kitao (2012), aimed to assess the efficacy of proposed Social Security reforms and its influence on sustainability of the US pension system. It was revealed that Social Security reform to raise retirement ages could have a large impact on the sustainability of the US pension system through changes in life-cycle savings and labor supply. In particular, labor force participation of older workers and their benefit take-up behavior react strongly to certain reforms. The study revealed that an increase in the earliest retirement age would not have any significant effect on the budget of the Social Security system since the benefits will be permanently raised by forcing individuals to postpone retirement. The study suggested that policies that encourage the participation and work effort of older workers as well as individuals' own saving for retirement could help enhance the sustainability of the

A study conducted on Middle East and North African (MENA) countries addressed the question of removing deficits about equity, efficiency, and financial sustainability and how the prospects are for more profound reforms in the future. The study revealed that the countries of MENA have hardly conducted any noteworthy pension reforms in the past and claim that this reluctance is because of political considerations of the ruling regimes and to the fact that most countries of MENA until now were able to finance the deficits of their pension schemes (Loewe, 2014). It was concluded, "The prospects of reforms that go beyond simple changes in contribution rates or pension formulas remain bleak."

Based on the above discussions, it is important to note that most of the research conducted to investigate the sustainability of social security systems are based on questionnaires and statistically analyze the collected data. Few researches used real time series data to address the concern specifically FCMs. The main advantage in the analysis of the use of FCM's is they incorporate learning algorithms, which time series do not.

In recent years, the use of FCMs in time series forecasting has been noticeable due to the transparency of FCM-based models. For example, the approaches proposed in Pedrycz (2010), Pedrycz et al. (2016), Lu et al. (2014), and Froelich and

Pedrycz (2017) rely on fuzzy information granules to forecast the time series with high accuracy. The values of the learning part of the time series are clustered, where the number of clusters is a user-defined parameter, using the fuzzy *c*-means algorithm (Bezdek et al., 1984). At each time iteration, the value of the time series fits to each cluster with some membership degree with

presumption that every cluster (i.e., concept) plays the role of a fuzzy set. On the other hand, other algorithms proposed a low-level approach where neurons represent attributes instead of comprising information granules (Froelich and Salmeron, 2014; Poczęta and Yastrebov, 2015; Papageorgiou, et al., 2015; Salmeron and Froelich, 2016). Nevertheless, these methods suffer from the same drawback that is; there is no guarantee that the produced weight set encompasses a realistic interpretation for the system. Even though the model is able to achieve good prediction rates. Consequently, this implies that the modelled system cannot be interpreted, although the FCM reasoning is still transparent.

In this paper, we investigate this complex issue using a real case study data extracted from the Jordanian Social Security System (JSS). More precisely, the contribution of this research is twofold. On the one hand, a FCM-based system is developed to forecast the social security revenues and expenses in Jordan. This will allow experts to forecast the revenues in next years and understand the underlying behaviour behind such predictions. On the other hand, to investigate the trade-off between interpretability and accuracy of FCMs, as we include expert knowledge to the system. To achieve this first, an initial FCM will be developed without any restrictions, and second, a FCM model will be developed using knowledge coming from experts.

3 FUZZY COGNITIVE MAPS

Cognitive mapping has become a convenient knowledge-based tool for modelling and simulation (Kosko, 1986). FCMs can be thought of as recurrent neural networks with learning capabilities, consisting of concepts and weighted relations among them. Concepts denote entities or variables, which are equivalent to neurons in neural network models. While weights associated to connections denote the causality among such nodes or concepts. Each link takes values in the range [-1, 1], denoting the causality degree between two concepts because of the quantification of a fuzzy linguistic variable, which is often assigned by experts during the

modeling phase (Nápoles, et al., 2016). The activation value of neurons is also fuzzy in nature and regularly takes values in the range [0, 1]. The higher the activation value of a neuron, the stronger its influence over the investigated system, offering decision-makers an overall picture of the systems behaviour.

