Devising Asymmetric Linguistic Hedges to Enhance the Accuracy of NEFCLASS for Datasets with Highly Skewed Feature Values

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Abstract: This paper presents a model to address the skewness problem in the NEFCLASS classifier by devising several novel asymmetric linguistic hedges within the classifier. NEFCLASS is a common example of the construction of a NEURO-FUZZY system. The NEFCLASS performs increasingly poorly as data skewness increases. This poses a challenge for the classification of biological data that commonly exhibits feature value skewness. The objective of this paper is to device several novel asymmetric linguistic hedges to modify the shape of membership functions, hence improving the accuracy of NEFCLASS. This study demonstrated that devising an appropriate asymmetric linguistic hedge significantly improves the accuracy of NEFCLASS for skewed data.

1 INTRODUCTION

NEURO-FUZZY systems are common machine learning approaches in healthcare because of their ability to learn and formulate rules from training data and represent the knowledge in an understandable way. NEFCLASS is a popular NEURO-FUZZY classifier in clinical research. The NEFCLASS classifier performs poorly on skewed datasets (Yousefi and Hamilton-Wright, 2016). This poses a challenge for the classification of biological data which commonly exhibits positive skewness. Addressing skewness in medical diagnosis systems is vital for finding rare events, such as rare diseases (Gao et al., 2010).

This is an extension of our previous studies published at the 9th and 11th International Joint Conference on Computational Intelligence (Yousefi and Hamilton-Wright, 2016; Yousefi et al., 2019). In the previous paper, we performed a study to analyze the relationship between skewness and the classification accuracy of classifiers. The study showed that the misclassification percentages of five examined classifiers from different families, i.e., NEFCLASS, ANFIS, BP-ANN, PD, and SVM, are significantly increased as the level of skewness is increased in the datasets. Also, the study indicated that the behaviour of NEFCLASS can dramatically change depending on the underlying data distribution. Further analysis of the NEFCLASS behaviour showed that the choice of discretization method affects the classification accuracy of the NEFCLASS classifier and that this effect was very strong in skewed datasets. The study demonstrated that the fuzzy sets constructed by the EQUAL-WIDTH discretization method, used in NEFCLASS, do not reflect the data distribution. This fault resulted in a classification accuracy that is lower for the NEFCLASS classifiers than for other techniques, especially when the feature values of the training and testing datasets exhibit significant skew. This motivated us to use methods such as Maximum Marginal Entropy (MME) and Class Attribute Interdependence Maximization (CAIM) which take into account the statistical information of data. Our study proved that utilizing MME and CAIM discretization methods in NEFCLASS significantly improved the classification accuracy for highly skewed data (Yousefi and Hamilton-Wright, 2016).

In this paper, we present a novel approach to modify the shape of membership functions where their shape resembles the skewness in the data. This leads to minimizing the effect of bias within the data, hence improving the accuracy of the NEFCLASS classifier for skewed datasets. NEURO-FUZZY systems store their knowledge as linguistic values between input neurons and rule layers. We hypothesized that adding weights to the connections between features with skewed distribution and rules increases the influence of those features on the decision making pro-

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cess. This motivated us to use asymmetric linguistic hedges for increasing connection weights between neurons, hence increasing the membership values of the skewed features. We show that this novel hybrid model based on the combination of an appropriate discretization method and an appropriate asymmetric hedge significantly improves the accuracy of NEFCLASS when dealing with positive skewed datasets (Yousefi, 2018).

The model is trained on several synthetically generated datasets with different levels of feature values skewness. Besides, we conducted a set of experiments to evaluate the effectiveness of our approaches for two real-world datasets, Electromyography and Wisconsin Diagnostic Breast Cancer datasets, which are known to have highly skewed feature values. We evaluated the performance of the classifiers using misclassification percentages and the number of rules.

The next section of this paper contains a short review of the NEFCLASS classifier, discretization methods, and linguistic hedges that will be used to modify the NEFCLASS classifier. Section 3 describes the methodology of our study, and in section 4 the experimental results and statistical analysis are given. Finally, conclusions are presented.

