Regularization in Higher-order Photometric Stereo Inspection for Non-Lambertian Reflections

Doris Antensteiner[a] and Svorad Štolc
Austrian Institute of Technology, Giefinggasse 4, Vienna, Austria
{*doris.antensteiner, svorad.stolc*}*@ait.ac.at*

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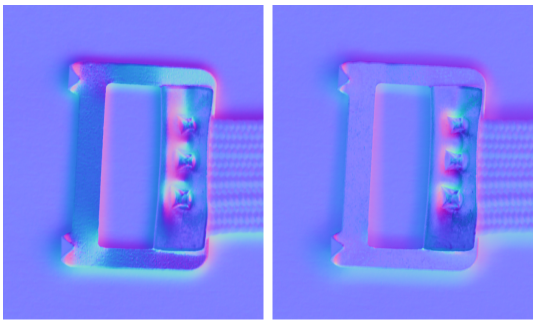
Abstract: In this paper we present and compare two regularized higher-order photometric stereo approaches for the reconstruction of varying albedos and surface normals of non-Lambertian materials. We evaluate the two different higher-order polynomial methods, which we additionally regularize with Tikhonov's method. The reconstruction of surface properties is essential for a vast amount of industrial applications, such as the identification of surface defects, the analysis of security features or the detection of forged documents. For the reconstruction of Lambertian objects, lower order models can be used to achieve an accurate representation, while higher-order models allow the description of non-Lambertian behaviors accurately. Qualitative and quantitative results on a ground truth dataset as well as on real-world data show that the use of a regularized higher-order polynomial model can significantly improve the surface normal and albedo reconstructions.

1 INTRODUCTION

Analyzing the surface normals and illumination independent reflectance (albedo) from multiple reflections of an object presents one of the most important problems in computer vision. Especially in industrial applications, the surface normals and albedo give essential information for a number of challenges. Such applications lie in the field of defect detection for inline production setups, the analysis of complex print structures (e.g. optically variable security features) as well as in the inspection and detection of forged documents and banknotes. However, such surface reflections behave usually in a complex non-Lambertian manner, while a vast amount of reconstruction algorithms only consider diffuse reflection elements. This leads to systematic errors in reflective regions. We tackle this issue by formulating two second-order polynomial models and pairing them with a Tikhonov regularization to analyze the non-Lambertian reflectance. The Tikhonov regularization is a widely-used regularization approach that was invented independently in various fields, e.g. (Tikhonov, 1943). We compare both higher-order polynomial models and explain their differences, along with comparing the photometric stereo reconstructions with standard Lambertian models.

[a] https://orcid.org/0000-0003-2083-0135



(a) Lambertian model. (b) HO-Poly model.

Figure 1: Illustration of the suppression of errors in surface normals due to a shiny metal surface on a bandage-pin using regularized second-order polynomials (HO-Poly) compared to the reconstruction with a standard Lambertian model. Note the compensation of the strong non-smooth deviations along the metal piece in HO-Poly in contrast to the non-compensated Lambertian reference method.

Photometric stereo is a well-studied computer vision approach which emerged in the early 80s (Woodham, 1980). It describes using the Lambertian reflectance model to recover the surface normals and albedo, with several images taken from a fixed scene under various calibrated illumination conditions. However, real world surface reflections which show specularities, benefit from higher-order models. Alternatively, those specular reflections can be treated as outliers by a lower order approximation (Mallick et al., 2005).

Polynomial approximations were previously used on texture maps (PTMs) (Malzbender et al., 2001) in order to achieve increased photo-realism of reconstructed surface textures. These are second-order models defined on the xy-domain. We build on that idea and advance this approach to a second-order polynomial defined on the full xyz-domain, which naturally expands upon the standard Lambertian model. We label this approach as higher-order polynomial (HO-Poly). This formulation is a natural higher-order extension of the Lambertian model formulation.

