Multiple Ellipse Detection by using RANSAC and DBSCAN Method

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Abstract: In this paper we consider one and multiple ellipse (represented as a Mahalanobis circle) detection problem on the basis of data points coming from one or several ellipses not known in advance. For solving one ellipse detection problem two methods are mentioned. These methods are used by solving the multiple ellipse detection problem. The method proposed in this paper is based on the well-known RANSAC method using the parameters *MinPts* and ε from the DBSCAN method. In this way the efficiency of choosing the best ellipse among N ellipses given by the RANSAC method is improved. The local density $\hat{\rho} = \frac{|\hat{\pi}|}{|\hat{E}|}$ is determined for each obtained ellipse $\hat{E}$ with circumference $|\hat{E}|$ and corresponding cluster $\hat{\pi}$ with $|\hat{\pi}|$ elements . If local density $\hat{\rho}$ is smaller than lower bound $\frac{MinPts}{2\varepsilon}$ of the local density of the whole set $\mathcal{A}$, the ellipse $\hat{E}$ will be dropped. In order to obtain the final solution, an Adaptive Mahalanobis k-means algorithm is applied on the remaining ellipses. The method is illustrated on several examples with artificial data point sets and also on a few real images.

1 INTRODUCTION

Ellipse detection problem plays a specifically significant role in different applications such as pattern recognition and computer vision (Akinlar and Topal, 2013), agriculture, astronomical and geological shape segmentation, images analysis in medicine (Grbić et al., 2016), robotics and object detection, and other image processing (Moshtaghi et al., 2011; Prasad et al., 2013), etc.

In our paper we consider one and multiple ellipse detection problem on the basis of a data point set coming from a number of ellipses in the plane, whereby the edges of these ellipses do not have to be exact or clear and the number of ellipses does not have to be known in advance.

The paper is organized as follows. In the next section the statement of the problem and DBSCAN-parameters are defined. In Section 3 one ellipse detection problem is considered. 2 Section 4 considers the multiple ellipse detection problem. The method based on the RANSAC method with parameters from the DBSCAN-algorithm (Ester et al., 1996) is proposed. Two illustrative examples are given. Finally, some conclusions are given in Section 5.

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2 STATEMENT OF THE PROBLEM

Given is the set of data points in the plane $\mathcal{A} = \{a^i = (x_i, y_i)^T : i = 1, \dots, m\} \subset \Delta$, $\Delta = [a,b] \times [c,d] \subset \mathbb{R}^2$, $a < b, c < d$ which is considered to be scattered along multiple ellipses not known in advance and which should be reconstructed or detected. Additionally, we suppose that the subset of data points $\pi(E) \subset \mathcal{A}$ coming from some ellipse E satisfies the "homogeneity property", i.e. we assume that the set $\pi(E)$ is uniformly scattered around ellipse E, and the number

$$\rho(\pi) = \frac{|\pi(E)|}{|E|}, \tag{1}$$

where $|E|$ is the length of the ellipse E, will be called the local density of data point set $\pi(E)$.

By using the parameters *MinPts* and $\varepsilon(\mathcal{A})$ from the DBSCAN method (Ester et al., 1996) it is possible to estimate the lower bound for the local density of the whole set $\mathcal{A}$. According to (Scitovski and Sabo, 2019b) we determine the parameter $MinPts = \lfloor \log |\mathcal{A}| \rfloor$ and parameter $\varepsilon(\mathcal{A})$ in the following way. For each $a \in \mathcal{A}$ we determine radius $\varepsilon_a > 0$ of the smallest disc centered at a and containing at least *MinPts* elements of the set $\mathcal{A}$. Then 99.5% quantile of the set $\{\varepsilon_a : a \in \mathcal{A}\}$ is marked with $\varepsilon(\mathcal{A})$.

Remark 1. *Since for almost all points $a \in \mathcal{A}$ the corresponding disc with the center a and the radius $\varepsilon(\mathcal{A})$*

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contains at least MinPts elements of the set $\mathcal{A}$, the lower bound of the local density $\rho(\mathcal{A})$ of the whole set $\mathcal{A}$ can be estimated with $\frac{MinPts}{2\varepsilon(\mathcal{A})}$.

