# Intelligent Algorithms for Non-parametric Robot Calibration

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Abstract: In this paper, a novel method for non-parametric robot calibration which uses intelligent algorithms is proposed. The non-parametric calibration should prove very useful, because it does not need to identify the geometric parameters of the robot as is the case in parametric calibration. Instead, only the position measurements need to be provided. This could potentially lead to a cheaper and faster calibration process which could simplify its application on different and unique robot geometries. The biggest issue of using neural networks is that they require a lot of data, while for the process of robot calibration a very limited number of measurements is usually collected. In this experiment, the improvement of the hyperparameters of the neural network was attempted by using the genetic algorithms. Simulations also showed that the parametric optimization converges faster and that feed-forward back-propagating neural networks could not correctly simulate the behaviour of complex robots, or problems which used small datasets. However, for simple robot geometries and massive datasets, the neural network successfully simulated the behaviour of the robot. Although the number of measurements needed was well beyond the scope for real world applications, a few possible improvements were suggested for future research.

## 1 INTRODUCTION

Robot calibration is a process which can significantly improve the accuracy of a robot by correcting its positioning errors. The main objective is to establish an accurate mapping between the theoretical model of an idealized robot and an actual measured position. The inconsistencies in predicted and realized locations of a robot are arising from many error sources (for example, manufacturing tolerances, wear and tear, transmission errors, compliance and set-up errors).

The most common method of calibration is parametric calibration, and it focuses on constructing a model of a robot and determining the actual parameters of the robot, thereby improving the positioning accuracy. This type of calibration was extensively researched over the past decades and many solutions have already been given (Gang et al., 2014). The parametric calibration has in most cases four basic steps. First, the geometric model of the robot which describes the relationship between the robot joint space and the actuator space is made. Afterwards, the actual robot locations are measured by using an external measurement device, which is followed by parameter identification based on the differences between the measured and the expected positions. Finally, the implementation of the modified geometric model is made, which provides better positioning of the robot (Švaco et al., 2014).

In recent years, intelligent optimization techniques such as neural networks and genetic algorithms are increasing in popularity in a variety of applications. Genetic algorithms were inspired by the process of evolution and there are many approaches in the calibration techniques which are making use of genetic programming to minimize the difference in the actual and ideal robot positions by identifying the geometric parameters of a robot.

The differential evolution algorithm proved to be very effective and robust in the case of a serial-parallel hybrid robot, and the calibration was giving good results on the simulation of 15 measurements and all of the 54 geometrical error parameters were successfully identified. The precision of the final optimization function approached the scale of $10^{-22}$ (Wang et al., 2012). Even with a smaller number of measurements, the different genetic algorithms were able to com-

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pensate for as much as 87% of the positioning error. The number of measured points was 12, and the performance of the four different algorithms was compared (Barati et al., 2011). Improved quantum particle swarm optimization (IQPSO) also proved as an effective algorithm for the identification of the robot parameters (Wang et al., 2016). The mean squared error (MSE) of 260 positions measured by the laser tracker was 2.80 mm, and after IQPSO optimization, the MSE was only 0.07 mm. Compared with the ordinary particle swarm optimization algorithm (Alici et al., 2006), the convergence speed was improved by 200%. The genetic algorithm can also be used to establish and identify the whole geometric model of a robot (Dolinsky et al., 2007). The mapping between the ideal and the realized robot positions were based on a dataset of 30 robot measurements. Another 30 positions were used for the validation of the calibration and the MSE was improved from 1.85 mm prior to calibration to 0.77 mm after calibration.

Parametric calibration has fast convergence, the computation cost is low and the insight into the error sources is provided. Geometrical errors are usually the main cause of robot inaccuracy and they are responsible for up to 90% of the total positioning errors (Judd and Knasinski, 1990). However, non-geometric error sources (such as joint compliance errors caused by the robot weight and the payload, link deflection errors, backlash in gear transmission and thermal effects) make a smaller but still a significant contribution to the positional error of the robot (Elatta et al., 2004). It is difficult to model these effects parametrically because their number is too great to consider every one of them, but they can be corrected with non-parametric calibration. This type of calibration is independent of the robot model, and one of the methods used for non-parametric calibration is optimization with neural networks. It consists of three basic steps: measurement and the recording of the real and expected position of the robot, training and testing the neural network which simulates the behaviour of the robot and using the neural network output for the compensation of the error.