Spherical harmonics follow the same idea with different second-order basis functions and were previously studied in the context of photometric stereo. The projection onto a spherical basis to derive harmonics was used in (Basri et al., 2007). Later, face shapes were analyzed using spherical harmonics (Kemelmacher-Shlizerman and Basri, 2011), where the light coefficients and albedo were estimated from one reference face shape. We utilize and regularize the higher-order spherical harmonics (HO-SH) to reach a more accurate non-Lambertian model. This is a more established formulation compared to our HO-Poly and benefits from the orthogonal behavior of the basis functions. In contrast, HO-Poly does not have full orthogonality but in contrast offers one more degree of freedom for the second-order case.

We demonstrate two $L2$ regularized polynomial representations (HO-Poly and HO-SH) of the light source vectors, formulated to achieve a more precise photometric stereo representation. Their benefit in regards to the surface normal calculation is illustrated in Fig. 1 in comparison to a full Lambertian model. The shown bandage-pin was acquired with a light dome using 32 illumination sources. The surface normals show a strong bias on the highly reflective metal pin element in the image center. This effect is strongly suppressed using a regularized higher-order model.

In this paper, differences and properties of the two proposed polynomial representations are discussed. We outline the importance of regularization using higher-order photometric stereo models. For both methods, we propose and investigate polynomial representations consisting of up to second-order components. The zero-order elements represent the ambient component which can help dealing with illumination behaviours such as stray light. The first-order components of the light polynomials are utilized to extract the varying albedo and surface normals of the object. Second-order elements comprise higher-order information, such as specular lobes. We use a Tikhonov regularization to restrict the behaviour of the second-order components as well as the ambient zero-order component simultaneously.

2 REFLECTANCE REPRESENTATION

For Lambertian surfaces, low order spherical harmonics can be utilized to represent the illumination conditions accurately. A simple extension for non-Lambertian surfaces can be approximated by filtering specular regions (Shashua, 1997). Another approach of formulating the irradiance in terms of spherical harmonic coefficients of the incident illumination was explored in (Ramamoorthi and Hanrahan, 2001), building on that work, low order spherical harmonics were utilized by (Basri et al., 2007). Non-Lambertian surfaces require higher-order models to allow for an accurate representation.

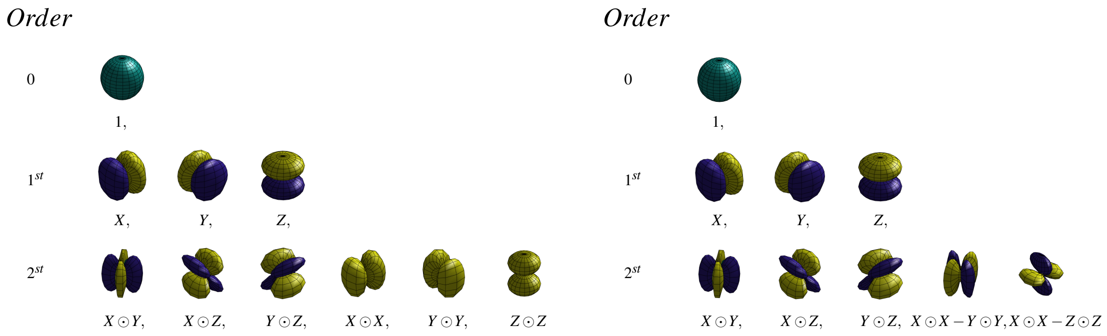
In this paper we formulate polynomial functions which were frequently used in the fields of optics, modelling surface reflectance and textures in order to synthesize geometric details from images.

Zernike polynomials (Zernike, 1934) are an orthogonal polynomial sequence which are used in optics for modelling optical aberrations. Since they are represented on a unit disk they are not directly applicable to our task. Instead, we formulate spherical harmonic polynomials which can be used to represent smooth surface reflectance functions on a sphere (MacRobert, 1948; Haindl and Filip, 2013).

For polynomial methods, the representation of specular peaks can not be estimated fully, if solely the lower-order polynomial function is considered. Higher-order functions are required for more accurate representations. In the field of photometric stereo, such higher-order representations are used to reach a compact formulation and stable analysis of the observed reflectance.