Now, let us define the Multiple Ellipse Detection problem (Akinlar and Topal, 2013; Grbić et al., 2016; Marošević and Scitovski, 2015). If the number $k \geq 1$ of ellipses is known in advance, the problem is considered as follows. For the given set of data points $\mathcal{A}$ one should estimate the unknown parameters of ellipses $E_j(p_j, q_j, \xi_j, \eta_j, \vartheta_j)$, $j = 1, \ldots, k$:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} p_j \\ q_j \end{bmatrix} + U(\vartheta) \begin{bmatrix} \xi_j \cos t \\ \eta_j \sin t \end{bmatrix}, t \in [0, 2\pi], \quad (2)$$

where $S_j = (p_j, q_j)^T$ are the centers, $\xi_j, \eta_j > 0$ are the lengths of semiaxes, ϑ_j are the angles and $U(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}$ are the matrices of rotation, by solving the following global optimization problem

$$\underset{p,q,\xi,\eta,\vartheta}{\operatorname{argmin}} F(p, q, \xi, \eta, \vartheta), \quad (3)$$

$$F(p, q, \xi, \eta, \vartheta) = \sum_{i=1}^{m} \min_{1 \leq j \leq k} \mathfrak{D}(a^i, E_j(p_j, q_j, \xi_j, \eta_j, \vartheta_j)),$$

where $\mathfrak{D}$ is some distance-like function defining the distance from a point $a \in \mathcal{A}$ to the ellipse E_j (for example, like (11)).

If E is understood as the cluster center (E-cluster-center), then multiple ellipse detection problem (3) can be interpreted as the problem of searching for a k-globally optimal partition $\Pi = \{\pi_1, \ldots, \pi_k\}$

$$\underset{\Pi \in Part(\mathcal{A};k)}{\operatorname{argmin}} \mathcal{F}(\Pi), \quad (4)$$

$$\mathcal{F}(\Pi) = \sum_{j=1}^{k} \sum_{a^i \in \pi_j} \mathfrak{D}(a^i, E_j),$$

where E_j is perceived as the E-cluster-center of cluster π_j and $Part(\mathcal{A}; k)$ is the set of all k-partitions of the set $\mathcal{A}$ (Morales-Esteban et al., 2014; Scitovski and Scitovski, 2013).

The minimizing function F is nonconvex and nondifferentiable, but, similarly as in (Scitovski and Sabo, 2019a), it can prove to be Lipschitz-continuous. In order to solve the global optimization problem (3) we can apply one of the known global optimization methods (Horst and Tuy, 1996; Paulavičius and Žilinskas, 2014). For example, one could try to solve the problem by using the well-known `DIRECT` global optimization algorithm (Grbić et al., 2013; Jones et al., 1993). However, due to the property of the `DIRECT` algorithm to search for all points of the global minimum, using this algorithm would prove to be a very inefficient procedure (Scitovski and Sabo, 2019a).

In the case of an unknown number of ellipses using, for example, incremental method (see Grbić et al. (2016); Bagirov et al. (2011)), an optimal k-partition for $k = 1, 2, \ldots$ is searched for. After that, by using some indexes adapted for E-cluster-centers (see e.g. Grbić et al. (2016); Scitovski and Sabo (2020)), a partition with the most appropriate number of clusters can be determined.

There are several methods known for solving this problem in the literature. Most of them are based on the Hough transform (Mukhopadhyay and Chaudhuri, 2015), center based clustering (Marošević and Scitovski, 2015; Moshtaghi et al., 2011; Morales-Esteban et al., 2014) or geometric method (Isack and Boykov, 2012; Prasad et al., 2013). The method `EDCircles` proposed in Akinlar and Topal (2013) can be used in real-time applications.

3 DETECTION OF ONE ELLIPSE

We first consider one ellipse detection problem in the plane. An ellipse in the plane can be defined as a set of points $\{(x, y)^T \in \mathbb{R}^2 : P(x, y) = 0\}$, where (Fitzgibbon et al., 1999)

$$P(x, y) = ax^2 + 2bxy + cy^2 + dx + ey + f, \quad (5)$$

where $ac - b^2 = 1$. For a given set $\mathcal{A}$ the optimal values of parameters a, b, c, d, e, f, g can be obtained by solving a constrained linear Least Squares problem

$$\underset{\substack{a,b,c,d,e,f \in \mathbb{R} \\ ac-b^2=1}}{\operatorname{argmin}} \sum_{i=1}^{m} (P(x_i, y_i))^2, \quad (6)$$

and can be written in the standard form (2), as shown in the next subsection.

