

Linear Model Adjustment and Approximate Approach for Creating Minimal Overhead Wires Network for Vehicle Schedules

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Abstract: Nowadays, there is a significant effort in reducing the environmental impact caused by public transport. This goal can be achieved in many ways. One possible way is in using battery-assisted trolleybuses in cities. Therefore this paper deals with the question of how to create a minimum overhead contact network for such vehicles operation. The article presents the mathematical model of such a problem and tests the impact of the version with the modified condition. It also proposes a suboptimal way to propose needed wire network for operation selected vehicle schedules using individual routes. As benchmarks, some vehicle schedules in Zilina were selected.

1 INTRODUCTION

The European Union is currently calling for the reduction of the environmental impact caused by carbon-dioxide emissions. It can be assumed that the trend of implementation of battery-assisted trolleybuses in cities will grow soon. The technology of battery-assisted trolleybus combines the benefits of classical trolleybus with movement freedom of standard buses. The basic idea is illustrated in Figure 1.

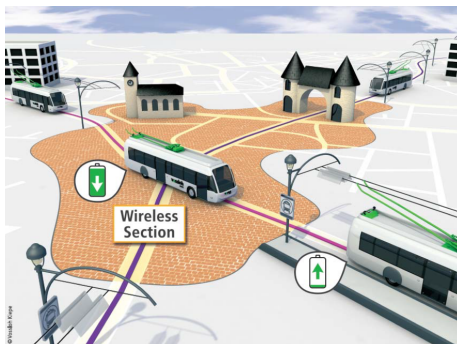


Figure 1: The basic idea of battery assisted trolleybus operation. [Vossloh Kiepe] (Bartomiejczyk, 2017).

We have developed the mathematical model to create a minimal network of overhead wires that would be sufficient for deployment and operation of

such battery-assisted trolleybuses for individual bus lines (Grygar and Kohani, 2019b). Next logical step in our research is to use this model to solve the problem with longer and more complex vehicle schedules. This paper introduces the mathematical model of exact optimization for individual routes. We are also dealing with the question, how to improve the effectivity of our mathematical model. Therefore, we will compare the adjusted version of the linear model with the original formulation.

With this model, we are able to optimize all of the service trips in the vehicle schedule individually. Using correct parameters, we ensure that a vehicle will be able to complete all tours in schedule (Grygar and Kohani, 2019a).

This approach can be considered as a heuristic algorithm extension to the previous research. Based on this approach, it is possible to optimize the schedule of a vehicle without regard to its length. Although obtained results are based on an exact approach to optimizing the individual routes, it is necessary to consider the coverage of individual schedules as a heuristic result. We are using real data from the public transport system. The data was obtained from the public transport provider (DPMZ) in Zilina (Grygar and Kohani, 2019a).

This paper is written with the following structure. First, we mention the current state of the art in Section 2. Then, in the next section of the paper, we formulate the problem. After that, Section 4 describes the data

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conversion approach. Section 5 explains the mathematical model of the problem and studied condition modification. Lastly, Section 6 introduces performed benchmarks and explains the collected results.

2 STATE OF THE ART

Traditional trolleybuses have been used for several years in public transport in some cities in Slovakia. To deploy trolleybuses, it is necessary to cover all routes with continuous overhead wires (Pisko, 2017). An interesting alternative is in using battery-assisted trolleybuses.

The battery-assisted trolleybuses are considered as the most cost-effective electrically powered public transport technology solution (Bergk et al., 2016), (Rogge et al., 2015) and (Yang et al., 2014).

In our previous research, we published the analysis in the topic of limiting factors of battery-assisted trolleybuses (Grygar et al., 2019). This study is based on valuable information papers from other authors. Vehicle technology is well covered in real deployment observations by (Bartomiejczyk, 2017) and (Bartomiejczyk et al., 2013). Information about lifespan of traction batteries and temperature impact can be found in (Ghlich et al., 2014), (Montoya et al., 2017), (Rice et al., 2017), (Ritter et al., 2016), (Rogge et al., 2015) and (Saft, 2016).