2 BACKGROUND

2.1 The NEFCLASS Classifier

NEFCLASS (Nauck et al., 1996; Nauck and Kruse, 1998; Klose et al., 1999) is a NEURO-FUZZY classifier that generate fuzzy rules and tune the shape of the membership functions to determine the correct class label for a given input. NEFCLASS consists of a three-layer fuzzy perceptron containing a heuristic learning algorithm based on fuzzy error propagation. A three-layer fuzzy perceptron has the same structure as a three-layer-feed-forward neural network, but the weights between the input neurons and the hidden neurons are modelled as fuzzy sets, and the links between hidden neurons and output neurons are unweighted. Fig. 1 shows a NEFCLASS model that classifies input data with two features into two output classes by using three fuzzy sets and two fuzzy rules. The fuzzy sets and the fuzzy rules are obtained from the training data through a supervised learning algorithm. Input features are supplied to the nodes at the bottom of the figure. These are then fuzzified, using a number of fuzzy sets. The sets used by a given rule are indicated by linkages between input nodes and rule nodes. If the same fuzzy set is used by multiple rules, these links are shown passing through an

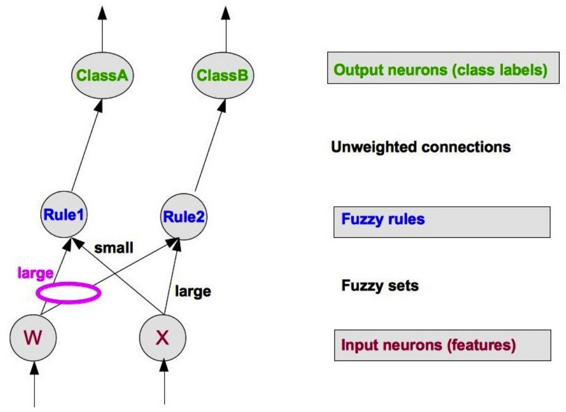


Figure 1: A NEFCLASS model with two inputs, two rules, and two output classes. The figure extracted from (Yousefi and Hamilton-Wright, 2016).

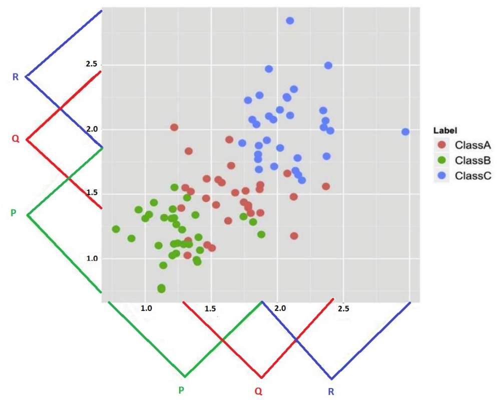
oval, such as the one marked "large" in Fig. 1. Rules directly imply an output classification, so these are shown by unweighted connections associating a rule with a given class. Multiple rules may support the same class, however that is not shown in this diagram.

In Fig. 2a, a set of initial fuzzy membership functions describing regions of the input space are shown, here for a two-dimensional problem in which the fuzzy sets are based on the initial discretization produced by the EQUAL-WIDTH algorithm. As will be demonstrated, NEFCLASS functions work best when these regions describe regions specific to each intended output class, as is shown here, and as is described in the presentation of a similar figure in the classic work describing this classifier (Nauck et al., 1996, pp. 239).

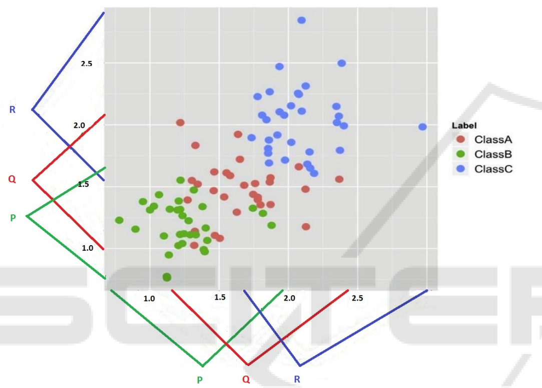
As is described in the NEFCLASS overview paper (Nauck and Kruse, 1998, pp. 184), a relationship is constructed through training data to maximize the association of the support of a single fuzzy set with a single outcome class. This implies both that the number of fuzzy sets must match the number of outcome classes exactly, and in addition, that there is an assumption that overlapping classes will drive the fuzzy sets to overlap as well.

Fig. 2a shows the input membership functions as they exist before membership function tuning performed by the original NEFCLASS algorithm, when the input space is partitioned into EQUAL-WIDTH fuzzy intervals.

Fig. 2b demonstrates that during the fuzzy set tuning process, the membership function is shifted and the support is reduced or enlarged, in order to better match the coverage of the data points belonging to the associated class, however as we will see later, this process is strongly informed by the initial conditions set up by the discretization to produce the initial fuzzy



(a) Initial fuzzy set membership functions in NEFCLASS, produced using EQUAL-WIDTH discretization



(b) Results of tuning the above membership functions to better represent class/membership function information

Figure 2: Fuzzy membership functions before and after training data based tuning using the NEFCLASS algorithm. The figure extracted from (Yousefi and Hamilton-Wright, 2016).

membership functions.