3.1 An Ellipse as a Mahalanobis Circle

An ellipse $E(S, \xi, \eta, \vartheta)$, $S = (p, q)^T$, written in the standard form (2), can be interpreted as a "unit" Mahalanobis circle (M-circle)

$$E = \{u \in \mathbb{R}^2 : (S - u)^T \Sigma^{-1} (S - u) = 1)\}, \quad (7)$$

where $\Sigma = (\sigma_{ij}) \in \mathbb{R}^{2\times2}$ is the positive definite matrix with eigenvalues ξ^2, η^2. Due to the insurance of the monotonicity property of the k-means algorithm, a normalized Mahalanobis distance-like function $d_M : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}_+$,

$$\begin{aligned} d_M(u, v; \Sigma) &:= \sqrt{\det \Sigma}\,(u - v)^T \Sigma^{-1} (u - v) \\ &= \|u - v\|_\Sigma^2 \end{aligned} \quad (8)$$

is introduced.

An ellipse $E(S,\xi,\eta,\vartheta)$ can be written as M-circle (Scitovski and Sabo, 2019a)

$$E(S,r,\Sigma) = \{u \in \mathbb{R}^2 : d_M(S,u;\Sigma) = r^2\}, \qquad (9)$$

where $r^2 = \sqrt{\det\Sigma} = \xi\eta$ and conversely, an M-circle corresponds to the ellipse $E(S,\xi,\eta,\vartheta)$, where

$$\operatorname{diag}(\xi^2,\eta^2) = U\left(\tfrac{r^2}{\sqrt{\det\Sigma}}\Sigma\right)U^T, \qquad (10)$$

$$U = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}, \quad \vartheta = \tfrac{1}{2}\arctan\tfrac{2\sigma_{12}}{\sigma_{11}-\sigma_{22}}.$$

Also, we can now define the algebraic distance from the point $a \in \mathbb{R}^2$ to the ellipse E (Morales-Esteban et al., 2014; Marošević and Scitovski, 2015)

$$\mathfrak{D}(a,E) = (\|S-a\|_\Sigma^2 - r^2)^2, \qquad (11)$$

where $\|S-a\|_\Sigma^2 = d_M(a,S;\Sigma)$. Similarly, we could define Total Least Squares distance and Least Absolute Deviations distance (Grbić et al., 2016).

3.2 An Ellipse Detection by using Locally Optimization Method

For recognizing an ellipse on the basis of given set of data points $\mathcal{A}$ which comes from an ellipse not known in advance we apply some local optimization method (Newton, Quasi-Newton (Dennis and Schnabel, 1996)). For that purpose, it is necessary to have a good initial approximation. A very good initial approximation is (see Scitovski and Sabo (2019b, 2020)):

$$S_0 = Mean[\mathcal{A}], \qquad (12)$$

where $\Sigma_0 = \frac{1}{m}\sum_{a\in\mathcal{A}}(S_0-a)(S_0-a)^T$ and

$$r_0 = \frac{1}{m}\sum_{a\in\mathcal{A}}\|S_0-a\|_{\Sigma_0}^2,$$

because

$$\sum_{a\in\mathcal{A}}(\|S_0-a\|_{\Sigma_0}^2 - r^2)^2 \geq \sum_{a\in\mathcal{A}}\left(\|S_0-a\|_{\Sigma_0}^2 - \frac{1}{m}\sum_{a\in\mathcal{A}}\|S_0-a\|_{\Sigma_0}^2\right)^2.$$

Note that the matrix Σ_0 is a covariance matrix written using the Kronecker product.

