

# Peer-to-peer Energy Trading for Smart Energy Communities

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**Abstract:** Local energy communities (LECs) comprise prosumers cooperating for the satisfaction of their energy needs. Prosumers are community members that can both produce and consume energy. LECs facilitate the integration of renewables and provide the potential for reducing energy costs. Peer-to-peer (P2P) energy trading allows direct energy exchange between members of a local energy community. The surplus of energy from renewables is traded to meet a local consumption demand so that costs and revenues stay within community avoiding transmission losses and stress on the grid. This study presents a design of the peer-to-peer market for local energy communities where prosumers are rational and self-interested agents acting selfishly in an attempt to optimize their trades. The market design aims to ensure self-consumption of locally produced energy and provides incentives for balancing supply and demand within community. The proposed design is investigated in comparison with existing ones using theoretical analysis and simulations. The obtained results are promising and reveal advantageous properties of the proposed P2P energy trading.

## 1 INTRODUCTION


Nowadays, the topics of energy and climate policies are crucial, and require a transition for energy systems from exhaustible energy sources, such as fossil and nuclear, to renewables (e.g. wind and solar). About this, several policies have been made in order to support installation and employment of renewable energy sources (RES). These policies reward RES producers for the excess of energy they export to the grid, for example by paying them money for unit of energy exported or by providing them energy at no cost later, so that they are repaid for their RES investment. These policies have been effective for increasing the share of energy generated from RES, which in 2016 already represented 30% of total energy generated in Europe<sup>1</sup>; however, the employment of this type of energy comes with stability problems to the grid (Mihaylov et al., 2019; Commission, 2013), and support policies are unable to solve this at the moment, so this limits their effectiveness. More precisely, renewable


energy production is difficult to control, and for this reason production usually does not match consumption.


In order to face these issues, in these years there have been new proposals for incentive mechanisms (Ilic et al., 2012; Mihaylov et al., 2014; Kok et al., 2005; Capodici et al., 2011). The key feature of these mechanisms is building the payment support functions for producers and consumers in such a way that, at every moment, energy consumption is encouraged to match energy production.

Some of these mechanisms use market-based approaches (Ilic et al., 2012; Kok et al., 2005), which consider a local market where prosumers sell the energy they produce and consumers buy it. Another work by (Mihaylov et al., 2014) proposes a mechanism called *NRG-X-Change*, which does not rely on an energy market, as locally produced renewable energy is simply injected into the grid and taken by consumers. Unlike traditional support policies, the *NRG-X-Change* mechanism offers incentives not only for producers but also consumers, depending on their contribution to the local energy balance.

The most prominent feature of *NRG-X-Change* is the employment of *NRGcoin*, a virtual currency whose exploitation offers some relevant benefits to

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energy trading. However, there are some important factors not taken into account by this mechanism, such as energy self-consumption and the management of congestion. More precisely, we want to ensure that prosumers always consume their produced energy before selling or buying the excess energy they produce or need. In other words, we want to actively discourage scenarios where the energy consumption or production is excessive. Therefore, the purpose of this work is to create a new incentive mechanism based on *NRG-X-Change* and the use of *NRGcoin* using as a baseline the works in (Mihaylov et al., 2014) and (Lilliu et al., 2019b), but further improving the critical points of the mechanism and studying its behavior in a context with selfish agents. The contributions of this paper are therefore the following:

- We analyze the *NRG-X-Change* project and identify its main issues (import and export price functions) and the situations where they appear;
- We design new import and export price functions that can work better within the *NRG-X-Change* mechanism by exploiting known results in game theory about Nash Equilibria, and provide theoretical background on those;
- We demonstrate the validity of our proposed functions through mathematical proof, and carry out an experimental evaluation on real data from a grid in Cardiff that measured the efficiency of our proposed functions.

The remainder of the paper is structured as follows. Section 2 describes the literature related to our work. Section 3 formulates the game theoretical model, and in Section 4 we present the baseline we work on top, and how we want to achieve the objectives of this paper. Section 5 validates our theoretical results and quantifies the efficiency of the described mechanisms via simulations using real data from a grid in Cardiff (UK). Finally Section 6 concludes the paper and describes possible future developments.

## 2 RELATED WORK

Although the *NRG-X-Change* mechanism has not yet been implemented in actual grids, there are some works in which it has been described in detail. The most important one is from (Mihaylov et al., 2014), where the authors explain the advantages of using a digital currency and the interactions between grid users, with or without outside parties. Also, (Mihaylov et al., 2015) created a scenario simulating a local grid, showing how *NRG-X-Change* performs in a realistic environment.