There are three different modes to be used for rule selection in NEFCLASS. These modes are based on the performance of a rule or on the coverage of the training data. The three options for the rule selection mode presented here are *Simple, Best* and *BestPerClass*. The *Simple* rule selection chooses the first generated rules until a predefined maximum number of rules is achieved. The *Best* rule selection is an algorithm that ranks the rules based on the number of patterns associated with each rule and select the rules from this list. The *BestPerClass* option is selection of rules by creating an equal number of rules for each class. This method uses the *Best* rule selection algorithm to ranks the rules.

After the construction of the fuzzy rules, a fuzzy set learning procedure is applied to the training data, so that the membership functions are tuned to better match the extent of the coverage of each individual class in the training data space (Nauck et al., 1996, pp. 239). Fuzzy membership functions will grow or shrink, as a result, depending on the degree of ambiguity between sets and the dataset coverage.

2.2 Discretization

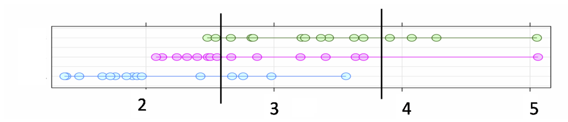
A discretization process divides a continuous numerical range into a number of covering intervals where data falling into each discretized interval is treated as being describable by the same nominal value in a reduced complexity discrete event space. In fuzzy work, such intervals are then typically used to define the support of fuzzy sets, and the precise placement in the interval is mapped to the degree of membership in such a set.

In the following discussion, we describe the EQUAL-WIDTH, MME, and CAIM discretization methods. For example, imagine a dataset formed of three overlapping distributions of 15 points each, as shown with the three coloured arrangements of points in Fig. 3. The points defining each class are shown in a horizontal band, and the points are connected together to indicate that they are part of the same class group. In parts 3a and 3b, the results of binning these points with two different discretization techniques are shown. The subfigures within Fig. 3 each show the same data, with the green, red and blue rows of dots (top, middle and bottom) within each figure describing the data for each class in the training data.

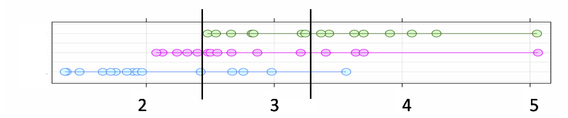
Fig. 3a demonstrates the partitioning using EQUAL-WIDTH intervals. Note that the intervals shown have different numbers of data points within each (21, 19 and 5 in this case).

2.2.1 Marginal Maximum Entropy

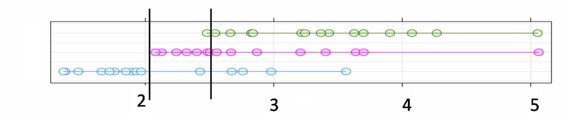
Marginal Maximum Entropy based discretization (MME) (Chau, 2001; Gokhale, 1999) divides the dataset into a number of intervals for each feature, where the number of points is made equal for all of the intervals, under the assumption that the information of each interval is expected to be equal. The intervals generated by this method have an inverse relationship with the points' density within them. Fig. 3b shows the MME intervals for the example three-class dataset. Note that the intervals in Fig. 3b do not cover the same fraction of the range of values (*i.e.*, the widths differ), being the most dense in regions where there are more points. The same number of points (15) occur in each interval. In both of these discretization strategies, class identity is ignored, so



(a) EQUAL-WIDTH



(b) MME



(c) CAIM

Figure 3: Three discretization techniques result in different intervals produced on the same three-class dataset. The figure extracted from (Yousefi and Hamilton-Wright, 2016).

there is likely no relationship between class label distribution and discretization boundary.

2.2.2 Class-Attribute Interdependence Maximization

CAIM (class-attribute interdependence maximization) discretizes the observed range of a feature into a class-based number of intervals and maximizes the inter-dependency between class and feature values (Kurgan and Cios, 2004). CAIM discretization algorithm divides the data space into partitions, which leads to preserve the distribution of the original data (Kurgan and Cios, 2004), and obtain decisions less biased to the training data.

Fig. 3c shows the three CAIM intervals for our sample data set. Note how the placement of the discretization boundaries is closely related to the points where the densest portion of the data for a particular class begins or ends.

2.3 The Modified NEFCLASS Classifier using Alternative Discretization Methods

The fuzzy sets constructed by the EQUAL-WIDTH discretization method do not reflect the data distribution. Modification of NEFCLASS through alternative discretization methods takes into account an important difference between the discretization methods and their effects on the classifier's accuracy. In our previous study (Yousefi and Hamilton-Wright, 2016), we implemented a modified NEFCLASS classifier, embedded with a choice of two alternative discretization methods, MME and CAIM, here are called NEF-MME and NEF-CAIM. We showed that NEF-MME and NEF-CAIM achieved greater classification accuracy when dealing with skewed distributed data. Since the accuracy of NEF-MME and NEF-CAIM were significantly higher than NEFCLASS, we used only NEF-MME and NEF-CAIM in this study, and NEFCLASS was discarded.