Example 1. *Data point set which comes from the ellipse $E((5,5),5,2,\frac{\pi}{4})$ is shown in Fig. 1a (Grbić et al., 2016). The corresponding initial approximation obtained by* (12) *gives the ellipse shown in Fig. 1b. The final solution obtained by Newton (Dennis and Schnabel, 1996) method is shown in Fig. 1c.*

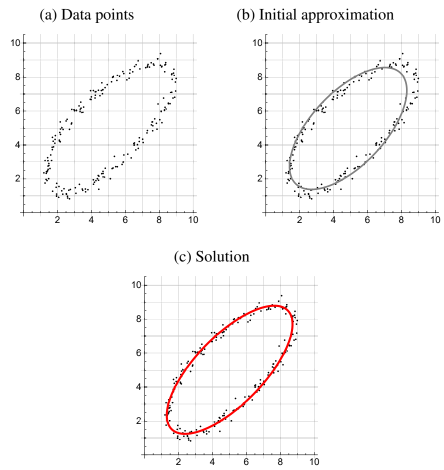

Figure 1: One ellipse detection using local optimization method.

3.3 Ellipse Detection using the RANSAC and the DBSCAN Method

RANSAC-method is proposed in the paper Fischler and Bolles (1981). There are different areas of applications with this method (see e.g. Cupec et al. (2009); Isack and Boykov (2012)). In our paper we also apply this method to solve one and multiple ellipse detection problem.

An ellipse in the form (5) can be determined on the basis of several data points from the set $\mathcal{A}$ by solving constrained linear Least Squares problem (6). For example, if five chosen points $(x_1,y_1)^T,\ldots,(x_5,y_5)^T \in \mathcal{A}$ do not lie on a line, i.e. if the rank of the matrix $\begin{bmatrix} 1 & x_1 & y_1 \\ \vdots & \vdots & \vdots \\ 1 & x_5 & y_5 \end{bmatrix}$ is equal to 3, we obtain an ellipse candidate E, which shall be written in the form of M-circle $E(S,r,\Sigma)$. Additionally, if $E \subset \Delta$, we assume to have gotten an acceptable candidate for the ellipse. Finally, in the $\varepsilon(\mathcal{A})$-neighborhood of the acceptable ellipse we determine the number of points from the set $\mathcal{A}$. We repeat the procedure N times (say, 10) and keep that ellipse $\hat{E}$ for which the corresponding set of points is the largest.

Ellipse $\hat{E}$, written in the form of M-circle $\hat{E}(\hat{S},\hat{r},\hat{\Sigma})$, is a good initial approximation for the ellipse which will be searched for by solving the local optimization problem for the function

$$F(S,r,\Sigma) = \sum_{i=1}^{m}\mathfrak{D}(a^i,E(S,r,\Sigma)), \qquad (13)$$

with the initial approximation $\{\hat{S},\hat{r},\hat{\Sigma}\}$.

Example 2. *The data point set which comes from the ellipse $E((5,5),5,2,\frac{\pi}{4})$ is shown in Fig. 1a (see also*

Grbić et al. (2016)). After $N = 8$ iterations we got five points which resulted in the ellipse shown in Fig. 2a. The final solution obtained by the Newton method is shown in Fig. 2b and coincides with the ellipse obtained in Example 1.

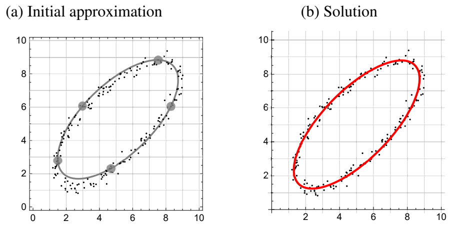

Figure 2: One ellipse detection using the `RANSAC` method.

4 THE MULTIPLE ELLIPSE DETECTION PROBLEM

Now, let us suppose that the data point set $\mathcal{A}$ comes from several ellipses in the plane not known in advance. We will search for the solution of the multiple ellipse detection problem by solving global optimization problem (3), where $\mathfrak{D}$ is the algebraic distance given by (11). Unfortunately, direct application of the well-known clustering method (see e.g. Domeniconi et al. (2016); Scitovski and Scitovski (2013)) is impossible. Also, the method described in Subsection 3.2 cannot be applied in this case. For solving this problem we show a generalization of the method described in Subsection 3.3. The corresponding algorithm will be called `RANSAC` for multiple ellipse detection problem (or abbreviated, `RM`-algorithm) and will be described in the following way.