A survey on incentive mechanisms has been done by (Mihaylov et al., 2019), where issues about net metering and feed-in tariffs are discussed, such as overconsumption in some periods of the year, overpayment for the energy provider, and underpayment to energy producers. In that work, there is a comparison of some new support mechanisms which aim to solve these problems. One of them is the auction-based mechanism Nobel (Ilic et al., 2012), which trades energy at a local level based on the stock exchange model. Another mechanism is PowerMatcher (Kok et al., 2005), which is based on a market structure regulated by software agents balancing demand and supply in a tree-shaped hierarchical structure, and are called *SD-matchers*. There is also a work proposed by (Capodiecici et al., 2011), which is a negotiation-based mechanism where prosumers compete with energy generating companies (*Gencos*) for selling energy, and negotiate the price with buyers. It is then shown how this mechanism affects the energy price, and also what happens with the implementation of a learning strategy for the buyers. However, all these above-cited works focus on market mechanisms which require final prosumers to get involved into complex bidding processes.

Literature on employment of game theory for the smart grid context is extensive. (Saad et al., 2012) present a survey which describes comprehensively how game theory methods have been used for smart grid. The work done by (Rad et al., 2010) is probably one of the closest to the work we propose here in terms of employment of game theory, although its focus is mainly devoted to manage demand response whereas ours is on the incentive mechanism. The game theory usage done by (Nguyen et al., 2012) and (Soliman and Leon-Garcia, 2014) has also been of inspiration for us, since the model we will propose in this work has been heavily influenced by their game theory modeling in the smart grid context. The work from (Loni and Parand, 2017) contains another survey describing the use of game theory for smart grid, although it focuses on collaborative games, whereas our work describes mainly a situation where each agent is selfish.

## 3 MODELIZATION OF THE PROBLEM

In this section the modelization of the problem will be discussed. We built a model from a game theory perspective in order to explain the effect of the incentive mechanism on the agents. Our construction was inspired by (Nguyen et al., 2012) and (Soliman and

Leon-Garcia, 2014).

### 3.1 Game Theory Approach to Our Smart Grid Problems

First, we introduce our notation for game theory. We define a game  $G = (U, S, Q)$  in the following way:

- $U = \{U_1, \dots, U_N\}$  is the set of players.
- $S = \{S_1, \dots, S_N\}$ , where for any  $i \in \{1, \dots, N\}$ ,  $S_i$  is the set of player  $U_i$ 's strategies.
- $Q = \{q_1, \dots, q_N\}$ , where for any  $i \in \{1, \dots, N\}$ ,  $q_i : \prod_{j=1}^N S_j \rightarrow \mathbf{R}$  is the payoff function for player  $U_i$ .

Let us consider a grid with  $N$  users: we will now define a game on it. The players are the grid users, which will thus be denoted as  $U_i$ , for  $i \in \{1, \dots, N\}$ . In what follows we describe the users' payoff functions and the strategies they are allowed to choose.

For each user  $U_i$ , we define a vector  $\mathbf{c}_i$  which indicates the energy *consumption* of  $U_i$ , and a vector  $\mathbf{p}_i$  which indicates the energy *production* of  $U_i$ . The size of these vectors is the number of intervals in which the considered time horizon is divided: for example, in a scenario with a 24 hour time horizon with 15 minutes intervals, the size of  $\mathbf{c}_i$  and  $\mathbf{p}_i$  is 96. Each component of both  $\mathbf{c}_i$  and  $\mathbf{p}_i$  is a non-negative real number. We will denote this number of time intervals as  $T$ .

One important consideration has to be made about the values of  $\mathbf{c}_i$  and  $\mathbf{p}_i$ : in this work we assume that each user self-consumes all the energy she<sup>2</sup> needs before exporting it or imports the extra energy she needs, and the values of  $\mathbf{c}_i$  and  $\mathbf{p}_i$  are considered after self-consumption is taken into account. This, in formal terms, denoting as  $\mathbf{c}_i(t)$  the  $t$ -th element of  $\mathbf{c}_i$ , is written as

$$\mathbf{c}_i(t) \cdot \mathbf{p}_i(t) = 0 \quad \forall t \in 1, \dots, T \quad (1)$$

for each  $i \in \{1, \dots, N\}$ . Of course, if  $U_i$  is a consumer which does not produce energy,  $\mathbf{p}_i$  is the zero vector. Later in this work, we will ensure that all the payment functions we propose actively encourage energy producers to self-consume their produced energy.