Having sufficient data, another characterization can be tailored by learning the reflectance behavior using convolutional neural networks. This allows weights to capture even complex reflectance distribution functions (Haindl and Filip, 2013; Antensteiner and Štolc, 2017).

In our work we propose regularized higher-order formulations. Using regularization for higher-order models is essential to control the higher-order components. In our presented methods we propose the use of a Tikhonov regularizer. This traditional regularization allows for a high performance implementation while remaining within a least-squares approach. A local per-pixel computation does not require any it-



(a) HO-Poly (b) HO-SH

Figure 2: Basis functions of the proposed regularized polynomial HO-Poly and the spherical harmonics formulation HO-SH, represented are the elements of the polynomials as described in Eq. 2 and Eq. 3 . These plots show the basis functions as they would appear for a large number of elements n.

erative optimization or sorting as it can be solved via a closed-form method. This makes our method apt for industrial inline applications which require real-time processing. We will evaluate a Tikhonov regularization on both, such polynomials which are a natural extension of the Lambertian formulation and PTMs (HO-Poly) as well as a spherical harmonics representation (HO-SH). These approaches are well-fitting for the reconstruction of surface normals and the varying albedo of non-Lambertian surfaces. We compare the results of our proposed regularized methods to the outcome reached by a standard Lambertian model.

3 PHOTOMETRIC STEREO

Photometric stereo methods reconstruct the surface orientation and albedo of an object, acquired under varying lighting conditions. In standard photometric stereo we consider a Lambertian surface capture of the dimensions of $M \times N$ pixels, with surface normals $N_{i,j} \in \mathbb{R}^3$ at the discretized locations $(i, j) \in M \times N$ and an albedo $\rho_{i,j} \in \mathbb{R}$. These are reconstructed under defined illumination sources $L \in \mathbb{R}^{n \times 3}$ with n observed intensities $E_{i,j} \in \mathbb{R}^n$. The matrix $M_{i,j} = \rho_{i,j} N_{i,j}$ with vectors at each location, representing albedo scaled surface normal vectors can be recovered using a standard least squares (LS) formulation, solved with a conjugate gradient approach:

$$\min_{M_{i,j}} \frac{1}{2} ||L \cdot M_{i,j} - E_{i,j}||^2. \tag{1}$$

The depth consequently can be recovered using e.g. the (generalized) method of Nehab, which was previously described in (Nehab et al., 2005; Antensteiner et al., 2018).

4 OUR REGULARIZED HIGHER-ORDER PHOTOMETRIC STEREO

We analyze two different higher-order functions for the photometric stereo light source modelling. First, we approach the description by a natural extension of the Lambertian formulation by higher-order terms with utilizing the Tikhonov regularization for behaving the second-order and zero-order terms (HO-Poly). We illustrate (Fig. 2a), that the corresponding basis functions show partial non-orthogonal behavior and hence possibly result in ambiguities. We evaluate this behavior and solve the problem by evaluating the more common spherical harmonics second approach with our proposed Tikhonov regularization extension (HO-SH). The spherical harmonics model is consisting of orthogonal basis functions (see Fig. 2b), while HO-Poly has the benefit of boasting one more degree of freedom.

4.1 Higher-order Polynomial

We are expressing the regularized higher-order polynomial HO-Poly in Cartesian coordinates. The terms of the HO-Poly are expressed as $P \in \mathbb{R}^{n \times 10}$, which we limit to the reconstruction of up to second-order elements. We define our light vector L by it's components $\{X, Y, Z\} \in \mathbb{R}^{n \times 3}$ as follows:

$$\begin{aligned} L &= [X, Y, Z], \quad \text{with} \\ X &= [x_1, \ldots, x_n], \\ Y &= [y_1, \ldots, y_n], \quad \text{and} \\ Z &= [z_1, \ldots, z_n]. \end{aligned} \tag{2}$$

We utilize the light vector components to formulate our polynomial as follows:

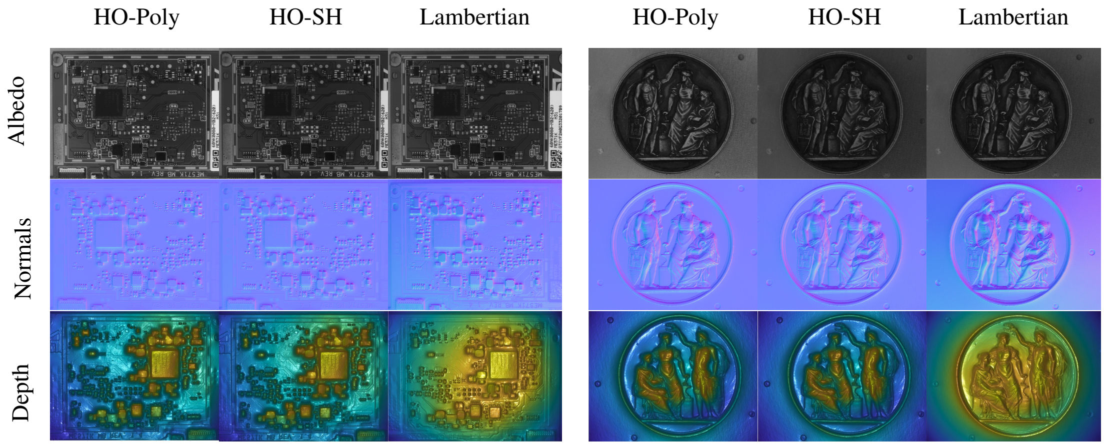


Figure 3: Comparison of the reconstruction using the proposed regularized higher-order methods HO-Poly and HO-SH in reference to the standard Lambertian model. Both on the PCB board (left) and the coin (right) the systematic surface normal errors (and as a result errors in the depth reconstruction) in the Lambertian reconstruction are suppressed by the use of higher-order models.

$$\begin{aligned} P &= [P_2, P_1, P_0], \quad \text{where} \\ P_2 &= [X \odot X, Y \odot Y, Z \odot Z, \ldots \\ &\quad X \odot Y, X \odot Z, Y \odot Z], \\ P_1 &= [X, Y, Z], \\ P_0 &= [1], \end{aligned} \tag{3}$$

with 1 defining a one-valued vector of length n and $\odot$ denoting the Hadamard product. We formulate an energy function to solve for our extended $M_{i,j} \in \mathbb{R}^{10}$.

$$\min_{M_{i,j}} \frac{1}{2} ||P \cdot M_{i,j} - E_{i,j}||^2 + \lambda ||\Gamma \cdot M_{i,j}||^2 \tag{4}$$

A Tikhonov regularizer $\Gamma \in \mathbb{R}^{7\times10}$ represents an identity matrix for $[P_2, P_0]$ with a linear scaling factor λ.

We extract the albedo and surface normals using the first-order components of our light polynomial P_1. The second-order components of the polynomial are holding non-Lambertian elements, which we filter out. The length of the vector $M_{i,j,l}$ is defined by the albedo, as per definition all normals are unit vectors, hence we can extract the albedo and normals simultaneously:

$$\sqrt{N_{i,j,x}^2 + N_{i,j,y}^2 + N_{i,j,z}^2} = 1, \tag{5}$$

$$\rho_{i,j} = \sqrt{M_{i,j,l_1}^2 + M_{i,j,l_2}^2 + M_{i,j,l_3}^2}. \tag{6}$$

The basis elements of this method (Eq. 3) are illustrated in Fig. 2a. We show that the last three second-order polynomials are not behaving perfectly orthogonal to the first-order components. Therefore, an overlap of these terms in the achieved representation is possible. To overcome these limitations, we propose the use of spherical harmonic polynomials, which fulfill the orthogonality of the basis functions but have a disadvantage in containing one degree of freedom less.