Neural networks were mostly used as an additional step in the calibration process for compensation of the non-geometric errors after the kinematic parameters have been determined (Aoyagi et al., 2010). The first experiments in using neural networks for robot calibration started three decades ago, and the artificial neural network managed to reduce the absolute positioning error by 1/3 for a 6-DOF manipulator (Takanashi, 1990). This was succeeded with a small dataset of 25 measured points. A Recurrent Neural Network (RNN) was also used for both the simulated and the experimental calibration of a 6-DOF robot (Xiao-Lin Zhong and Lewis, 1995). Only the internal joint measurements were used while the manipulator was in contact with the constraint plane, which generated the identification equations. A Hopfield type RNN was used for solving these equations and kinematic parameters were extracted. In total, 120 points have been used and position accuracy has been improved to the level of robot repeatability. A back-propagating neural network was used to compensate the joint transmitting error, but only after the geometric calibration of the robot (Liu et al., 2007). The input values for the neural network were joint angles and the information about the rotation direction, and the output was the angle by which the motors should rotate. The number of measured points was 19, and the robot workspace size was 15 x 15 x 15 cm. Experimental verification showed the MSE decreased from 3.7 mm prior, to 0.5 mm after the calibration process. More extensive data was provided with automated measuring system, which collected more than 10.000 robot positions and configurations (Zhao et al., 2019). The two-step calibration process consisted of a parametric calibration which identified the geometric parameters, and a non-parametric calibration which identified the nonlinear residual errors by using deep neural networks. The MSE was reduced from 1.81 mm to 0.10 mm. The repeatability accuracy of the robot was 0.05 mm. Other compensations of non-parametric effects were made with neural networks after identification of a robots geometric parameters by using the extended Kalman filter algorithm (Nguyen et al., 2015) and joint angle division (Wang et al., 2019). Experimental validations of both methods confirmed the enhanced position accuracy, which was increased from 3.59 mm to 0.42 mm in the first, and from 17 cm to 4.5 cm in the second example.

Shallow neural networks were used on a simulated dataset, which demonstrated the superior performance of a non-parametric calibration with a neural network in comparison with bilinear and fuzzy interpolations (Bai and Wang, 2019). The absolute position accuracy of a drilling robot was also improved by using the algorithm based on the extreme learning machine (Yuan et al., 2018). The input of the neural network was ideal position of a robot and the output was positional error measured by a laser tracker. The robot controller was directed to compensate for the predicted positional errors. By using this method, the absolute position was improved by 75.89%. It was also shown that choosing different hyperparameters of a neural network could increase the accuracy.

The main advantage of the non-parametric calibration lies in the possibility of simple application to

many different geometries of a robot, which could potentially lower the cost and time needed for the calibration of tailor-made and unique robots. Since the geometric model does not have to be taken into consideration, calibration is not limited to a pre-defined model of a robot. The biggest drawback of this approach is the neural networks tendency to be data-hungry and the need for a large amount of measurements. Furthermore, there are many network hyperparameters which can be adjusted, and this can generate unwanted errors if the hyperparameters are not tuned appropriately for the given optimization problem.

In this paper, we propose a novel method of an intelligent algorithm which could further improve the process of calibration by searching for the best hyperparameters of a back-propagating feed-forward neural network.