Now, we can define the utility function  $q_i$  for each grid user as the sum of the utility in the different time intervals, namely:

$$q_i = \sum_{t=1}^T q_i(t) \quad (2)$$

where the utility of a user  $U_i$  at time interval  $t$  is defined as:

$$q_i(t) = g(\mathbf{p}_i(t), t_p, t_c) - h(\mathbf{c}_i(t), t_p, t_c). \quad (3)$$

<sup>2</sup>from now on, this is to be read as he/she.

where  $g$  and  $h$  are two pre-determined functions with the property that  $g(0, a, b) = h(0, a, b) = 0$  for every  $a, b \in \mathbf{R}$ , and represent respectively the reward for the energy produced, and the cost for the energy consumed. Fixed a time unit  $t$ , the aforementioned quantities  $t_p$  and  $t_c$  are defined as

$$\begin{aligned} t_p &= \sum_{i=1}^N \mathbf{p}_i(t) \\ t_c &= \sum_{i=1}^N \mathbf{c}_i(t) \end{aligned} \quad (4)$$

and represent the total production and the total consumption through the grid at that given time. In particular, at a given time  $t$ ,  $t_p$  depends on every  $\mathbf{p}_i(t)$  and  $t_c$  on every  $\mathbf{c}_i(t)$ .

The formulation of costs and rewards based on functions which take into account the total production and the total consumption at a certain time has been inspired by (Mihaylov et al., 2014): this has been the work we used as baseline to build on top.

Next, we show the possible strategies for the players. More precisely, how the loads can be modeled in our scheme. An example of this has been seen in the work from (Soliman and Leon-Garcia, 2014). In the following we will indicate the loads that we have taken into consideration for our work.

- **Production:** This is  $\mathbf{p}_i$ . It represents the energy production of the user  $U_i$ , and it is a fixed vector unless the energy production is somehow modifiable, which is not our case.
- **Fixed Consumption:** This is a fixed vector  $\mathbf{f}_i$ . It represents the fixed part of the energy consumption of the user  $U_i$ . In this work it will be treated as a known vector, although it has to be forecast and may present uncertainty: a possible approach to this issue can be the one used by (Lilliu et al., 2019a), which treat the components of the vector as probability distributions.
- **Shiftable Load:** This is a vector  $\mathbf{h}_i^j$ , which represents a consumption load of user  $U_i$  which can be shifted in time. The user can control how many time units this vector can be shifted by, although sometimes they have restrictions on this. Given the vector  $\mathbf{h}_i^j$ , the notation  $r_k(\mathbf{h}_i^j)$  will indicate that vector shifted by  $k$  places. In other words,  $r_k$  indicates the rotation by  $k$  places: if  $k = 0$  there is no rotation, therefore  $r_0(\mathbf{h}_i^j) = \mathbf{h}_i^j$ .

Thus, if a user has  $n$  shiftable loads  $\{\mathbf{h}_i^1, \dots, \mathbf{h}_i^n\}$ , and decides to shift each  $\mathbf{h}_i^j$  by a quantity  $k_j$ , we have

$$\mathbf{c}_i = \mathbf{f}_i + \sum_{j=1}^n r_{k_j}(\mathbf{h}_i^j). \quad (5)$$

The set of possible strategies of  $U_i$  is then the set of all her possible vectors  $\mathbf{p}_i$  and  $\mathbf{c}_i$ , depending on how the loads are shifted.

## 4 PEER-TO-PEER MARKET DESIGN

This section presents the proposed peer-to-peer market for energy trading.

### 4.1 Purpose

The goal of this paper is to modelize an incentive mechanism for renewable energy, using as a baseline the NRG-X-Change (Mihaylov et al., 2014) mechanism, but improving some of its critical points. More precisely, these points are:

1. Ensuring that prosumers are not incentivized to curtail their own energy production, unless this is necessary for the grid stability.
2. Taking into account the possibility of a congestion, and actively attempting to prevent it.
3. Encouraging prosumers to self-consume the energy they produce before selling, or before buying from the grid the excess energy they need.
4. Analyzing the behavior of the users if they modulate selfishly their production and consumption, and ensure that they reach an equilibrium.

in their work, (Lilliu et al., 2019b) have addressed the first three points by creating functions which describe rewards for energy production and costs for energy consumption respectively. The scope of this work is to create new functions which address all of the four points mentioned above.

### 4.2 The NRG-X-Change Incentive Mechanism

First, we describe the baseline on which we have built our work. The first thing to be described is NRG-X-Change: it is an incentive mechanism based on the usage of a virtual currency.(Mihaylov et al., 2014)

Its functioning is described as follows. At each time unit, grid users may produce and consume energy. Suppose that at a certain time unit, the user  $U_i$  produces energy and injects an amount  $x$  of energy into the grid. The user is then rewarded a certain amount of NRGcoins for the energy produced. This amount is regulated by a function, which is defined as

$$g(x, t_p, t_c) = x \cdot \frac{q}{e^{\frac{(t_p - t_c)^2}{a}}} \quad (6)$$

where  $q$  and  $a$  are two parameters, the first indicating the maximum value of the tariff, the second calibrating the function.

Similarly, if at a certain time unit the user  $U_i$  consumes an amount  $y$  of energy from the grid, she will pay a certain amount of NRGcoins for that. This amount is regulated by a function, which is defined as

$$h(y, t_p, t_c) = y \cdot \frac{r \cdot t_c}{t_c + t_p} \quad (7)$$

where  $r$  is a parameter which determines the maximum possible value of the tariff.