Unfortunately, there are no studies available to address the problem of building an optimal overhead wires network for battery-assisted trolleybuses in a big graph. We can get inspiration from articles dealing with the deployment of induction lines for passenger cars. This task is considered similar (He et al., 2016), (Hwang et al., 2018) and (Ushijima-Mwesigwa et al., 2017). Our proposed model is based on a modification of the model presented in (Ushijima-Mwesigwa et al., 2017).

There are published many works about scheduling electric vehicles in public transport. For example paper (Kooten Niekerk et al., 2017) extends traditional scheduling problem by using electric vehicles. Furthermore, there is better paper (Janovec and Kohni, 2019) also deals with this topic. Authors use more realistic linear model and it is tested with real word benchmarks from Zilina, Slovakia.

Our problem can be basically explained as location problem on edges. There are many papers dealing with all sorts of standard location problems (Jaosikova and Jankovic, 2018), (Janosikova et al., 2017), (Kvet, 2019) or (Janacek and Kvet, 2016). We may explore options on how to use a similar approach to our problem in future research.

3 THE PROBLEM FORMULATION

In our research, we often deal with the problem of designing network systems, such as rescue or distribution systems. In such tasks, we can meet with an unlimited or limited capacity problem. When solving a standard location problem, we are usually selecting locations for centres in network nodes (Janek and Kovaikov, 1999). These can be tasks oriented to placing shops, warehouses, stations, hospitals and so on. The centres capacity or distribution times are represented as model conditions. Objective function is often oriented to minimizing the total cost (Janek et al., 2015).

Our problem can be formulated as a location problem on edges. So we are trying to create a minimal network of overhead wires that would be enough for deployment and operation of battery-assisted trolleybuses technology. The building cost of required overhead contact wires represents a big part of the cost of the entire transport system. Therefore, we are about to minimize it (Grygar et al., 2019).

In the current state of our research, we have to explore options on how to cover whole vehicle schedules with overhead contact lines using current model formulation. So, we introduce the suboptimal approach to optimize selected vehicle schedules. From a computational point of view, the proposed mathematical model is inadequate for solving large scale tasks with commercial solver like XPRESS IP. The options of efficient conditions modifications need to be also explored.

The data conversion process is an important part of solving the formulated problem. After data conversion, we can start the process of finding exact solutions for different datasets using selected IP solver.

4 DATA CONVERSION

To prepare data for solving using formulated model, we need to perform the transformation of the road network to another form. The process converts the road network graph to road segment graph for each route. In this transformation, we also convert edges form road segment graph to nodes in the road segment graph and vice versa. This transformation is illustrated in Figure 2 (Grygar and Kohani, 2019b).

State of Charge (SOC) represents actual state energy in a vehicles battery pack in %. Battery assisted trolleybus starts a journey with an initial SOC. SOC of a vehicle is calculated depending on whether a charging wire is built on the road segment (Grygar and Kohani, 2019b).

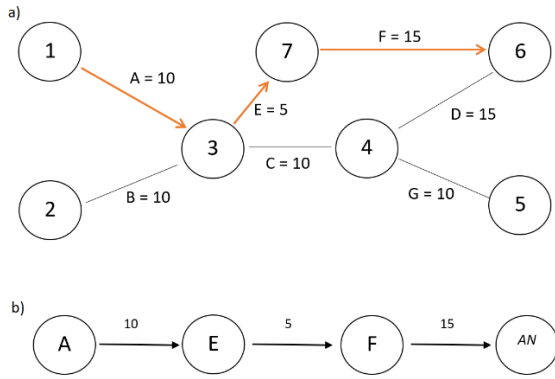


Figure 2: Creation of road segments graph from road network graph. a) Example of the road network graph. Bus line (with stops 1, 3, 7, 6) is marked with the orange line. Nodes are bus stops and edges are road segments. b) road segment graph created for the mentioned bus line. Nodes are road segments, AN is artificial node representing SOC of a vehicle after completing the trip (Grygar and Kohani, 2019b).

In the next phase, the state of charge graph is created for each line. Vehicle SOC graph illustration is in Figure 3. The mentioned graph represents all possible combinations of vehicles SOC. These options are combinations of individual segments coverage of a bus line by overhead charging wires. SOC graph contains only these combinations where the vehicle can pass whole route (Grygar and Kohani, 2019b).