Mathematically FCMs are used to demonstrate and to model the knowledge on the examining system in terms of concepts. It can be defined by using a 4tuple (C, W, A, f) where $C = \{C1, C2, ..., CM\}$ denotes a set of M neural processing entities, W: (Ci, $(Ci) \rightarrow wij$ is a function that associates a causal weight $wij \in [-1,1]$ to each pair of neurons (Ci, Cl). Similarly, A: $(Ci) \rightarrow Ai$ is a function that associates the activation degree $Ai \in \mathbb{R}$ to the Ci neuron at each iteration-step moment t (t = 1,2, ..., T). Finally, a transformation function $f: \mathbb{R} \to [0,1]$ is used to keep the neurons' activation value in the allowed interval. Equation (1) portrays the inference mechanism attached to an FCM-based system, using the (0) vector as the initial activation. This neural procedure is repeated until either a fixed-point attractor is discovered or a maximal number of iterations is reached.

$$A_{i}^{(t+1)} = f\left(\sum_{j=1}^{M} w_{ji} A_{j}^{(t)} + w_{ii} A_{i}^{(t)}\right), i \neq j$$
 (1)

The three most widely used threshold functions are the bivalent function, the trivalent function, and the sigmoid variants. Bivalent or trivalent are discrete FCMs, which cannot properly represent the degree of an increase or a decrease of a concept. Nevertheless, since discrete FCM are deterministic models they can always converge to a fixed-point attractor or limit cycle. Thus, the number of distinct states is finite. Continuous FCM (e.g. sigmoid FCM) on the other hand are able to simulate numerical changes of the activation value of concepts. They are consequently recommended for both qualitative and quantitative scenarios, as their prediction capability is much higher than discrete FCM (Nápoles, et. al, 2016). The increase of the numerical precision of predictions however may also lead to fully chaotic behavior offering no guarantee of convergence. Furthermore, as reported by Bueno and Salmeron (2009), the results revealed that the sigmoid function outperformed other functions using the same decision model. Consequently, the threshold function is a crucial issue for the system behaviour and performance. Therefore, in this research we will focus on Sigmoid FCMs.

FCMs can be constructed either using the knowledge coming from domain experts or using a learning method. In the next sub-section, an evolutionary procedure to derive the network structure in a supervised fashion is described.

3.1 Fuzzy Cognitive Maps Learning

The literature indicates that, when forecasting using time series, the use of the appropriate regression is of the highest importance. Likewise, the use of the right concepts in the FCM will influence the forecasting accuracy of the models (Salmeron and Froelich, 2016). This signifies that the weight matrix constructed in the FCM will be of the highest importance when obtaining forecasting accuracy. The learning process of the FCM itself or the opinion of experts will be the most important factor in this regard, resulting in an optimal weight matrix. The reason is that FCMs derive their strength from their simplicity. Thus, an expert opinion has to be present to predict accurately.

Assume that $Z(t) = [Z_1(t), Z_2(t), ..., Z_N(t)]$ is the desired system response for the (t-1) activation vector, $A(t) = [A_1(t), A_2(t), ..., A_N(t)]$ is the FCM output for the (t-1) initial vector, while T is the number of the learning records. Equation (2) displays an error function used in the context of time series forecasting, where W represents the candidate weight matrix, N is the number of neurons, while t indexes the iteration steps (i.e., the learning records). In short, this learning scheme attempt minimizing the dissimilarity between the expected outputs and the predicted ones.

$$E(W) = \frac{1}{(T-1)n} \sum_{t=1}^{T-1} \sum_{i=1}^{n} (Z_i(t) - X_i(t))^2$$
 (2)

In this supervised learning model, a continuous search method (i.e., Particle Swarm Optimization, Genetic Algorithms) generates the weights matrices to be evaluated by the algorithm. Equation (3) shows the structure of the weight set.

$$W' = \left[W_{1,2}, \dots, W_{1,n}, W_{2,1}, \dots, W_{2,n}, \dots, W_{n,n-1}\right]^T \enskip (3)$$

In this research, we adopt the Real-Coded Genetic Algorithm (RCGA) as standard continuous optimizer. The RCGA is an evolutionary search method that codifies genes directly as real numbers and can be used to optimize parametrical problems for continuous variables (Herrera et. al, 1998). Therefore, each chromosome involves a vector of

floating point numbers that involves a candidate solution. The size of the chromosomes is identical to the length of the vector, which is the solution to the optimization problem. In this way, each gene represents a variable of the problem, i.e. a weight component. Genetic operators only have to observe the fact that the values of the genes remain within the interval established by the variables they represent.

Each chromosome in the population is evaluated based on a fitness function according to the error function. Because Genetic Algorithms are frequently expressed like maximization-type problems, the previous error function is expressed in terms of a fitness function, which is formalizes as follows:

$$F(W) = \frac{1}{a * E(W) + 1} \tag{4}$$

Where $\alpha > 0$ is a user-specified parameter, W is the candidate weight set computed by the RCGA optimizer, while (W) is the error function. During the optimization, parents are selected and new population of chromosomes are generated with some probability. In this research, we use the well-known roulette wheel method as standard selection operator.