## 2 METHODS

Neural networks are universal approximators for nonlinear functions and their behaviour is loosely modeled in resemblance to the interconnected neurons in a biological brain, which transmit and process the data. The problems are usually formulated in a way so that the loss function (the measure of a discrepancy between the solution predicted by the network and the solution which is expected and given by the training set) is minimized on a training dataset, by using an optimization function which provides the means for minimization, for a set number of epochs. The reason for good performance on the various types of problems is the usage of different activation functions which introduce nonlinearities in the algorithm. The challenge which arises from using neural networks for the robot calibration is the amount of data needed for successful optimization. Usually, not many measurements for the calibration process are obtained, because such measurements are time-consuming and expensive. This can be partially compensated by choosing a network architecture and hyperparameters suitable for the specific problem.

While the non-parametric approach with neural networks is overshadowed by their usage as an additional step in parametric calibration, there is little research in their direct applications in the process of calibration. The objective of this experiment was to show if neural networks can behave in the same way as the robot that needs to be calibrated, simulating the exact same errors in the robots resulting orientation and position. However, no insights can be gained about the error sources as in a parametric calibration; with its hundreds or thousands parameters neural networks virtually behave equivalent to a black box because of the sheer complexity of the code. Unlike the parametric calibration, this process does not require both the robot model and the measurements. Instead, only the measurements should be provided.

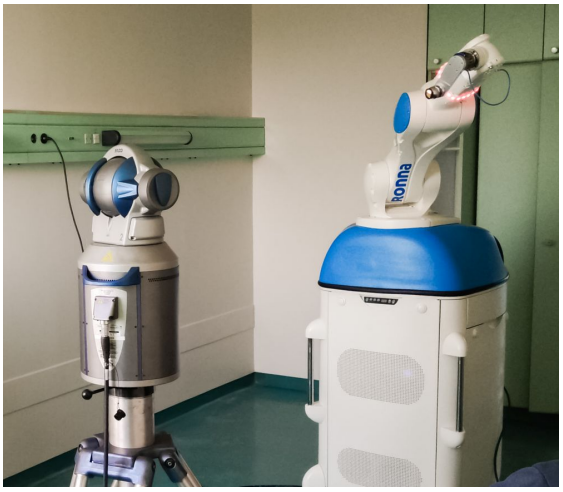


Figure 1: Measurement of the robot positions with a laser tracker.

The positions of the 6-DOF KUKA KR6 R900 robot were measured with a high-accuracy FARO ION laser tracker (Figure 1), with the mean percentage error of 0.018 mm. The theoretical locations which the robot was commanded to reach were recorded as information about the idealized positions ($P_i$), in opposition to measured positions of the robot ($P_m$), and the difference in those two values equals the robot positioning error. A total of 366 robot positions were measured. The data was divided into two groups - the group of idealized positions of the robot ($P_i$) was used as the input for the neural network, and the group of associated measured positions of the robot ($P_m$) was used as the output for the neural network. The dataset was then divided into two sets - the training set with 315 input/output pairs ($P_i$, $P_m$) and the test set with 51 input/output pairs ($P_i$, $P_m$). Every position $P$ consisted of six values, three of which denote spatial coordinates $(x, y, z)$ and the other three which denote the orientation $(r, p, y)$ of the robot.

The neural network used was a feed-forward, back-propagating network, with two hidden layers and interconnected neurons in each layer (the number of neurons was 50). Linear and hyperbolic tangent functions were used as activation functions, mean absolute error was used for the loss function and the Adam optimization (Kingma and Ba, 2014) for minimizing the loss function. The learning rate was adaptive - the staircase exponential decay was used, and the input and output pairs were normalized.

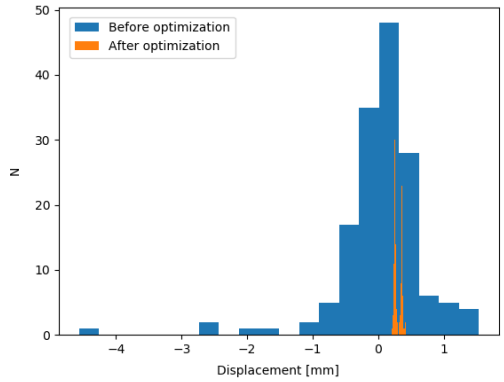
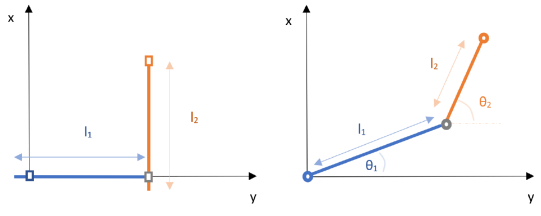



Figure 2: Displacement of the robot manipulator for the KUKA KR6 R900 before and after optimization.