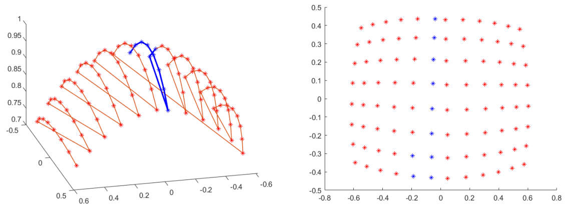
4.2 Spherical Harmonic Polynomials

We are expressing our regularized spherical higher-order harmonics (HO-SH) in Cartesian coordinates as homogeneous polynomials (Gallier, 2009). For the photometric stereo estimation, we use bases of the higher-order spherical harmonic polynomials up to the second-order with a polynomial $P \in \mathbb{R}^{n\times9}$ as follows:

$$\begin{aligned} P_n &= [P_2, P_1, P_0], \quad \text{where} \\ P_2 &= [X \odot X - Z \odot Z, Y \odot Y - Z \odot Z, \ldots \\ &\quad X \odot Y, X \odot Z, Y \odot Z], \\ P_1 &= [X, Y, Z], \quad \text{and} \\ P_0 &= [1]. \end{aligned} \tag{7}$$

Contrary to our polynomial in Eq. 3, spherical harmonics show orthogonality in the basis functions. This shows in the second-order polynomials or higher-order polynomial functions. Note that the total number of polynomial elements is reduced by one compared to our HO-Poly approach.

We are solving for the extended $M_{i,j} \in \mathbb{R}^9$ containing the surface normals and albedo using a squared energy function (properties described in Eq. 5 and Eq. 6) with a Tikhonov regularization as described in Eq. 4, albeit with distinct dimensions with $\Gamma \in \mathbb{R}^{7\times9}$. The regularizer Γ is representing an identity matrix for $[P_2, P_0]$ with a linear scaling factor λ. The first-



Side Top

Figure 4: Illumination positions of the DiLiGenT dataset, used for our quantitative evaluation. Of the given 96 total positions, we chose 10 of the extended central arch (marked in blue). The positions are shown from a side-view with connected arches to indicate the scan pattern as well as from the top view.

order components of our light polynomial P_1 are used to extract the albedo and surface normals.

5 RESULTS

We analyzed the difference between two regularized polynomial representations of higher-order photometric stereo inspection, namely a natural extension of the Lambertian formulation with higher-order terms (HO-Poly) and a spherical harmonic polynomial representation (HO-SH), which we described in Sec. 4, quantitatively and qualitatively. We show that regularization in higher-order photometric stereo formulations is essential and evaluate it's influence using a Tikhonov regularizer. In our model, we activate the regularization on the second-order and zero-order components. The latter represent the ambient component, which is needed to deal with effects stemming from stray light.

Lower order photometric stereo descriptions can represent diffuse surface reflections. Higher-order estimates are necessary for surfaces which show non-Lambertian behavior. This is the case for most real-world materials. We showed the difference between our HO-Poly light source representation and the HO-SH in Fig. 2a and Fig. 2b. Where the former can result in ambiguous representations due to a partial non-orthogonal behavior of the basis but has one more degree of freedom and the latter has orthogonal basis functions. We investigate this difference and the influence of the regularization.

5.1 Quantitative Evaluation

We use the DiLiGenT dataset (Shi et al., 2019) to quantitatively evaluate our regularized higher-order methods in reference to a Lambertian photometric stereo model. The quality is measured by the mean squared error (MSE) between the given ground truth (GT) surface normals and our results. Note that surface normals vectors have unit length by definition, which allows a quite accurate comparison using the MSE distance. The DiLiGenT non-Lambertian photometric stereo dataset contains general materials with various degrees of shine as well as ground truth surface normals which were acquired using a high precision laser scanner. It consists of 10 scenes (ball, cat, pot1, bear, pot2, buddha, goblet, reading, cow and harvest), each illuminated from 96 directions. These 96 directions are organized into 12 arches (see Fig. 4), each arch consisting of 8 illumination directions. We used the first 10 light directions and acquisitions from the dataset for each scene, since we usually deal with a limited number of illumination sources in industrial environments. These 10 light sources cover the whole central acquisition arch and two points of the neighbouring arch.