These tariffs have been chosen in order to regulate consumption, so that users are encouraged to consume energy when local production is higher than local consumption, and discouraged to consume when local production is lower. Also, production is rewarded more when local production and consumption are similar, in order to encourage balance between them.

However, this system has several flaws, which the paper from (Lilliu et al., 2019b) has pointed out and addressed by creating one new function for rewarding energy production and one for determining costs for energy consumption. These functions will be described now.

We indicate with  $t_p^{-i}$  and  $t_c^{-i}$  the values of  $t_p$  and  $t_c$  if  $x$  and  $y$  are both zero. In other words,

$$\begin{aligned} t_p^{-i} &= \sum_{j \neq i} \mathbf{p}_j(t) \\ t_c^{-i} &= \sum_{j \neq i} \mathbf{c}_j(t) \end{aligned} \quad (8)$$

The new selling function  $g$  is defined as

$$g(x, t_p, t_c) = P_{max} \cdot (g_1(t(x, t_p^{-i}, t_c^{-i})) - g_1(t(0, t_p^{-i}, t_c^{-i}))) - P(x, t_p^{-i}, t_c^{-i}) \quad (9)$$

where  $P(x, t_p^{-i}, t_c^{-i})$  is a penalty term, which is positive if  $t_p^{-i}$  and  $t_c^{-i}$  are such that a congestion will happen (i.e.  $|t_p^{-i} + x - t_c^{-i}| > B$ ), and zero otherwise, and  $P_{max}$  is a scaling factor used to determine the tariff. In this definition,  $g_1$  is a function defined as

$$g_1(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ \frac{1}{1 + e^{\frac{2t-1}{t^2-t}}} & \text{if } t \in (0, 1) \\ 1 & \text{if } t \geq 1 \end{cases} \quad (10)$$

which determines the shape, and  $t$  is a function defined as

$$t(x, t_p^{-i}, t_c^{-i}) = \frac{t_p^{-i} + x - t_c^{-i}}{2B} + \frac{1}{2} \quad (11)$$

where  $B$  is a constant, depending on the maximum difference between  $t_p^{-i}$  and  $t_c^{-i}$  which will not correspond to a congestion.

The new buying function  $h$  is defined as

$$h(y, t_c^{-i}, t_p^{-i}) = Q_{max} \cdot h_1\left(\frac{t_c^{-i} + y - t_p^{-i}}{B} + 1\right) \cdot y + P(y, t_c^{-i}, t_p^{-i}). \quad (12)$$

$Q_{max}$  is a parameter used in order to determine the tariff value, corresponding to the maximum cost per unit of energy;  $B$  is a parameter corresponding to the congestion threshold like in the definition of Eq. (11), and  $P(x, t_c^{-i}, t_p^{-i})$  is a penalty factor which is zero for  $t_c^{-i} + y - t_p^{-i} < B$  and positive otherwise.

In the above definition, the function  $h_1$  is defined as:

$$h_1(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ \sqrt{t} & \text{if } t \in (0, 1] \\ 2 - \sqrt{2-t} & \text{if } t \in [1, 2) \\ 2 & \text{if } t \geq 2. \end{cases} \quad (13)$$

These new functions solve many of the issues of the original NRG-X-Change work, but still have to be analyzed from a game theoretical perspective. In order to do so, we have to use the model defined in Section 3 and analyze the behavior of the players with the payoff functions relative to each system.

For the original NRG-X-Change work, it is not difficult to write  $q_i(t)$ : using Eq. 3, it can be written as

$$q_i(t) = x \cdot \frac{q}{e^{\frac{(t_p - t_c)^2}{a}}} \text{ if } y = 0 \quad (14)$$

$$q_i(t) = -y \cdot \frac{r \cdot t_c}{t_c + t_p} \text{ if } x = 0$$

keeping in mind that either  $x$  or  $y$  is equal to zero, as seen in Eq. 1.

As mentioned by (Lilliu et al., 2019b) the expression for  $q_i$  is more complicated to write, although it may be simplified if we consider different cases. Since at least one between  $x$  and  $y$  is equal to zero, substituting Eq. (10) and (11) in Eq. (9) and Eq. (13) in Eq. (12):

- **If**  $y = 0$ , i.e. the user's production is greater than the user's consumption, we have

$$q_i(t) = \frac{P_{max}}{1 + e^{\left(\frac{t_p^{-i} + x - t_c^{-i}}{B}\right)^2 - \frac{1}{4}}} - \frac{P_{max}}{1 + e^{\left(\frac{t_p^{-i} - t_c^{-i}}{B}\right)^2 - \frac{1}{4}}} \quad (15)$$

- **If**  $x = 0$ , i.e. the user's consumption is greater than the user's production, we have
  - If  $t_p^{-i} > t_c^{-i}$ ,

$$q_i(t) = -y \cdot Q_{max} \cdot \sqrt{\frac{t_c^{-i} + y - t_p^{-i}}{B} + 1} \quad (16)$$

- If  $t_p^{-i} < t_c^{-i} + y$ ,

$$q_i(t) = -y \cdot Q_{max} \cdot \left(2 - \sqrt{1 - \frac{t_c^{-i} + y - t_p^{-i}}{B}}\right) \quad (17)$$

An important remark has to be made: since the production suffers from no curtailment, but  $x$  is the value of production minus consumption, a prosumer may change this value by changing her consumption.