The distribution of the difference between the measured ($P_m$) and idealized positions ($P_i$) for $(x,y,z)$ components before optimization and the distribution of the difference between the expected and calculated positions after the optimization with neural network was calculated (Figure 2). Although the distribution of the error after optimization got smaller by an order of magnitude, the error was still quite large. The reason for such underperformance could perhaps be found in an inadequate choice of network hyperparameters.

For the validation of that assumption, the performance of the same neural network was then analyzed on various robot simulations, which were specially constructed to introduce datasets of robots with various complexity. The input/output pairs ($P_i$, $P_m$) were generated for:

- a planar linear robot with $N$ translational links
- a planar articulated robot with $N$ rotational joints




(a) (b)

Figure 3: Schematic representation of the linear (a) and articulated (b) planar robots in special case with two links.

The planar linear robot with $N$ links (Figure 3a) was generated by stacking new links perpendicularly to the formerly added link. The idealized position ($P_i$) was a set of random $x$ and $y$ positions, while the realized position ($P_m$) had an additional constant error ($\delta x$ and $\delta y$) for every link.

The data for the planar articulated robot with $N$ links (Figure 3b) was generated by choosing a random set of $N$ joint angles $(\theta_1, \theta_2, ..., \theta_N)$ and using the following equation for the idealized positions $P_i(x,y)$:

$$x = \sum_{i=1}^{N} l_i cos(\sum_{j=1}^{i} \theta_i), \quad y = \sum_{i=1}^{N} l_i sin(\sum_{j=1}^{i} \theta_i) \qquad (1)$$

The $l_i$ represents the link length for the $i$th joint. For the calculation of the realized robot positions $P_m(x_{error}, y_{error})$, errors in the link lengths ($\delta l$) as well as errors in the joint angles ($\delta\theta$) were introduced:

$$x_{error} = \sum_{i=1}^{N} (l_i + \delta l_i) cos(\sum_{j=1}^{i} (\theta_i + \delta\theta_i)) \qquad (2)$$

$$y_{error} = \sum_{i=1}^{N} (l_i + \delta l_i) sin(\sum_{j=1}^{i} (\theta_i + \delta\theta_i)) \qquad (3)$$

With these equations for the linear and articulated planar robots, it was possible to quickly generate the "ideal" and "measured" datasets and to examine the performance of a former neural network previously used for an optimization of a KUKA KR6 R900 dataset. The performance of the network was defined as the mean squared error of the difference in $P_m(x_{error}, y_{error})$ given by the dataset and $P'_m(x_{error}, y_{error})$ predicted by the neural network. The results were satisfactory in the case with one link for both robots, but in contrast to the linear robot which showed good performance (although very slowly decreasing as the new links were added), the performance of the neural network for the articulated robot with more than two links significantly decreased by two orders of magnitude (Figure 4). From this result it was possible to conclude that the choice of the neural network hyperparameters used on the specific problem of the 6-DOF articulated robot was inadequate.

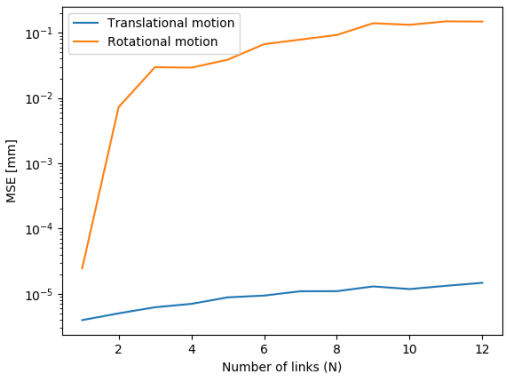



Figure 4: Comparison of the MSE of the linear and the articulated robot after optimization.