2.4 Linguistic Hedges

Membership function parameters can be defined and tuned in a number of ways. For one, the shape of membership functions can be slightly modified by using linguistic hedges (Zadeh, 1965). Linguistic hedges are fuzzy operators that increase or decrease the membership degrees of the associated fuzzy sets. A new membership function, for example, can be obtained by applying a power or square root to the existing membership function. In other words, linguistic hedges modify the meaning of a membership function. For example, the linguistic hedge *"very"* changes the meaning of the linguistic variable *"tall"* to *"very tall"*. This approach allows a composite linguistic variable to be generated from the primary terms (Huynh et al., 2002).

Using linguistic hedges allows for the description of more complex relationships among variables, hence, leading to an improvement in the efficiently of a fuzzy system. Using linguistic hedges helps to tune the membership functions, which can lead to an increase in the classification accuracy. Zadeh (Zadeh, 1965) categorized linguistic hedges into three different operations: concentration, dilation, and contrast intensification. The concentration and dilation hedges that used in this work are defined as follow.

- **Concentration Hedge**
  Applying a concentration hedge to a fuzzy set P decreases the degree of the membership function of x in the fuzzy set P while retaining the same support. The hedge operation of $Con(\mu_P(x))$ is defined as :

$$Con(\mu_P(x)) = \mu_P^{\alpha}(x); \alpha > 1. \quad (1)$$

  Based on the concentration definition, some hedge operations, such as *"more", "much more", "very", "extremely"*, and *"absolutely"* can be defined by specifying the values of α in the equation 1 as 1.25, 1.5, 2, 3, and 4, respectively.

- **Dilation Hedge**
  Applying dilation hedge to a fuzzy set P increases the degree of the membership function of x in the

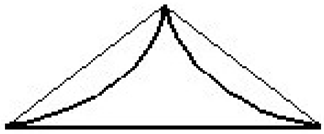
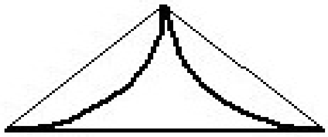
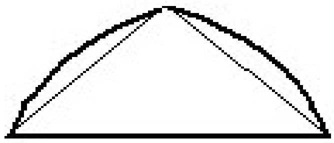
Hedge	Mathematical Expression	Graphical Representation
VERY	$Con(\mu_P(x)) = \mu_P^2(x)$	
ABSOLUTELY	$Con(\mu_P(x)) = \mu_P^4(x)$	
FAIRLY	$Dil(\mu_P(x)) = \mu_P^{\frac{1}{2}}(x)$	

Figure 4: Examples of linguistic hedges.

fuzzy set P while retaining the same support. The dilation hedge operation of $Dil(\mu_P(x))$ is defined as:

$$Dil(\mu_P(x)) = \mu_P^{\alpha}(x); 0 < \alpha < 1. \qquad (2)$$

Based on the dilation definition, some hedge operations, such as *"fairly"*, *"somewhat"*, and *"slightly"*, can be defined by specifying the values of α in the equation 2 as $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively.

Figure 4 shows three examples of concentration and dilation hedges.

In this work, we build several asymmetric linguistic hedges, based on the concentration and dilation, and incorporate them into a neuro-fuzzy system to address the skewness problem. In total six linguistic terms, including VERY, EXTREMELY, ABSOLUTELY, FAIRLY, SOMEWHAT, and SLIGHTLY will be used to build the asymmetric hedges. These hedges are discussed in more detail in Section 3.

3 METHODOLOGY

In this section, we present the system design, the experimental setup, the classifiers' configurations, the evaluation measures, and the datasets that are used in our experiments.

3.1 System Design

In this section we describe the modifications made to the NEFCLASS design. NEFCLASS consists of the following major components (the new design components are denoted as (**modified NEFCLASS**) :

- Initialization
  - Initialization of fuzzy sets using one of the three options:
    - ∗ EQUAL-WIDTH (This is the only choice provided by the original NEFCLASS)
    - ∗ Marginal Maximum Entropy (MME) (**modified NEFCLASS**)
    - ∗ Class Attribute Interdependent maximization (CAIM) (**modified NEFCLASS**)
  - Initialization of fuzzy rules
- Rule learning
- Fuzzy sets learning
- Fuzzy sets tuning by linguistic hedges (**modified NEFCLASS**)

The detailed of the above steps are discussed as follows:

Step1: Initialization

- Initialization of fuzzy sets (**modified NEFCLASS**): our proposed modified NEFCLASS begins by selecting one of three possible types of discretization methods, namely, EQUAL-WIDTH, MME, and CAIM methods.
- Initialization of fuzzy rules: after the initialization of the fuzzy sets, the initialization of fuzzy rules takes place. The fuzzy rules' antecedents are completed by adding fuzzy sets for each feature where triangular membership functions are used.