### 4.3 New Proposals

Among the points listed in Section 4.1, the fourth is the one which still has to be solved. Basically we want to check whether a Nash Equilibrium (NE) exists for the previously defined incentive systems, and in case it does not, to create new possible selling and buying functions which guarantee the existence of a NE while fulfilling the first three points listed in Section 4.1.

For the first two systems, our strategy has been the following: first, create a simulation of the described game in order to see empirically whether a NE exists or not, and if the answer was positive, find a mathematical proof of its existence. As we will see in Section 5, these functions do not guarantee the existence of a NE.

We wanted to create selling and buying functions which guarantee the existence of a NE in the game we described. For this, we exploited a well-known result: a game with a concave payoff function admits a pure NE (Nguyen et al., 2012). So, if we can create a concave selling function and a convex buying function, the utility function defined in Eq. 3 will be concave and the theorem proven by (Rosen, 1965) will guarantee the existence of a pure NE.

However, this may create conflicts with the other points, in particular with the condition for avoiding curtailment and the self-consumption condition. For the former, it is enough to ensure that the selling function  $g$  is a monotonic function with respect to  $x$ . In order to check the latter, however, we want to see that the following condition

$$g(x, t_p, t_c) < h(x, t_p, t_c) \quad (18)$$

holds for each  $x > 0$  and for each  $t_p, t_c \geq 0$ .

Some ideas for functions that respect the above constraints may be the following. We will denote

$$Z = t_p^{-i} - t_c^{-i} + B. \quad (19)$$

- For the selling function  $g$ :

- **[Logarithm]** One idea may be to base the function on logarithm. In this case, the function can be

$$g(x, t_p, t_c) = k_1 \cdot \left(\ln \frac{x + Z + a_1}{Z + a_1}\right) \quad (20)$$

where  $a_1$  and  $k_1$  are parameters, and  $a_1 > 0$ .

- **[Square root]** Another idea may be to use the square root function as a base. In this case, a possible candidate is

$$g(x, t_p, t_c) = k_1 \cdot (\sqrt{x + Z + a_1} - \sqrt{Z + a_1}) \quad (21)$$

where  $a_1$  and  $k_1$  are parameters, and  $a_1 > 0$ .

- For the buying function  $h$ :

- **[Quadratic]** One possibility is to use the quadratic function. In this case, a candidate may be

$$h(y, t_p, t_c) = k_2 \cdot ((y - Z + a_2)^2 - (a_2 - Z)^2) \quad (22)$$

where  $a_2$  and  $k_2$  are parameters, and  $a_2 > 2B$ .

- **[Square root]** Another possibility for this may be the negative square root function. In this case, our function may have the following form:

$$h(y, t_p, t_c) = k_2 \cdot (\sqrt{Z + a_2} - \sqrt{Z + a_2 - y}) \quad (23)$$

where  $a_2$  and  $k_2$  are parameters, and  $a_2 > 0$ .

To each of the proposed functions a term  $P$  has to be added. For selling functions,  $P$  is a function determined by  $x$ ,  $t_p$  and  $t_c$  such that its value is zero unless  $t_p - t_c > B$ , in which case it is negative. For buying functions,  $P$  depends on  $y$ ,  $t_p$  and  $t_c$  and its value is zero unless  $t_c - t_p > B$ , in which case it is positive. In other words,  $P$  is a penalty term which punishes overproduction/overconsumption when it causes a congestion. Notice that, with the used notation, a congestion occurs if and only if  $Z \notin (0, 2B)$ .

These functions have been designed to have a certain behavior in relation to the variable  $Z$ . When local energy consumption is higher than local energy production, the energy generated by producers is valuable since it helps covering the consumption needs of the community: in this case the selling functions assume higher values, and consequently producers are paid more for the energy they produce. On the other hand, when local energy production is higher than local energy consumption, the surplus energy injected by the producers is not needed by the consumers, so the selling functions assume lower values and energy producers are paid less. As far as the buying functions are concerned, when local energy consumption is higher than local energy production we want to discourage consumers from consuming energy, since energy has to be bought from outside the community: for this reason, buying functions assume higher values and the price for buying energy is higher for consumers. Conversely, when local energy consumption is lower than local energy production, we want to

encourage consumers to consume energy, since they would use the energy generated by the producers: for this reason, buying functions assume lower values and consequently energy costs less for the consumers.