One of the most challenging tasks will be to determine the best architecture and hyperparameters of

the neural network for solving the specific calibration problem when non-linearities are introduced. For the calibration attempt of the 6-DOF KUKA KR6 R900 robot, another computer-generated model was created, based on the aforementioned robot.

The robot position is calculated by forward kinematics, for which the Denavit-Hartenberg parameters of the robot $(\theta, \alpha, d, a)$ are used. Here, $\theta$ stands for the joint angle of the robot, while $\alpha, d$ and $a$ are inherent constants which describe the geometry of the robot and define the robot's kinematic chain. The model is also taking into account 24 different kinematic error parameters ($\delta\theta$, $\delta\alpha$, $\delta d$ and $\delta a$). At the beginning and the end of the kinematic chain, the robot base reference frame as well as robot flange reference frame were added. The position of the robot is calculated by multiplication of matrix transformations (Jerbić et al., 2020):

$$T = T_{base} T_1 T_2 T_3 T_4 T_5 T_6 T_{flange} \tag{4}$$

Each transformation for the modified Denavit-Hartenberg notation equals:

$$T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_i \\ s\theta_i c\alpha_i & c\theta_i c\alpha_i & s\alpha_i & d_i s\alpha_i \\ s\theta_i s\alpha_i & c\theta_i s\alpha_i & c\alpha_i & d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, i = 1, ..., 6 \tag{5}$$

The abbreviations for the sine and cosine functions were used ($c = cos$ and $s = sin$). The coordinates and orientations in ($P_i$, $P_m$) dataset can be calculated from $T$.

To find the best hyperparameters of the neural network, the genetic algorithm, which is searching for the best solution from the population of all potential solutions, was constructed. At the beginning, a random population of the individuals was created. In every new generation, the selected individuals are the best performing ones, and the new possible solutions were added to the population by recombinations and mutations of the best-performing individuals. Since the population is generated completely at random, genetic algorithms have slow convergence speed.

The pseudocode is given as follows:

```
t = 0
Population initialization P(t)
Evaluation of the population P(t)
while (t < T):
    t = t + 1
    P'(t) = Selection of the P(t-1)
    P''(t) = Mutation of the P'(t)
    P'''(t) = Recombination of the P''(t)
    Evaluation of the new population P'''(t)
    P(t) = P'''(t)
```

In this paper we consider the novel method for robot calibration by using the neural networks carefully selected by the genetic algorithm and investigate its potential in the future usage.

# 3 RESULTS

The same genetic algorithm was used for both the parametric and the non-parametric optimization, but the population for each problem was generated in a different way. The individual solutions for the parametric optimization represented the correction of the DH parameters used for the calculation of the forward kinematics. Every individual of the population consisted of 24 numbers in the range of $[10^{-8}, 10^{-2}]$, and the data was distributed evenly across the whole range. The input and output pairs were formed by computer-generated datasets for the KUKA KR6 R900, and the data was generated for three datasets with 10, 100 and 1000 pairs of ideal and associated realized locations of the robot. Best-performing individuals were selected and added to the new population by means of crossovers and mutation. A significant improvement in the accuracy was obtained (Figure 5), which confirmed the validity of the usage of genetic algorithms in the parameter identification problems. Although there were no new insights in using the genetic algorithms in robot calibration, the reference for the best obtained accuracy was provided, as well as the time needed for the optimization (Table 1).

Although the number of the individuals in the population was relatively low (30 in each new generation), all of the three different datasets eventually reached the satisfactory accuracy. As expected, the convergence was faster for the largest dataset. Considering the computation time, a significantly less time was needed for the dataset with 100 points. The trade-off for a small increase in precision, as in the optimization of the biggest dataset with 1000 robot positions, means the computation time might be several times longer. However, with different randomly generated initial population, the convergence speed could be a lot faster or slower.