Step 2: Rule Learning

After constructing the initial fuzzy sets and the antecedents of fuzzy rules, the fuzzy rule learning procedure is applied to the training data. At this phase, the activation of a rule unit and the activation of the output unit are computed for each pattern.

Step 3: Fuzzy Sets Learning

After the construction of the fuzzy rules, a fuzzy sets learning procedure is applied to the training data such that the membership functions are tuned to better match the extent of the coverage of each individual class in the training data space, as shown in Figure 2.1.

Step 4: Fuzzy Sets Tuning by Asymmetric Linguistic Hedges (modified NEFCLASS)

At this phase, a linguistic hedge can be selected to adjust the membership functions. The objective of this step is to increase or decrease the membership functions using linguistic hedges. The linguistic hedge parameter takes the type of the hedge. The parameter can be set to NONE or one of the 11 asymmetric linguistic hedges provided. The effect of the various settings of the parameter is the main focus of this work.

3.2 Improving Accuracy by Asymmetric Linguistic Hedges

NEURO-FUZZY systems stores their knowledge as linguistic values between neurons of input and rule layers (Bargiela and Pedrycz, 2001, p. 276). Adding weights to the connections between features and their associated rules increases the influence of those features on the decision making process. This motivates us to use asymmetric linguistic hedges for increasing connection weights between neurons, hence increasing the membership values of the skewed features. We argue that asymmetric hedges can be used to express the information distribution and bias membership functions toward bias within data. We hypothesize that if the shape of a membership function resembles the skewness in the data, the information distribution will be similar to data distribution; it will minimize the effect of bias within data, thus improving the accuracy of the classifier. In particular, we examine the treatment of positively skewed data. However, this approach can be extended and modified for treatment of negative skewness. Our design modification aims to improve the accuracy of NEFCLASS, uses asymmetric linguistic hedges to tune and optimize the membership functions. Hence, the objective of this paper is to answer the research question as follows:

Does devising asymmetric linguistic hedges improve the accuracy of the NEFCLASS classifier for skewed datasets?

- **Null Hypothesis:** There will be no significant decrease in the misclassification percentage of the NEFCLASS classifier after applying the asymmetric hedges.
- **Alternative Hypothesis:** Applying asymmetric linguistic hedges to the membership functions significantly reduces the misclassification percentage of NEFCLASS for skewed data.

Our asymmetric hedges apply different hedges to each side of a membership function. The effect of asymmetric hedges results in the skewing of a membership function in a positive or negative direction (Bargiela and Pedrycz, 2001). Table 1 displays the name, the mathematical operation, and the type of the 11 asymmetric hedges that are defined for our experiments. The name assigned to each asymmetric hedge has been chosen to reflect the type of operation and the amount of change that are applied to each side of the membership functions.

Five asymmetric hedges are defined to change the right side of a membership function, while the left side remains unchanged. For example, the BIG-CONCAVERIGHT hedge applies ABSOLUTELY to the right side of the membership function, which results in a big decrease (concavity) on the right side, while the left side remains unchanged. The other six hedges apply a concentration operation on one side and a dilation operation on the other side. For example, BIG-CONVEXLEFT-CONCAVERIGHT hedge applies SLIGHTLY to the left side, and ABSOLUTELY to the right side of the triangular fuzzy set, which results in a big increase of the membership function in the left side (convexity), and a big decrease in the right side (concavity).

In this work we will use the terms $MF^{(2)}$, $MF^{(3)}$, $MF^{(4)}$, $MF^{(\frac{1}{2})}$, $MF^{(\frac{1}{3})}$, and $MF^{(\frac{1}{4})}$ to denote VERY, EXTREMELY, ABSOLUTELY, FAIRLY, SOMEWHAT, and SLIGHTLY, respectively. Note that in tables and figures, linguistic hedges have been replaced with these terms for the sake of clarity of the operations and to save space.

3.3 Synthesized Datasets

Three synthesized datasets were used for experiments. The synthesized datasets were produced by randomly generating numbers following the F-DISTRIBUTION with different degrees of freedom

Table 1: List of asymmetric hedges used in the modified NEFCLASS.