First, we randomly choose 5 points from the set $\mathcal{A}$ not lying on the line. The ellipse determined on the basis of these points by (6) and contained in rectangle Δ is an acceptable candidate for the searched ellipse. We write this ellipse in the form of `M`-circle $E(S,r,\Sigma)$. By repeating the procedure, we assume that we have found N candidates. In this case the choice of ellipse in whose $\varepsilon(\mathcal{A})$-neighborhood is the largest number of points from the set $\mathcal{A}$ is no longer an acceptable criterion for the best ellipse as it was in the case of one ellipse detection (Subsection 3.3).

In multiple ellipse-case we will suppose that the best ellipse $\hat{E}$ has the largest local density (1) of points in its $\varepsilon(\mathcal{A})$-neighborhood. The cluster $\hat{\pi} := \{a \in \mathcal{A} \colon \mathfrak{D}(a,\hat{E}) < \varepsilon(\mathcal{A})\} \subset \mathcal{A}$ of points from this $\varepsilon(\mathcal{A})$-neighborhood should be dropped from the set $\mathcal{A}$ and the procedure should be repeated on the rest of the set $\mathcal{A} \setminus \hat{\pi}$.

We repeat the whole procedure until the number of the remaining sets becomes smaller than some number given in advance (for example, $5\,MinPts$). In that way we obtain κ ellipses $\hat{E}_j$, $j = 1,\dots,\kappa$.

Furthermore, we determine the local density $\hat{\rho}_j(\hat{E}_j) = \frac{|\hat{\pi}_j|}{|\hat{E}_j|}$ for each pair $(\hat{\pi}_j, \hat{E}_j)$, where $|\hat{\pi}_j|$ is the number of points in the cluster $\hat{\pi}_j$, and $|\hat{E}_j|$ is the length (circumference) of the ellipse $\hat{E}_j$ which can be estimated using the well-known *Ramanujan approximation*

$$|\hat{E}_j| \approx \pi(\hat{\xi}_j + \hat{\eta}_j)\big(1 + \tfrac{3h}{10+\sqrt{4-3h}}\big), \qquad (14)$$

where $h = \frac{(\hat{\xi}_j - \hat{\eta}_j)^2}{(\hat{\xi}_j + \hat{\eta}_j)^2}$ Note that, for that purpose, it will be necessary to write the ellipse $\hat{E}_j$ in the standard form $\hat{E}_j(\hat{S}_j, \hat{\xi}_j, \hat{\eta}_j, \hat{\vartheta}_j)$ (see Subsection 3.1).

Using the lower bound for the local density of the set $\mathcal{A}$ (see Section 2), the ellipses, for which (see Remark 1)

$$\hat{\rho}_j(\hat{E}_j) < \tfrac{MinPts}{2\varepsilon(\mathcal{A})}, \qquad (15)$$

will be dropped. We apply the Adaptive Mahalanobis k-means algorithm to all the remaining ellipses (Grbić et al., 2016; Marošević and Scitovski, 2015; Morales-Esteban et al., 2014). The algorithm can be described in following two steps which are repeated iteratively:

Step A: (Assignment step) For each set of mutually different M-circles $E_1(S_1, r_1, \Sigma_1)$, ..., $E_k(S_k, r_k, \Sigma_k)$, the set $\mathcal{A}$ should be divided into k disjoint nonempty clusters $\pi_1, \dots, \pi_k$ by using the minimal distance principle;

Step B: (Update step) Given a partition $\Pi\{\pi_1, \dots, \pi_k\}$ of the set $\mathcal{A}$, one can define the corresponding M-circle-centers $\hat{E}_j(\hat{S}_j, \hat{r}_j, \hat{\Sigma}_j)$ $j = 1,\dots,k$ by using the methods described in Subsection 3.2 or Subsection 3.3;
Set $E_j(S_j, r_j, \Sigma_j) = \hat{E}_j(\hat{S}_j, \hat{r}_j, \hat{\Sigma}_j)$ for $j = 1,\dots,k$;

Remark 2. *Note that, in this way, the remaining ellipses will be similar to the original ones. Therefore, it will not be necessary to use indexes for detecting the most appropriate partition with ellipses-cluster-centers in the* `RM`*-algorithm.*

Note also that searching for new five randomly chosen points in the `RM`*-algorithm can be repeated so long until the local density for obtained ellipse becomes greater than the lower bound* $\frac{MinPts}{2\varepsilon(\mathcal{A})}$.