The MSE after optimization was tested on a completely new dataset specifically created for measuring the performance of the algorithm, which is the reason the error does not necessarily decrease in time. Since the generation number varies for every dataset, for comparison were depicted only the first 1641 generations (Figure 5), although the datasets with 10 and 100 points also eventually reached the satisfactory accuracy (Table 1).

The non-parametric optimization had a different

Table 1: The comparison of the mean squared errors after optimization with respect to number of generations and optimization time for various datasets.

| Dataset size | 1000 | 100 | 10 |
|---|---|---|---|
| MSE before optimization [mm] | 7.92 | 8.96 | 14.26 |
| MSE after optimization [mm] | $1.34 \times 10^{-4}$ | $2.04 \times 10^{-4}$ | $5.06 \times 10^{-4}$ |
| Number of generations | 1641 | 4451 | 99996 |
| Optimization time [min] | 490 | 134 | 366 |

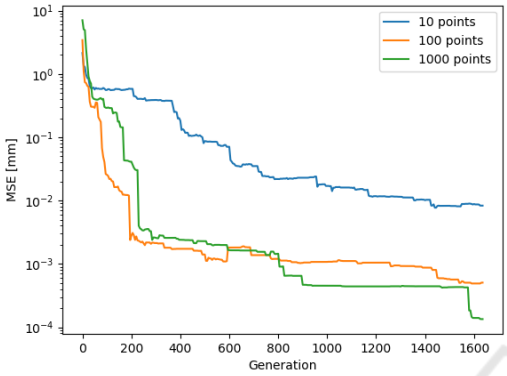



Figure 5: The behaviour of the mean squared errors for the parametric optimization by using the genetic algorithm for three different-sized datasets.

set of initialized population, whose individuals consisted of a variable number of layers, a random number of neurons for each layer, randomly chosen activation functions, optimization functions and loss functions. The batch number was also set to random as well as the normalization range. Sometimes the ability of the algorithm to generalize well worsens with time and longer training time does not always guarantee better results. For that reason, the number of epochs of the neural network was also random.

The performance of the different neural network architectures with different hyperparameters was tested, and the genetic algorithm once again selected the best-performing individuals and added new population of similar solutions by using crossovers and mutations. This process seemed very promising considering the success of both the genetic algorithms and neural networks in robot calibration. However, because of the duration of the optimization process, no new improvement was accomplished (Figure 6). Time needed for reaching the 500th generation was 166 minutes. This would not be considered long given the significant improvement in accuracy. But in comparison to parametric calibration, the search for the best neural network architecture was underperforming. The mean squared error for the best-ranked neural network after 500 epochs was 0.73 mm, and for the best-ranked parametric calibration for the 10, 100 and 1000 points were $5.06 \times 10^{-4}$ mm, $2.04 \times 10^{-4}$ mm and $1.34 \times 10^{-4}$ mm, respectively.

The reason for this might be found in the behaviour of the loss function in the parametric space. It was generally believed that neural network should search for global optima and the local optima solutions were not seemed as important. However, the new research is starting to show that the parametric space is very densely populated by a large number of local minima which give results almost as good as those of the global minimum (Kawaguchi et al., 2019). No matter what parameters we choose, there is a great probability the neural network's solution found in the local minima will be good enough for practical applications. In this light, the usage of the genetic algorithm for choosing the right neural network architecture is not the best way the robot calibration could be done.

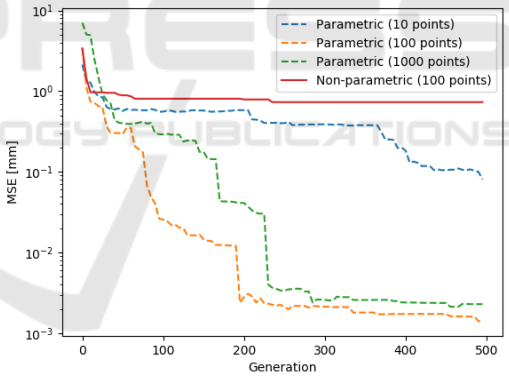



Figure 6: The comparison of the MSE for the parametric and non-parametric optimization.