Asymmetric Hedge	Operation	Left Side Hedge	Right Side Hedge
NONE *	NONE	NONE	NONE
SMALL-CONCAVERIGHT	NONE-$MF^{(2)}$	NONE	VERY
BIG-CONCAVERIGHT	NONE-$MF^{(4)}$	NONE	ABSOLUTELY
SMALL-CONVEXRIGHT	NONE-$MF^{(\frac{1}{2})}$	NONE	FAIRLY
MODERATE-CONVEXRIGHT	NONE-$MF^{(\frac{1}{3})}$	NONE	SOMEWHAT
BIG-CONVEXRIGHT	NONE-$MF^{(\frac{1}{4})}$	NONE	SLIGHTLY
SMALL-CONCAVELEFT-CONVEXRIGHT	$MF^{(2)}$-$MF^{(\frac{1}{2})}$	VERY	FAIRLY
SMALL-CONVEXLEFT-CONCAVERIGHT	$MF^{(\frac{1}{2})}$-$MF^{(2)}$	FAIRLY	VERY
BIG-CONVEXLEFT-CONCAVERIGHT	$MF^{(\frac{1}{4})}$-$MF^{(4)}$	SLIGHTLY	ABSOLUTELY
BIG-CONCAVELEFT-CONVEXRIGHT	$MF^{(4)}$-$MF^{(\frac{1}{4})}$	ABSOLUTELY	SLIGHTLY
MODERATE-CONVEXLEFT-CONCAVERIGHT	$MF^{(\frac{1}{3})}$-$MF^{(3)}$	SOMEWHAT	EXTREMELY
MODERATE-CONCAVELEFT-CONVEXRIGHT	$MF^{(3)}$-$MF^{(\frac{1}{3})}$	EXTREMELY	SOMEWHAT

* No hedge is applied. This is the default.

chosen to control skew. The F-DISTRIBUTION (Natrella, 2003) has been chosen as the synthesis model because the degree of skew within an F-DISTRIBUTION is controlled by the pairs of degrees of freedom specified as a pair of distribution control parameters. This allows for a spectrum of skewed data distributions to be constructed. We designed the datasets to present different levels of skewness with increasing skew levels. Three pairs of degrees of freedom parameters have been used to generate datasets with different levels of skewness, including low, medium, and high-skewed feature values. After initial experiments datasets with degrees of freedom $(100,100)$ was chosen to provide data close to a normal distribution, $(100,20)$ provides moderate skew, and $(35,8)$ provides high skew.

A synthesized dataset consisting of 1000 randomly generated examples consisting of four-feature (W, X, Y, Z). F-DISTRIBUTION data for each of three classes was created. The three classes (ClassA, ClassB and ClassC) overlap, and are skewed in the same direction. We have taken care to ensure that all datasets used have a similar degree of overlap, and same degree of variability. The size of datasets were designed to explore the effect of skewness when enough data is available to clearly ascertain dataset properties. Ten-fold cross validation was used to divide each dataset into training (2700) and testing (300 point) sets in which an equal number of each class is represented. This method provides a better estimate of median performance, as well as a measure of variability.

Table 2 shows the minimum and maximum values of skewness for each feature based on 10 jackknife-based datasets (*i.e.*, dataset LOW-100,100, with the degree of skewness between 0.582 and 0.907, is low-skewed and dataset HIGH-35,8, with the degree of skewness between 1.289 and 3.764, is highly skewed). The high variability of the skewness values shown in Table 2 is due to the fact that these values are average, minimum, and maximum over three class labels for each feature.

3.4 Real-world Datasets

To show the pertinence of this analysis to real-world data problems, we ran all tests on two publicly available datasets: the Wisconsin Diagnostic Breast Cancer Dataset (WBDC) from UCI Machine Learning Repository, and a clinically applicable world of quantitative electromyography (EMG). The characteristic information of these datasets is presented in the following sections.

- **Electromyography Dataset (EMG)**: QEMG is the study of the electrical potentials observed from contracting muscles as seen through the framework of quantitative measurement. QEMG is used in research and diagnostic study (Stashuk and Brown, 2002). EMG datasets are known to contain features with highly skewed value distributions (Enoka and Fuglevand, 2001).

  The EMG dataset used here contains seven features of MUP templates (Amplitude, Duration, Phases, Turns, AAR, SizeIndex, and MeanMU-Voltage) observed on 791 examples representing three classes (Myopathy, Neuropathy, Healthy), collected through a number of contractions, and used in previous work (Varga et al., 2014).

- **Wisconsin Diagnostic Breast Cancer Dataset (WDBC)** : The Wisconsin Diagnostic Breast Cancer Dataset (WDBC) dataset contains 30 features

Table 2: Summary of skewness for the F-DISTRIBUTED and CIRCULAR-UNIFORM-DISTRIBUTED datasets.

Dataset	Minimum skewness				Maximum skewness			
	W	X	Y	Z	W	X	Y	Z
UNIFORM	−0.020	−0.050	−0.048	−0.060	−0.006	0.074	0.017	0.093
LOW-100,100	0.582	0.432	0.443	0.679	0.799	0.618	0.536	0.907
MED-100,20	1.354	1.178	1.198	1.403	1.947	2.144	1.547	1.721
HIGH-35,8	1.289	2.038	2.506	2.746	2.247	3.081	3.534	3.764

and two classes (benign and malignant). The WDBC dataset, observed from 569 examples, contains a class distribution of 357 benign and 212 malignant. Features have been extracted from a digitized image of a fine needle aspirate of a breast mass. This dataset is characterized by high dimensionality, very precise values, and almost no missing data. In examining the normality of the features, we found that seven features were highly positively skewed.