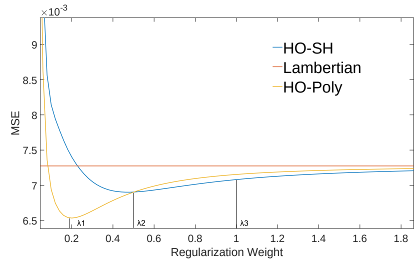


Figure 5: Influence of the regularization weight λ. Higher-order photometric stereo polynomials (HO-SH and HO-Poly) require a regularization to weight the Lambertian influence. The best result was reached at $\lambda_1 = 0.19$ for HO-Poly, both representations show equal results at $\lambda_2 = 0.5$. We show numerical evaluation at both points λ_1 and λ_2 as well as for $\lambda_3 = 1$ in Tab. 1.

Our quantitative evaluation of the influence of the regularization parameter λ on our higher-order models is represented in Fig. 5. We show the importance of the regularization for higher-order models. Numeric results for the chosen regularization parameter values $\lambda_1 = 0.19, \lambda_2 = 0.5, \lambda_3 = 1$ are presented in Tab. 1. A smaller MSE value indicates a result closer to the GT surface normals.

Over all examples in the dataset, the sufficiently regularized HO-SH and HO-Poly performed superior to the traditional Lambertian model. For a regularization of $\lambda > 0.5$ the HO-SH model performs best. We explain this by the orthogonality of the basis functions. While ambiguous results can occur using the HO-Poly functions, HO-SH allows for a non-ambiguous solution. Though, for a lower regularization of $0.05 < \lambda < 0.5$ the best result is achieved by the HO-Poly model. This model has one additional degree of freedom which allows for a superior re-

Table 1: Quantitative evaluation of the proposed algorithms using the DiLiGenT dataset. For both proposed regularized methods as well as the standard Lambertian reconstruction we evaluate the MSE to the ground truth surface normals.

	MSE	ball	cat	pot1	bear	pot2	buddha	goblet	reading	cow	harvest	mean
λ_1	Lambertian	0.0192	0.0201	0.0280	0.0321	0.0627	0.0495	0.0889	0.1046	0.1385	0.1838	0.0727
	HO-SH	0.0128	0.0255	0.0284	0.0257	0.0601	0.0527	0.1043	0.1127	0.1329	0.1903	0.0745
	HO-Poly	**0.0122**	**0.0197**	**0.0231**	**0.0226**	**0.0558**	**0.0438**	**0.0856**	**0.0943**	**0.1243**	**0.1724**	**0.0654**
λ_2	Lambertian	0.0192	0.0201	0.0280	0.0321	0.0627	0.0495	0.0889	0.1046	0.1385	0.1838	0.0727
	HO-SH	**0.0148**	0.0198	0.0264	**0.0246**	**0.0586**	0.0475	0.0876	0.1024	**0.1296**	0.1792	**0.0690**
	HO-Poly	0.0165	**0.0191**	**0.0259**	0.0269	0.0593	**0.0465**	**0.0852**	**0.0999**	0.1321	**0.1790**	**0.0690**
λ_3	Lambertian	0.0192	0.0201	0.0280	0.0321	0.0627	0.0495	0.0889	0.1046	0.1385	0.1838	0.0727
	HO-SH	**0.0174**	**0.0196**	**0.0272**	**0.0287**	**0.0607**	**0.0484**	**0.0874**	**0.1031**	**0.1344**	**0.1811**	**0.0708**
	HO-Poly	0.0184	0.0197	0.0274	0.0304	0.0616	0.0486	0.0876	0.1031	0.1365	0.1822	0.0715

sult in a low-regularized setting. Regularizing too low however, when dealing with higher-order photometric stereo methods, results in a reconstruction performance far below the one achieved by a simple Lambertian model. By this we can clearly show the importance of regularization for higher-order models. The function of the regularization is to enforce the higher-order and zero-order components to contain lower values.