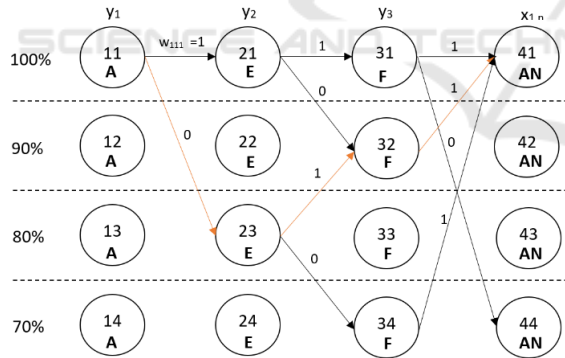


Figure 3: State of charge graph. Rows represent individual charge level and nodes are road segments. Transitions between charge level are marked with 1 if the vehicle is charged from wires and with 0 if not. The $y(s)$ and $x(r,i)$ are decision variables, and $w(r,s,i)$ input data (individual feasible combinations of charging). (Grygar and Kohani, 2019b).

5 MATHEMATICAL MODEL FORMULATION

In this section, we describe the original version of the

linear model of our problem. Then we will describe the proposed adjustment of the model.

We define set of all routes R , and set I_r as feasible alternatives for route r and then set off all used segments S . The number of segments n and routes m .

After that $x_{r,i} \in \{0, 1\}$ is decision variable of selection of alternative i for route r . And is decision variable $y_s \in \{0, 1\}$ of segment s in graph will or won't be covered by overhead charging wire.

Input combinations $w_{r,s,i} \in \{0, 1\}$ for route r on segment s for alternative i is charging line needed. Lastly the number of feasible alternatives a_r for route r and cost of building charging wires c_s for segment s .

$$\min \sum_{s=1}^n c_s \cdot y_s, \quad (1)$$

$$\sum_{i=1}^{a(r)} x_{r,i} = 1 \quad \text{for } r = 1..m, \quad (2)$$

$$n \cdot y_s \geq \sum_{r=1}^m \sum_{i=1}^{a(r)} x_{r,i} \cdot w_{r,s,i} \quad \text{for } s = 1..n, \quad (3)$$

$$y_s \in \{0, 1\} \quad \text{for } s = 1..n, \quad (4)$$

$$x_{r,i} \in \{0, 1\} \quad \text{for } r = 1..m; \quad i = 1..a(r). \quad (5)$$

The objective function 1 for the problem of minimizing the total building cost. Constraint 2 serve for choosing one combination for all routes on the line. Constraint 3 ensures that we install a charging line if at least one route requires an installation. The obligatory constraints are 4 and 5.

5.1 Edited Mathematical Model

As we have presented in the already published research, this formulation of the mathematical model is not suitable for larger-scale tasks. Therefore, we decided to make a change to the condition 3. Original conditions 3 were formulated for each route.

Modification of the condition 3 results in the following condition 6.

$$y_s \geq \sum_{i=1}^{a(r)} x_{r,i} \cdot w_{r,s,i} \quad \text{for } s = 1..n; \quad r = 1..m. \quad (6)$$

This change does not change the meaning of the model. Conditions 6 are formulated for each service trip and also for each route. It results in an increase in the number of conditions when solving a task using IP-solver XPRESS. We will study the impact of this change on the solving process of the problem using the following computational tests.

6 THE BENCHMARKS AND COMPUTATIONAL TESTS

We obtained data from the public transport company (DPMZ) in Zilina, Slovakia. Two vehicle schedules were selected. For each benchmark, two-vehicle schedules were selected.

Our model is able to solve the problem for single bus routes only. Therefore, it was necessary to ensure that the vehicle starts and ends individual journeys on the line with the same level of SOC. Therefore, we choose the rule that every service trip must end with SOC of at least 60 %, and every service trip start's with SOC of 60 %. The solution found using this parameter setting will mean that a vehicle is able to overcome the entire schedule. However, it is not guaranteed to find the optimal solution to the original problem. All input parameters related to vehicle are listed in Table 1.