4 RESULTS AND DISCUSSIONS

This section presents the results and discussion. Section 4.1 presents the results and discussions for experiments using the synthesized data. Section 4.2 gives the results for the real-world data.

4.1 Experiments using Synthesized Data

In Table 1, all of the full names of the hedges are given. As these names are quite long, and the strength of the operation is apparent from the value of the power, the mathematical terms are used here for brevity. Table 3 show the misclassification percentages (as median± IQR) obtained by NEF-MME and NEF-CAIM classifiers using the 11 asymmetric hedges, as well as without application of a hedge (shown as NONE in the table). Comparisons between classifiers were performed as follows: for each discretization method and each dataset, results obtained without applying hedge (shown as NONE) were compared with those obtained from the application of each hedge. We conducted one-tailed M-W-W tests at a 0.05 significance level.

- **Comparison of Misclassification Percentages:** Table 4 reports the M-W-W test results. Note that we reported results only for those hedges, listed in Table 3, whose application led to a significant improvement in accuracy. The P-VALUES obtained for non significant results were greater than or equal to 0.23 (not shown in the table).

  As shown in Table 4, in the case of medium and high skewed data, the test yielded a main effect for applying four asymmetric hedges, such that the misclassification percentages were significantly lower for hedges with a root of 3 or 4 on the right side. These four hedges are shown as grey shade in Table 3. In the case of HIGH-35,8, a smaller interquartile range in the misclassification percentage was achieved for these four hedges, compared to NONE. Since there is no significant improvement in the accuracy of the low-skewed data, there is no apparent penalty when applying the hedge; however, as skew increases the utility of the hedge becomes significant.

  Hedges with higher roots increase membership functions more than those with lower roots. Hence, hedges with root 3 or 4 cause a higher degree of skewness in membership functions than hedges with root 2. For example, the changes in membership functions by "SOMEWHAT" and "SLIGHTLY" are more pronounced than the change by "FAIRLY". In particular, the hedges with roots of 3 or 4 on the right side, *i.e.*, NONE-$MF^{(\frac{1}{4})}$, NONE-$MF^{(\frac{1}{3})}$, $MF^{(3)}$-$MF^{(\frac{1}{3})}$ and $MF^{(4)}$-$MF^{(\frac{1}{4})}$, cause a greater increase in the membership value of inputs presented in the right side of the triangular membership functions. Hence, assigning a higher membership value puts more emphasis on inputs presented on the right side of the membership functions. which leads to the inclusion of more information in the decision-making process.

  As shown in table 3, the results obtained using the NONE-$MF^{(\frac{1}{4})}$ and $MF^{(4)}$-$MF^{(\frac{1}{4})}$ hedges were similar. Given the fact that these two hedges have different operators on the left side, the similarity in their obtained accuracy suggests that applying $MF^{(\frac{1}{4})}$ to the right side of the membership function could be the main reason for the increase in accuracy. As shown in the table, results obtained using the NONE-$MF^{(\frac{1}{3})}$ and $MF^{(3)}$-$MF^{(\frac{1}{3})}$ hedges provides further evidence to support this suggestion. Furthermore, the results show that hedges with concentration operator in the right side, *i.e.*, NONE-$MF^{(4)}$, $MF^{(\frac{1}{3})}$-$MF^{(3)}$, and $MF^{(\frac{1}{4})}$-$MF^{(4)}$ did not improve accuracy, when data is positively skewed (see also Ta-

Table 3: Misclassification percentages (Median ± IQR) obtained from NEF-MME and NEF-CAIM with and without the application of hedges.