It is not difficult to check that the proposed selling functions are monotonic: the more energy a user produces, the more she is paid for. For this reason the user is discouraged from curtailing her own energy production, since curtailment would decrease her profits. The only exception is the case of congestion for overproduction, where the penalty term discourages producers from generating energy by lowering their profits. This is the first condition described at the beginning of Section 4.1, which is then respected by our functions: we want to ensure that the functions we create satisfy all of them. The second condition that has to be fulfilled is the fact that both selling and buying functions actively consider the possibility of congestion: since the functions we are creating have a penalty term for congestion, they fulfill this condition. We will prove in Section 5.1 that the proposed functions respect also the third condition, while the last condition holds because of the theorem proven by (Rosen, 1965). This theorem implies that our game guarantees the existence of a Nash Equilibrium if the payoff functions are concave; it is easy to check that this is true from Eq. 3, since the proposed selling functions are concave and the proposed buying functions are convex.

## 5 SIMULATIONS AND RESULTS

This section is divided in two parts. In Section 5.1 the constraint for encouraging self-consumption due to pricing is proven to hold for the newly proposed functions, while in Section 5.2 it is proven that the baseline functions do not guarantee the existence of a NE, while the new proposed functions always reach a NE as expected from the theoretical results.

### 5.1 Parameter Settings

The goal of this section is to verify that the functions proposed in this work respect the self-consumption condition, which is expressed in Eq. 18. We will analyze the behavior of the functions, and check if there is a choice for the parameters such that the condition is fulfilled.

### 5.1.1 Candidate 1: Logarithm Selling, Quadratic Buying

This is the case where the selling function  $g$  is the one described in Eq. 20, while the proposed buying function  $h$  is the one described in Eq. 22.

We want to check that Eq. 18 holds for some choice of the parameters of these two functions. By rewriting this condition with the two functions, we want to prove that there is a choice for the parameters  $k_1$ ,  $a_1$ ,  $k_2$  and  $a_2$  such that

$$k_1 \cdot \left( \ln \frac{x+Z+a_1}{Z+a_1} \right) < k_2 \cdot \left( (x-Z+a_2)^2 - (a_2-Z)^2 \right)$$

holds for each  $x > 0$  and  $Z \in (0, 2B)$ .

**Lemma 1.** For each  $x > 0$ ,  $Z > 0$ ,  $a_1 \geq 1$  and  $a_2 \geq 2B + 1$ , and choosing  $k_1 = k_2 > 0$ , the inequality

$$\ln \left( 1 + \frac{x}{Z+a_1} \right)^{k_1} < k_2 \cdot x \cdot (x - 2Z + 2a_2) \quad (24)$$

holds.

*Proof.* Considering the exponent, Eq. 24 becomes

$$\left( 1 + \frac{x}{Z+a_1} \right)^{k_1} < e^{k_2 \cdot x \cdot (x - 2Z + 2a_2)}$$

Since  $k_1 = k_2$ , we can remove the equal exponents and the inequality that we want to prove becomes

$$1 + \frac{x}{Z+a_1} < e^{x \cdot (x - 2Z + 2a_2)}$$

Thus, exploiting the properties  $a_1 \geq 1$  and  $a_2 \geq 2B + 1$ , the chain of inequalities

$$1 + \frac{x}{Z+a_1} \leq 1 + \frac{x}{Z+1} < 1+x < e^{x^2+x} \leq e^{x \cdot (x - 2Z + 2a_2)}$$

completes the proof, since it holds for every value of  $x > 0$  and  $Z \in (0, 2B)$ .  $\square$

**Proposition 1.** The function defined in Eq. 20 with  $a_1 = 1$ , together with the function defined in Eq. 22 with  $a_2 = 2B + 1$ , satisfies the self-consumption condition defined in Eq. 18 if  $k_1 = k_2 > 0$ .

*Proof.* Thanks to Lemma 1, with the aforementioned choice of parameters, the chain of inequalities

$$\begin{aligned} k_1 \cdot \ln \left( \frac{x+Z+a_1}{Z+a_1} \right) &= \ln \left( \frac{x+Z+a_1}{Z+a_1} \right)^{k_1} = \\ &= \ln \left( 1 + \frac{x}{Z+a_1} \right)^{k_1} < k_2 \cdot x \cdot (x - 2Z + 2a_2) = \\ &k_2 \cdot \left( (x-Z+a_2)^2 - (a_2-Z)^2 \right). \end{aligned}$$

holds for each  $x > 0$ ,  $Z \in (0, 2B)$ .  $\square$

### 5.1.2 Candidate 2: Square Root Selling, Negative Square Root Buying

This is the case where the selling function  $g$  is the one described in Eq. 21, while the proposed buying function  $h$  is the one described in Eq. 23.