4 SUSTAINABILITY OF SOCIAL SECURITY SYSTEM IN JORDAN

The aim of this research is focused on predicting the revenue and expense values and understanding the underlying interrelation between concepts; the latter is the main motivation to use cognitive mapping models. To achieve that, we introduce a case study case of forecasting social security revenues and expenses in Jordan. The dataset used for analysis are extracted from the JSS system, related to revenues and expense in Jordan for the period of 120 months (from 2006 until 2015). Each data set contains a set of variable that contribute to the overall value (i.e. revenues or expenses). The data set variables (map concepts) for both revenues and expenses are listed in Table 1.

Table 1: Revenues and Expenses Datasets.

Dataset	Variables	
Revenues	C1: Aging subscription C2: Work related injuries C3: Maternity insurance C4: Years earlier service	
	C5: Optional Subscriptions C6: Miscellaneous revenue C7:Stamps	
Expenses	C1: Pensions C2: One time	
	C3: Work injuries C4: Maternity insurance	

Table 2 reports descriptive statistics about the aggregated revenues and expenses and Figure 1 below depicts the trend of the aggregated revenues and expenses. Over the past 120 months, the revenues were higher than expenses, which implies that the JSS is financially sustainable. Furthermore, the gap between the trend lines of the revenues and expenses has increased in recent months.

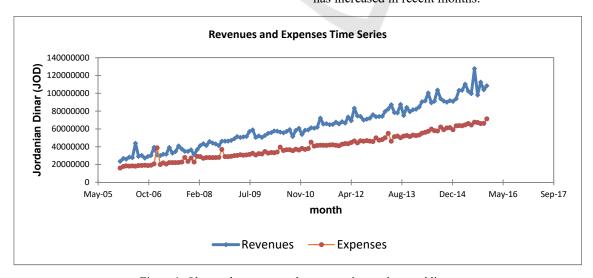


Figure 1: Observed revenues and expenses time series trend lines.

Table 2: Revenues and expenses descriptive statistics.

Variable	Revenues	Expenses
Observations	120	120
missing data	0	0
Minimum	23579730	16040461
Maximum	127734545	71242897
Mean	62829093	39864540
Std. deviation	23903670	14576539

In order to activate the neurons with values inside the activation interval the data were normalized to the [0,1] range using the min-max normalization method. Furthermore, the datasets were randomly divided into two disjoint subsets: the learning set (95%) and the test set (5%). Aiming at evaluating the quality of the FCM-base forecasting model, we adopt the Mean squared Error (MSE).

Due to the stochastic nature of the heuristic search methods, we perform 20 trials and average the quality measures. Moreover, we use the following parametric setting in all the simulations: 50 chromosomes as in the artificial population, 200 generations, the α parameter is set to 10, the crossover probability is set to 0.8, and the mutation probability is set to 0.1, while the Fmax parameter is set to 0.999. In addition, the models will be experimented using two different configurations. In the first one, genes can take values in the [-1, 1] range, and this will be referred to as the unrestricted model. Whereas, in the second configuration we include expert knowledge related to the sign of causal weights which be referred to as the restricted model. For example, considering the revenues, the longer an individual is subscribed with age (C1: Aging subscription) the value of the optional subscription (C5: Optional Subscriptions) will be increased, which implies a positive casualty between these two concepts. On the other hand, it is obvious that the fewer the years an individual is subscribed the lower

his optional subscription (i.e. there is a negative casualty relation between C4: year's early service and C5: Optional subscription.

It is important to note that the goal is not to increase performance but to preserve the coherence in the causal cognitive network. Weights are freely estimated in the [-1, 1] range which may actually produce improved prediction rates due to the attached freedom degree. However, there is no way to ensure that weights produced by a heuristic search method contains a causal meaning for the problem under investigation. In the next sub-section, the results of

the FCM analysis for both configurations are presented.

4.1 Analysis of the Results

Table 3 depicts the MSE error rates for the revenues and expenses models. The average MSE of the test set of the revenues model for the unrestricted model are 0.0431, while for the restricted model, the reported MSE is 0.1992. As for the expenses model, the unrestricted model reported lower MSE measure as well. As a result, one can note that promoting the FCM interpretability leads to higher forecasting errors. Thus, the question that arises here is that why when attempting to promote the interpretability of the FCMs leads to higher forecasting error, since in principle, a neuron naturally produces a times series?