As it was mentioned before, we also studied the impact of model adjustment. We tested both models using the same benchmarks.

Table 1: Battery assisted trolleybus parameters settings selected for all benchmark runs.

Parameter	Value	Unit
Min SOC	20	%
Max SOC	80	%
Battery capacity	50	kWh
Energy consumption	1.5	kWh/km
Initial SOC	60	%
Minimal SOC on route end	60	%
Charging time	30	s/1kWh
Travel speed	30	km/h

We used workstation with the following hardware specifications. Processor Intel Core i5-7200U 2,5Ghz with 3,10Ghz turbo boost (two cores and four threads), paired with 16 GB of DDR4 2133MHz RAM. We selected IP-solver XPRESS IVE as software tool for solving this problem.

6.1 Benchmark 4_4a and 5_1a

In the first benchmark, we selected two schedules. The first schedule of a vehicle named 4.4a, which serves 10 service strips. Mainly trips of lines 4 and 14, but the schedule is starting with a shortened service strip of line 1 and once also servers line 5. The second schedule 5.1a serves 10 service trips on line 5. Selected schedules are presented in Table 2. Individual routes from selected schedules are illustrated in Figure 4.

Table 2: Selected schedules for the benchmark 4.4a and 5.1a. The lines covered by schedules are presented.

Vehicle schedule	Line	Stops count
4.4a	4	20
	14	20
	1m	13
	5a	16
	5b	15
5.1a	5a	16
	5b	15

According to the optimal solution of the benchmark, 7477m of road segments were selected for building overhead charging lines. The total length of all road segments were 35566m, which means that approximately 21% were selected. Selected road segments are listed in Table 3 and visually illustrated in Figure 4.

Table 3: Selected road segments in optimal solution.

Starting node	Ending node
Halkova	Policia
Policia	Hurbanova
Hurbanova	Zeleznicna stanica
Sv. Cyrila a Metoda	Obchodna
Jasenova	Limbova
Polna	Hlinska
Matice slovenskej	Fatranska
Stefanikovo namestie	Zeleznicna stanica
Hurbanova	Policia
Mostna	Hlinska
Polna	Smrekova
Smrekova	Limbova
Limbova	Jasenova

According to benchmark results in Table 4, it is clear, that adjustment of the model had a positive impact on the process of solving the given problem. As we can see the number of constraints increased, while the number of variables stays the same. After the pre-solve process, the number of variables decreased significantly specifically using edited model formulation. Pre-solve in IP solver attempts to simplify the problem by detecting and removing redundant constraints, tightening variable bounds, etc.

Moreover, using the original model formulation the solver was unable to successfully finish the benchmark run, because of lack of available RAM. On the other hand, the optimal solution was found using the edited model. Running time was also shorter using the edited model.

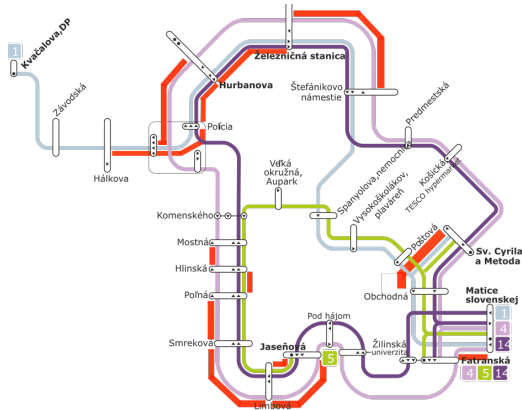


Figure 4: Illustration of bus lines 1, 4, 5 and 14 in Zilina. Selected edges for installing overhead charging wires are marked as red.

Table 4: Models comparison.

	Original model	Edited model
Constraints (initial)	76	336
Constraints (pre-solved)	76	336
Variables (initial)	3816093	3816093
Variables (pre-solved)	2099867	1482018
Running time (s)	6134	3248
Objective function value	7478	7477
Optimal solution	out of memory	yes

6.2 Benchmark 24.5 and 26.3

In second benchmark we also selected two schedules presented in Table 5.