**Proposition 2.** There is a choice for the parameters  $k_1$  and  $a_1$  for the selling function defined in Eq. 21, and for the parameters  $k_2$  and  $a_2$  for the buying function defined in Eq. 23, such that these two functions fulfill the condition defined in Eq. 18. More specifically, this happens if  $k_1 = k_2 > 0$  and if  $a_1 \geq a_2 + 2B$ .

*Proof.* The self-consumption condition can be written as

$$\begin{aligned} k_1 \cdot \left( \sqrt{x+Z+a_1} - \sqrt{Z+a_1} \right) < \\ k_2 \cdot \left( \sqrt{Z+a_2} - \sqrt{Z+a_2-x} \right). \end{aligned}$$

Substituting  $k_1 = k_2$  and canceling them out, it becomes

$$\sqrt{x+Z+a_1} - \sqrt{Z+a_1} < \sqrt{Z+a_2} - \sqrt{Z+a_2-x}.$$

Rationalizing the denominators, this holds if and only if

$$\frac{x}{\sqrt{x+Z+a_1} + \sqrt{Z+a_1}} < \frac{x}{\sqrt{Z+a_2} + \sqrt{Z+a_2-x}}$$

and considering denominators, since we want to prove it for  $x > 0$ , this is true if and only if

$$\sqrt{x+Z+a_1} + \sqrt{Z+a_1} > \sqrt{Z+a_2} + \sqrt{Z+a_2-x}.$$

Now, if  $a_1 \geq 2B + a_2$ , we have the chain of inequalities

$$\begin{aligned} \sqrt{x+Z+a_1} + \sqrt{Z+a_1} &> 2\sqrt{a_1} \geq \\ 2\sqrt{2B+a_2} &> \sqrt{Z+a_2} + \sqrt{Z+a_2-x} \end{aligned}$$

which holds for every  $x > 0$ ,  $Z \in (0, 2B)$ , and this completes the proof.  $\square$

## 5.2 Results

In this section, we show two important facts. First, the NRG-X-Change incentive system and the one described by (Lilliu et al., 2019b) do not guarantee the existence of a NE in the game described in Section 3. Second, we will see that a NE always exists with the functions we have defined in the end of Section 4, as expected by the known theoretical results.

A script in *Python* language has been created in order to simulate this framework. For this, real data relative to a Cardiff grid with 184 users, 40 of which

are prosumers, have been used. More detailed information about the grid can be found in (Sisinni et al., 2016)<sup>3</sup>.

This is how the game has been simulated:

1. A set of  $N$  users,  $U_1, \dots, U_N$ , is randomly chosen from the 184 Cardiff users: depending on the scenario that we want to simulate,  $N$  is predetermined, and it is the number of users that are prosumers. The predicted consumption and production of each user  $U_i$  is known. Each user has exactly one shiftable load, which is randomly allocated at the beginning.
2. This step is repeated sequentially for each user  $U_i$ , i.e. starting from  $U_1$ , then  $U_2$ , and arriving to  $U_N$ . Each user considers that the values of  $t_p$  and  $t_c$  may have been modified by the actions of previous users. The user  $U_i$  now has the possibility to choose where to allocate her shiftable load. For each possibility, her payoff function is calculated, taking into account the total production and consumption through the grid at each time unit. After this,  $U_i$  allocates her shiftable load in the time unit which maximizes her payoff function  $q_i$ . Performing this step on all the users of the considered set is called an *iteration*.
3. For each user, a check is performed in order to see if any user has changed the allocation of her shiftable load compared to the previous iteration. If none has, the simulation ends. If at least one user has changed it, the game goes back to step 2 with the current allocations.
4. After a certain maximum number of iterations, or if the configuration of the shiftable loads is identical to the one reached in a previous iteration, the game ends.

If the game ends at step 3, the configuration of shiftable loads has reached a Nash equilibrium, since the most profitable strategy for each user is to leave the load in the allocation where it already was. If the game ends at step 4 for finding a configuration already known in the past, the game will never reach a Nash equilibrium, since it will go on a loop. It is not necessary to establish a maximum number of iterations, as the number of configurations is finite and therefore the game will eventually end in one of the two ways that have been just described. However, if computational time gets too high, it may be convenient to choose a limit for those.

These are the cases that have been simulated in order to see whether the algorithm just described converges or not.

<sup>3</sup>To obtain the data please send an email to the MAS<sup>2</sup>TERING coordinator <https://www.mas2tering.eu/>

Table 1: Number of cases in which Nash Equilibrium is reached depending on the used selling/buying functions. The number of simulations carried out is 245 for each case and couple of functions; the size of the grid varies, ranging from 10 to 70 users. The leftmost column indicates the percentage of grid users who produce energy. The functions are, in order from left to right: the original NRG-X-Change functions, the functions proposed in (Lilliu et al., 2019b), the logarithm selling and square buying functions, and the square root selling and negative square root buying functions.