Since the attempt at the optimization did not reach the expected values (Figure 2), and no new insights into the architecture of the neural network were gained, the validity of the usage of neural networks for optimization in robot calibration was questioned. The generated input and output pairs $(P_i, P_m)$ for the KUKA KR6 R900 robot were used for verification of the universal approximation theorem, which claims that the neural networks should be able to approximate arbitrary and continuous non-linear functions to any desired accuracy (Kidger and Lyons, 2019). To verify this theory, the random neural network architecture was generated, as well as the data in certain

volume range. For the first dataset, the points were generated in the $2m^3$ volume. For the second dataset, the points were generated in the $2dm^3$ volume, and for the third dataset in the $2cm^3$ volume. The various dataset sizes were used.

The randomly chosen parameters of the neural network were generated, and the theorem was confirmed - the neural network reached the desired accuracy for the dataset in the small volume because of the largest density of the data (Figure 7). With more data, the optimization process would perform even better. However, for the purpose of the non-theoretical robot calibration, the number of measured points should be below four-digit number.

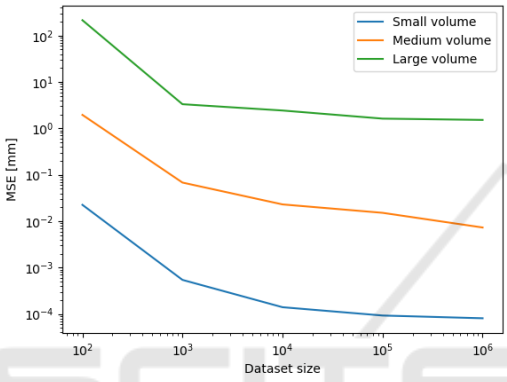



Figure 7: The behaviour of the MSE with respect to the size of the input and output datasets, for different volumes.

## 4 DISCUSSION AND CONSLUSION

Since the non-parametric robot calibration by using neural networks was sparsely researched, the possibility of calibration was immediately tested on the KUKA KR6 R900 robot. The usage of the neural networks in the calibration problems showed unsatisfactory results in simulating the behaviour of a 6-DOF articulated robot. For that reason, computer-simulated datasets for two planar robots with different complexities were generated. For the linear robot the calibration proved possible, as opposed to the articulated robot for which the optimization proved impossible as soon as additional complexity was introduced.

There are many hyperparameters of the neural network which can be tuned. Otherwise the network does not produce good approximations for the given optimization problem. In this paper, a novel method which uses the genetic algorithms to find an appropriate network architecture was proposed, since the choice of appropriate hyperparameters is of extreme importance.

Besides the non-parametric, a parametric-optimization method was tested for reference. The genetic algorithm was also used, but in this case, the population consisted of the geometric parameters which were needed to be identified. The comparison of the non-parametric and the parametric optimization was made on a simulated dataset, and the faster convergence and better performance of the parametric optimization was demonstrated.

The validity of the non-parametric calibration attempt was also questioned and tested with massive datasets. For massive datasets, the feed-forward back-propagating neural network proved as a good optimizer. However, for the simulation of the complex robot behaviour, more specific types of network structures should be researched. Preferably, the number of the measured positions should be as small as possible to speed up both the measuring and the calibration process, but large enough for the model to be able to generalize well on the whole working space of the robot.

The solutions considered in this paper are time-invariant, which does not hold true for the real robot. Because of the possibility for some manipulators to reach certain points by alternative joint configurations, the positional errors will be different. It should prove valuable to consider, in the future research, how the former movement of the robot affects the error and if it is possible to make a correction with the neural networks. By using recurrent networks such as the LSTM (Long Short-Term Memory), which process entire sequences of data, it might be possible to address this issue. The use of Bayesian Neural Networks might also produce better approximations on smaller datasets because they could extract more information than other neural networks.

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