Table 3: Error rates of the revenues and expenses models (restricted and unrestricted models).

Dataset	Configuration (Model type)	MSE
Revenues -	Unrestricted Model	0.0431
	Restricted Model	0.1992
Expenses	Unrestricted Model	0.026
	Restricted Model	0.064

To answer this question, we must take note of the convergence properties of the causal network. In most FCM-based systems, ensuring convergence is mandatory, otherwise making reliable decisions is not possible (Nápoles et. al, 2016; Homenda et. al, 2014). Nevertheless, in the time series context, convergence is not desirable since it decreases the network's capability of computing both short and long-term predictions. Consequently, producing truly interpretable casual cognitive models may be produced on the expense of performance. This is the price we pay to produce truly interpretable causal cognitive models. Nevertheless, whether such forecasted values are acceptable for this real-world problem is questionable.

Figures 2 and 3 shows the actual and fitted values for the unrestricted and restricted models of revenues and expenses respectively. The horizontal axis represents time steps, while the vertical axis shows the activation values. The results depicted in the figures confirms our hypothesis, the convergence properties of the FCM-based system affect the

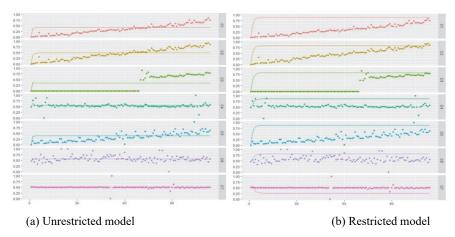


Figure 2: Actual versus forecasted revenues values using the using the current activation values to predict the next time series point.

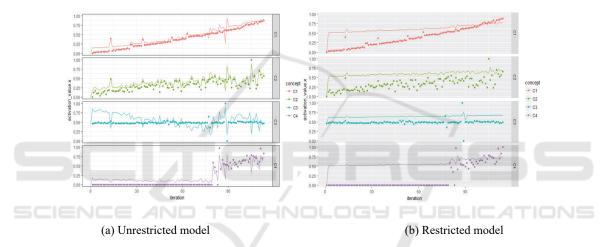


Figure 3: Actual versus forecasted expense values using the using the current activation values to predict the next time series point.

quality of forecasted values for both revenues and expenses. Forecasting error increases because of decreasing the degree of freedom. This can be shown in the forecasted values line trend for both models (revenues and expenses) noticeably deviates from the actual values.

At the aggregated level of revenues and expenses, Figure 4 shows the actual and forecasted models (C1 represents revenues and C2 represents expenses). The trend lines for the forecasted revenues and expenses (straight line) follows the same trend in the actual values. Furthermore, the revenues are higher than expenses with the gap between them increases with time. Thus, the JSS is financially sustainable.

5 CONCLUSIONS

In this paper, we proposed an FCM-based time series-forecasting model to investigate the factors affecting revenues and expenses of the social security in Jordan. The proposed model uses a *RealCode Genetic Algorithm* to learn the map structure together with well-known methods for FCM learning. As a first step, we allowed the algorithm to indicate a relationship value between the neurons to fluctuate between -1 and 1 without constraints, which was referred to the unrestricted model. However, the model occasionally computed a positive relation where it should be a negative one and vice versa, therefore producing a forecasting model that does not have a coherent meaning for this real-world problem.

In order to produce meaningful weights, we included domain knowledge related to the sign of each causal relation. The numerical simulations have shown that using the current value to forecast the revenue values leads to higher error rates since the model converges to an equilibrium attractor.

Furthermore, when using FCMs, it is key to promote the network's transparency, otherwise the model will behave like a black box and as a result, there is no reason to employ other (perhaps more accurate) forecasting models. In the context of the Jordanian Social Security financial sustainability, the resulting models predicted that with time, the revenues would still be higher than expenses. Future research will be focused on increasing the forecasting accuracy rates while retaining the network capability.

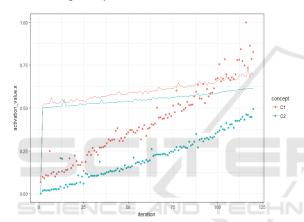


Figure 4: Actual and forecasted revenues and expenses.

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