Example 3. *Let us consider the data point set $\mathcal{A}$ shown in Fig. 3a which comes from three ellipses. The number of points is $|\mathcal{A}| = 563$, and* `DBSCAN`*-parameters are $MinPts = 6$, and $\varepsilon(\mathcal{A}) = 0.378$. The lower bound for the local density of whole set $\mathcal{A}$ in this case is* 15.9.

In order to detect ellipses from which the data point set $\mathcal{A}$ comes, we apply the RM-algorithm for ellipses detection. First, the red ellipse shown in Fig. 3a is obtained in 4^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 3b remains. After that, the next red ellipse shown in Fig. 3b is obtained in 6^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 3c remains. Finally, the third red ellipse shown in Fig. 3c is obtained in 5^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, not a single point of the set $\mathcal{A}$ remains. By applying Adaptive Mahalanobis k-means algorithm, three ellipses shown in Fig. 3d are detected.

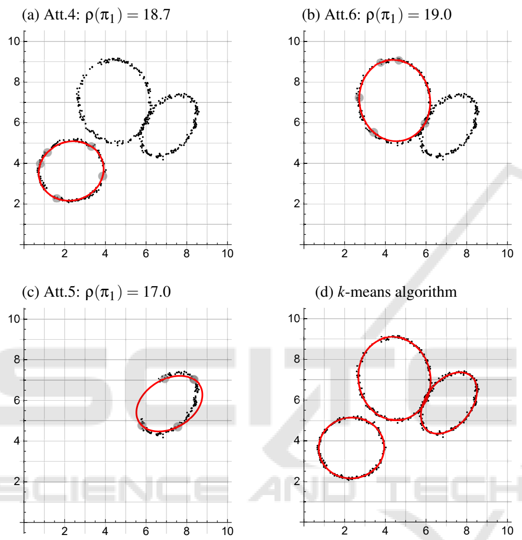

Figure 3: Three ellipses detection from Example 3.

Example 4. *Let us consider data point set* $\mathcal{A}$ *shown in Fig. 4a which comes from four ellipses. The number of points is* $|\mathcal{A}| = 669$ *and* DBSCAN*-parameters are* $MinPts = 6$ *and* $\varepsilon(\mathcal{A}) = 0.284$*. Let us show how the* RM*-algorithm can be applied in this case. The lower bound for the local density in that case is* 10.6*.*

In order to detect the ellipses, from which the data point set $\mathcal{A}$ comes, we apply the RM-algorithm for ellipses detection. First, the red ellipse shown in Fig. 4a is obtained in 61^{st} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 4b remains. After that, the next red ellipse shown in Fig. 4b is obtained in 96^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 4c remains. Furthermore, the next red ellipse shown in Fig 4d is obtained in 103^{rd} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 4e remains. After that, the next red ellipse shown in Fig 4e is obtained in 17^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, the set of points shown in Fig. 4f remains. Finally, the next red ellipse shown in Fig 4f is obtained in 5^{th} attempt. By dropping the points in its $\varepsilon(\mathcal{A})$-neighborhood, 5 points of the set $\mathcal{A}$ remain remaining and theRM-algorithm is finished.