%	Original	Improved	Log/Sq	SqRs
10%	20	203	245	245
30%	114	232	245	245
50%	242	245	245	245

Table 2: Results of the simulations in terms of convergence and self-consumption. The first column describes the functions, the second column indicates the average number of iterations for reaching a NE, and the third column reports how much, on average, the amount of self-consumed energy increases with our optimization.

Functions	Iterations	SelfCons
Original	4.7255	15.7248
Improved	4.8554	20.2876
Log/Sq	4.0370	23.2854
SqRs	3.8872	23.3021

- Payoff functions are determined by the original NRG-X-Change selling and buying functions. (**Original**)
- Payoff functions are determined by the selling and buying functions defined in (Lilliu et al., 2019b). (**Improved**)
- Payoff functions are determined by the selling function in Eq. 20 and the buying function in Eq. 22. (**Log/Sq**)
- Payoff functions are determined by the selling function in Eq. 21 and the buying function in Eq. 23. (**SqRs**)

Since the functions' parameters are arbitrary, they have been chosen so that for  $t_p = t_c$  all the selling functions have the same value, and all the buying functions have the same value, which roughly correspond to the average selling/buying (respectively) tariffs already existing for the Cardiff grid. In Table 1 we report the number of times the game converges to a NE depending on the percentage of energy producers through the grid. Table 2 describes the average number of iterations needed to obtain convergence with each couple of functions (**Iterations**), and the average increase of self-consumed energy through the grid after performing the game (**SelfCons**).

From these results, these considerations follow.



- The functions labeled as **Original** and **Improved** do not guarantee that the algorithm converges to a NE. On the other hand, as expected from the theoretical results, the new proposed functions always converge to a Nash equilibrium.
- The **Improved** functions perform better than the **Original** functions in terms of convergence, for every considered grid size and percentage of prosumers. They also provide better results in terms of increase of self-consumption, although the average number of iterations required for convergence is slightly higher.
- The new proposed functions (**SqRs** and **Log/Sq**) outperform the old functions (**Original** and **Improved**) on all the considered aspects. They always guarantee convergence to a NE, need less iterations to converge compared to the old functions, and also improve self-consumption through the grid.
- Among the new proposed functions, the *SqRs* functions seems to perform better than the *Log/Sq* functions in terms of convergence speed. Also, the amount of self-consumed energy is, on average, slightly higher.

Finally, we report in graphical form the consequences of the game to the daily net consumption through the grid, which achieves an effect of peak-shaving/valley-filling. We took as example the case with 4 prosumers and 36 consumers, although all cases present the same behavior. In Figure 1 the difference between production and consumption of energy is shown at each time unit, before and after optimization.

## 6 CONCLUSIONS AND FUTURE WORK

Local energy communities and peer-to-peer energy trading are expected to be important components of future energy system. This study investigated different mechanisms for regulating energy exchanges between agents in local energy communities. The agents are assumed to act in their own self-interest when optimizing energy usage. The market mechanisms are designed to incentivize production and self-consumption of locally produced renewable energy while matching supply and demand.

The proposed design exhibits advantageous theoretical properties when compared with some existing market mechanisms. The results obtained by simulations using real-world data reveal that incentives

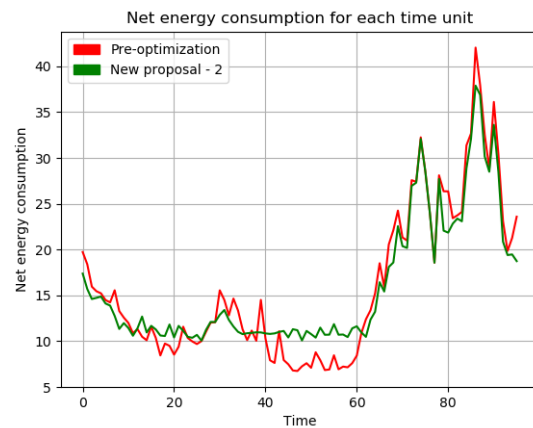


Figure 1: Graph of net consumption of energy before and after the optimization. The  $x$  axis indicates the time units through the day, while on the  $y$  axis is indicated the difference between total consumption and production through the grid. We have depicted the values before the optimization (in red) and after the optimization (in green). It has to be remarked that the shiftable loads were randomly allocated through the day in the initial configuration.

offered by the proposed mechanisms in the form of buying and selling prices lead more sustainable behaviours of energy communities. Consumers schedule their consumption in periods with an intensive injection of renewable energy whereas producers are given a high reward that help discouraging them from curtailment, which can appear under other approaches.

As future work we intend to investigate how the proposed mechanisms behave when diverse sources of flexibility are available including heating systems and storage units.

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