5.2 Qualitative Evaluation

Using a light dome with $n = 32$ known illumination sources (as illustrated in Fig. 6), we extracted the surface normals and albedos using the HO-Poly and HO-SH models. We compare the results obtained for these models to estimates stemming from the standard Lambertian formulation of Eq. 1. Reconstructions using the regularized higher-order polynomial (HO-Poly), the regularized higher-order spherical harmonics (HO-SH) and the Lambertian model are shown in Fig. 3. It is visible that the Lambertian reconstruction both on the PCB and the coin shows strong systematic offsets. This deviation can be observed in the surface normals and demonstrates its effects in the resulting depth map, achieved by the method of Nehab (Nehab et al., 2005). Considering the varying albedo, a more stable result can be subjectively observed using higher-order spherical harmonics compared to the other two methods. The surface normal reconstruction both as achieved by the HO-Poly and the HO-SH approach show similar behaviors when regularized sufficiently ($\lambda = 0.7$ was used for our example).

6 SUMMARY AND CONCLUSIONS

We proposed and compared two regularized higher-order photometric stereo approaches to reconstruct the varying albedo and surface normals from an object. Utilizing higher-order terms allows superior results when regularized sufficiently. We demonstrate this on two formulations, where one is a well-known compact model which boasts orthogonality of the basis functions (HO-SH) and the other one has an additional degree of freedom (HO-Poly). We demonstrate why regularization is essential in higher-order models and show an evaluation of both proposed regularized formulations in reference to a Lambertian model qualitatively and quantitatively.

Our regularization of choice is achieved by Tikhonov's method. Since this fits well in a least-squares formulation, we can solve it very fast and efficiently, which is essential for industrial applications. In the presence of shadows our formulation has additional benefits compared to an $L1$-formulation (such as presented by (Zhang and Drew, 2015)), since they behave non-sparse. In areas with sparse outliers such as spikes produced by glass or highly reflective surfaces, our approach might show numerical disadvantages compared to an $L1$ regularization.

We tested all methods on non-Lambertian real world data, acquired with a light dome with 32 illumination sources. Our experiments demonstrate that our regularized methods can significantly improve the reconstruction of the albedo, surface normals and depth compared to a Lambertian model, provided that sufficient regularization is used. We demonstrate that non-regularized higher-order models are not well-behaved and show high error rates. Additionally we show that while normals can be more accurately reconstructed using orthogonal basis functions and proper regularization (HO-SH), using our polynomial formulation with an additional degree of freedom (HO-Poly) with lower regularization shows the best numeric accuracy. We prove this by evaluating the approaches on the DiLiGenT dataset, which contains GT surface normal data. With our regularized HO-SH approach we can approximate a wide variety of surface reflectance be-

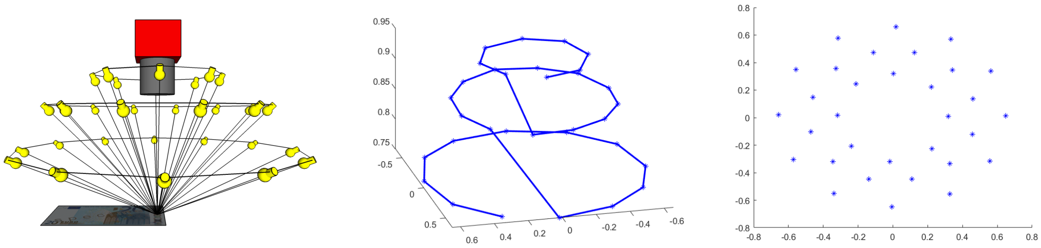


Figure 6: Illustration of our photometric light dome acquisition setup used in our real world qualitative experiments, with 32 illumination sources. The side view shows the connected light source positions to indicate the scan pattern, the top view marks single light source locations.

haviors.

Achieving highly accurate results on non-Lambertian surfaces is essential for industrial applications. Stable approaches allow highly accurate results in the field of detection and analysis of surface defects, material analysis as well as the inspection of security features and high precision prints.

In future projects we will consider extending regularized higher-order methods further beyond second-order models. Such an extension might lead to additional advantages for specific materials which are hard to fit using second-order models.

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