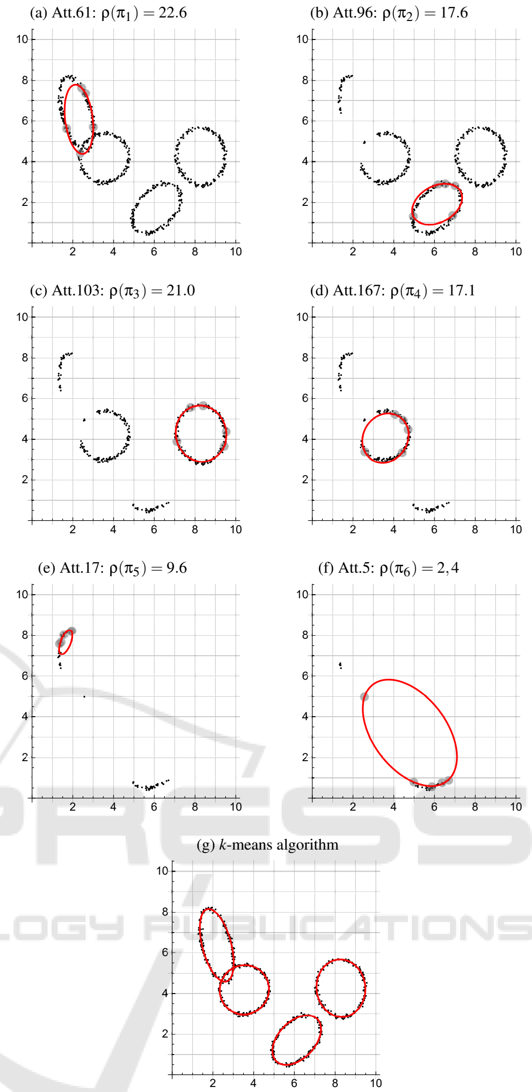

Figure 4: Based on the data point set from Example 4, the RM-algorithm has detected four ellipses.

The clusters corresponding to the first four ellipses have a local density greater than lower bound 10.6, and the last two ellipses have a local density less than the lower bound. Therefore, the last two ellipses will be dropped, and the k-means algorithm will be applied to the remaining four ellipses. The results are shown in Fig. 4g.

Note that the RM-algorithm does not require the use of indexes for recognizing the most appropriate partition with ellipse-cluster-centers. One can try to improve the efficiency of the method by multistarting the method.

4.1 Real-world Problems

The proposed method for solving the multiple ellipse detection problem can also be applied to real images. For that purpose, we carried out the preprocessing of the edge detection by using the Canny filter first (see Bradski (2000); Wolfram Research (2016)).

In Fig 5a a fetal head detection on an ultrasound image is shown. Corresponding edge curves obtained by Canny filter in Fig 5b are shown. The red ellipse in Fig 5a denotes the fetal head. Similarly, in Fig 6, a detection problem for several cups on the table is considered.

(a) Detection (b) Canny filter

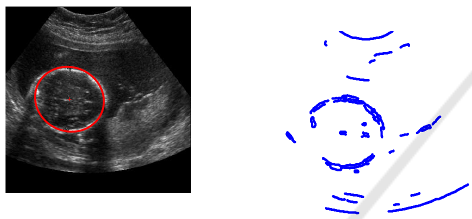

Figure 5: Real image.

(a) Detection (b) Canny filter

Figure 6: Real image.

5 CONCLUSIONS

Solving multiple ellipse detection problem is important in many applications. In our paper one and multiple ellipse detection problem are considered on the basis of a data point set coming from a number of ellipses with noisy edges in the plane. Thereby, we suppose that the subset of data points coming from some ellipse satisfies the “homogeneity property”. For that situation, a method based on the `RANSAC`-method is proposed, whereby the `DBSCAN`-parameters *MinPts* and ε play a significantly important role.

It is important to note that the `RM`-algorithm does not require the use of indexes for recognizing the most appropriate partition with ellipse-cluster-centers. This is the basic advantage of this method regarding the method `EDCircles` given in (Akinlar and Topal, 2013) and method given in (Grbić et al., 2016). Unlike our method, `EDCircles` does not recognize an ellipse with semi-axes (ξ,η), $\frac{\xi}{\eta} \geq 4$ and cannot detect a single ellipse with a clear edge if its shape departs significantly from a circular shape. However, our method requires more computing time than `EDCircles`.

The method proposed in our paper could be applied to the case of other geometrical objects too, but its application is also possible in $3D$.

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REFERENCES

Akinlar, C. and Topal, C. (2013). Edcircles: A real-time circle detector with a false detection control. *Pattern Recognition*, 46:725–740.

Bagirov, A. M., Ugon, J., and Webb, D. (2011). Fast modified global *k*-means algorithm for incremental cluster construction. *Pattern Recognition*, 44:866–876.