The first schedule of a vehicle named 24.5, which serves 14 service strips. Mainly trips of line 24, but also serves shortened service strip on line 30. The second schedule 26.3 serves 11 service trips on line 26. Individual routes from selected schedules are illustrated in Figure 5.

In this case, according to benchmark results in Table 6, we can tell that the optimal solution was found using both models. The optimization process was

Table 5: Selected vehicle schedules.

Vehicle schedule	Line	Stops count
25_5	24	20
	24	7
	30	6
26_3	26a	17
	26b	18
	26	9

Table 6: Models comparison.

	Original model	Edited model
Constraints (initial)	108	380
Constraints (pre-solved)	107	379
Variables (initial)	4020976	4020976
Variables (pre-solved)	2295378	1106005
Running time (s)	800	1299
Objective function value	10566	10566
Optimal solution	yes	yes

faster using the original model formulation.

The optimal solution of the second benchmark tells that 10566m of road segments were selected for building overhead charging lines. The total length of all road segments were 53984m, which means that approximately 19.6% were selected. Selected road segments are listed in Table 7 and visually illustrated in Figure 5.

Table 7: Selected road segments in optimal solution.

Starting node	Ending node
Priehradna	Furdekova
Hurbanova	Zeleznicna stanica
Zeleznicna stanica	Stefanikovo namestie
Predmestska	Kosicka TESCO hyp.
Kosicka TESCO hyp.	Pri celulozke
Potoky	Dolna trnovska
Pri celulozke	Kosicka TESCO hyp.
Predmestska	Stefanikovo namestie
Stefanikovo namestie	Zeleznicna stanica
Zeleznicna stanica	Hurbanova
Kysucka	Namestie hrdinov
Jastrabia	Vrania
Vrania	Jastrabia
Kysucka	Zeleznicna stanica
Internatna	Univerzitna
Internatna	Vysokoskolakov plavaren

7 CONCLUSIONS

We can expect that the demand for the inclusion of assisted trolleybuses technology in urban transport will have an increasing trend. In this case, reliable ways of solving related problems need to be researched (Grygar et al., 2019).

This paper briefly explains the required data conversion approach and mathematical model required for creating a minimal network of overhead wires that

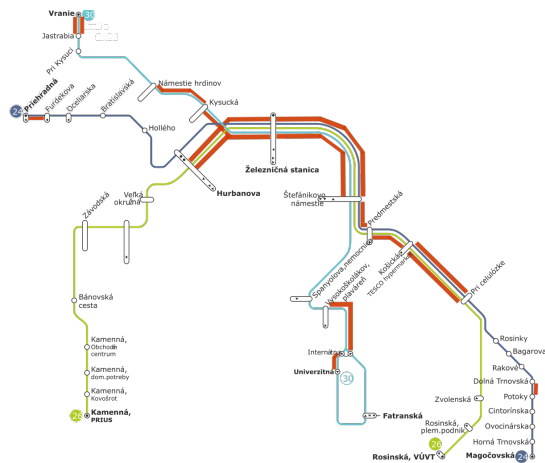


Figure 5: Illustration of bus lines 24, 26 and 30 in Zilina. Selected edges for installing overhead charging wires are marked as red.

would be enough for deployment and operation of such battery-assisted trolleybuses.

Performed computational tests demonstrate the suboptimal approach on how to solve this problem for selected vehicle schedules. According to the first benchmark, we can see, that performed condition adjustment in the model can be beneficial for solving similar problems. The presented solutions show that around 20% coverage was needed for the operation of such vehicles. We can see, that selected segments were mostly in the centre of the city. This is caused by the fact that most of the bus lines connect the city centre with other areas.

We will try to improve our approach to future research. Vehicle SOC needs to be preserved from the end of a route to start of next one. After that, we will be able to find optimal solutions for vehicle schedules.

Lastly, the route's nodes number can have a serious impact on the solvability of the problem. Therefore, in future research, we will try to combine nearby route segments or simplify the peripheral edges (Grygar and Kohani, 2019b). We will also investigate other methods of creating a more efficient model. Heuristic approach options may need to be considered for large scale tasks as well.

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