Bradski, G. (2000). *The opencv library, Dr. Dobb's Journal of Software Tools*.

Cupec, R., Grbić, R., Sabo, K., and Scitovski, R. (2009). Three points method for searching the best least absolute deviations plane. *Applied Mathematics and Computation*, 215:983–994.

Dennis, J. J. and Schnabel, R. (1996). *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. SIAM, Philadelphia.

Domeniconi, G., Semertzidis, K., Lopez, V., Daly, E. M., Kotoulas, S., and Moro, G. (2016). A novel method for unsupervised and supervised conversational message thread detection. In *Proceedings of the 5th International Conference on Data Management Technologies and Applications (DATA 2016) ISBN:*, pages 43–54.

Ester, M., Kriegel, H., and Sander, J. (1996). A density-based algorithm for discovering clusters in large spatial databases with noise. In *2nd International Conference on Knowledge Discovery and Data Mining (KDD-96)*, pages 226–231, Portland.

Fischler, M. and Bolles, R. (1981). Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography. *Communications of the ACM*, 24:381–395.

Fitzgibbon, A., Pilu, M., and Fisher, R. B. (1999). Direct least square fitting of ellipses. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21:476–480.

Grbić, R., Grahovac, D., and Scitovski, R. (2016). A method for solving the multiple ellipses detection problem. *Pattern Recognition*, 60:824–834.

Grbić, R., Nyarko, E. K., and Scitovski, R. (2013). A modification of the DIRECT method for Lipschitz global optimization for a symmetric function. *Journal of Global Optimization*, 57:1193–1212.

Horst, R. and Tuy, H. (1996). *Global Optimization: Deterministic Approach*. Springer, 3rd, revised and enlarged edition.

Isack, H. and Boykov, Y. (2012). Energy-based geometric multi-model fitting. *International Journal of Computer Vision*, 97:123–147.

Jones, D. R., Perttunen, C. D., and Stuckman, B. E. (1993). Lipschitzian optimization without the Lipschitz constant. *Journal of Optimization Theory and Applications*, 79:157–181.

Marošević, T. and Scitovski, R. (2015). Multiple ellipse fitting by center-based clustering. *Croatian Operational Research Review*, 6:43–53.

Morales-Esteban, A., Martínez-Álvarez, F., Scitovski, S., and Scitovski, R. (2014). A fast partitioning algorithm using adaptive Mahalanobis clustering with application to seismic zoning. *Computers & Geosciences*, 73:132–141.

Moshtaghi, M., Havens, T. C., Bezdek, J. C., Park, L., Leckie, C., Rajasegarar, S., Keller, J. M., and Palaniswami, M. (2011). Clustering ellipses for anomaly detection. *Pattern Recognition*, 44:55–69.

Mukhopadhyay, P. and Chaudhuri, B. B. (2015). A survey of Hough transform. *Pattern Recognition*, 48:993–1010.

Paulavičius, R. and Žilinskas, J. (2014). *Simplicial Global Optimization*. Springer.

Prasad, D. K., Leung, M. K. H., and Quek, C. (2013). Ellifit: An unconstrained, non-iterative, least squares based geometric ellipse fitting method. *Pattern Recognition*, 46:1449–1465.

Scitovski, R. and Sabo, K. (2019a). Application of the DIRECT algorithm to searching for an optimal *k*-partition of the set A and its application to the multiple circle detection problem. *Journal of Global Optimization*, 74(1),:63–77.

Scitovski, R. and Sabo, K. (2019b). DBSCAN-like clustering method for various data densities. *Pattern Analysis and Applications*. DOI: 10.1007/s10044-019-00809-z.

Scitovski, R. and Sabo, K. (2020). The adaptation of the k-means algorithm to solving the multiple ellipses detection problem by using an initial approximation obtained by the DIRECT global optimization algorithm. *Applications of Mathematics*. (accepted for publication).

Scitovski, R. and Scitovski, S. (2013). A fast partitioning algorithm and its application to earthquake investigation. *Computers & Geosciences*, 59:124–131.

Wolfram Research, I. (2016). *Mathematica*. Wolfram Research, Inc., Champaign, Illinois, version 11.0 edition.