Hedge	NEF-MME			NEF-CAIM		
	LOW-100,100	MED-100,20	HIGH-35,8	LOW-100,100	MED-100,20	HIGH-35,8
NONE	26.00 ± 1.75	34.16 ± 2.66	42.50 ± 4.25	24.33 ± 4.17	34.16 ± 2.60	41.67 ± 4.33
NONE-$MF^{(2)}$	32.30 ± 1.90	48.23 ± 2.90	52.83 ± 4.10	30.67 ± 2.08	48.43 ± 18.58	54.26 ± 12.08
NONE-$MF^{(4)}$	26.67 ± 0.66	37.80 ± 1.50	48.00 ± 7.20	26.00 ± 1.60	38.50 ± 0.92	46.00 ± 4.70
NONE-$MF^{(\frac{1}{2})}$	28.65 ± 2.42	39.50 ± 5.83	51.16 ± 3.58	27.16 ± 3.66	40.00 ± 4.75	50.35 ± 9.25
NONE-$MF^{(\frac{1}{3})}$	28.50 ± 1.16	27.66 ± 3.83	33.50 ± 2.68	28.67 ± 0.90	27.17± 2.60	34.50 ± 2.17
NONE-$MF^{(\frac{1}{4})}$	28.33 ± 1.15	27.66 ± 3.80	33.50 ± 2.67	28.00 ± 1.25	27.00 ± 3.25	34.50 ± 2.17
$MF^{(\frac{1}{2})}$-$MF^{(2)}$	26.00 ± 1.75	34.16 ± 2.66	42.50 ± 4.25	24.13 ± 4.16	34.30 ± 2.58	41.67 ± 4.16
$MF^{(\frac{1}{3})}$-$MF^{(3)}$	37.83 ± 5.40	64.83 ± 1.58	59.50 ± 5.83	33.00 ± 10.50	64.15 ± 0.70	67.33 ± 24.33
$MF^{(\frac{1}{4})}$-$MF^{(4)}$	26.50 ± 1.00	33.83 ± 2.40	41.65 ± 3.90	25.00 ± 3.40	33.67 ± 2.75	41.67 ± 2.66
$MF^{(2)}$-$MF^{(\frac{1}{2})}$	26.00 ± 1.75	34.16 ± 2.70	42.50 ± 4.25	24.13 ± 4.16	34.30 ± 2.58	41.67 ± 4.16
$MF^{(3)}$-$MF^{(\frac{1}{3})}$	28.50 ± 1.16	27.66 ± 3.83	33.50 ± 2.68	28.67 ± 0.90	27.17± 2.60	34.50 ± 2.17
$MF^{(4)}$-$MF^{(\frac{1}{4})}$	28.33 ± 1.16	27.66 ± 3.83	33.68 ± 2.66	27.83 ± 2.50	27.17 ± 3.12	34.50 ± 2.17

Table 4: Results of the M-W-W test to compare the misclassification percentages obtained by NEF-MME and NEF-CAIM between NONE and each hedge.

Classifier	Hege1 *vs*. Hedge2	Dataset		
		LOW-100,100	MED-100,20	HIGH-35,8
NEF-MME	NONE *vs*. NONE-$MF^{(\frac{1}{4})}$	0.98	***	***
	NONE *vs*. $MF^{(4)}$-$MF^{(\frac{1}{4})}$	0.98	***	***
	NONE *vs*. NONE-$MF^{(\frac{1}{3})}$	0.98	***	***
	NONE *vs*. $MF^{(3)}$-$MF^{(\frac{1}{3})}$	0.98	***	***
NEF-CAIM	NONE *vs*. NONE-$MF^{(\frac{1}{4})}$	0.98	***	***
	NONE *vs*. $MF^{(4)}$-$MF^{(\frac{1}{4})}$	0.98	***	***
	NONE *vs*. NONE-$MF^{(\frac{1}{3})}$	0.98	***	***
	NONE *vs*. $MF^{(3)}$-$MF^{(\frac{1}{3})}$	0.98	***	***

*** significant at 95% confidence ($p < .05$)

Table 5: Results of the M-W-W test to compare the misclassification percentages between each pair of datasets for all baseline classifiers and the NEFCLASS classifiers with employment of the NONE-$MF^{(\frac{1}{4})}$ and $MF^{(4)}$-$MF^{(\frac{1}{4})}$ hedges.

Classifier	Datasets		
	LOW-100,100 vs. MED-100,20	LOW-100,100 vs. HIGH-35,8	MED-100,20 vs. HIGH-35,8
NEF-ORG	***	***	.93
NEF-MME-NONE-$MF^{(\frac{1}{4})}$	.57	***	***
NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	.57	***	***
NEF-CAIM-NONE-$MF^{(\frac{1}{4})}$	.84	***	***
NEF-CAIM-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	1.00	***	***

*** significant at 95% confidence ($p < .05$)

ble 3). The reinforcing of a concentration operator to a membership function results in the reduction of magnitude to the grade of membership in which it is relatively large for those with low membership. This leads to the exclusion of those points presented on the skewed side from the decision making process.

In light of these findings, it can be concluded that applying dilation hedges with the root of 3 or 4 to the right side of membership functions significantly improved accuracy, when data is positively skewed. In contrast, applying concentration hedges to the right side of the membership functions did not have a positive effect on accuracy. Therefore, a higher accuracy can be achieved by means of an appropriate choice of a hedge. We suggest that it is beneficial to consider choosing an appropriate hedge based on the amount and direction of skew.

- **Comparison of Number of Rules:** the test did not identify a significant increase in the number of rules obtained by each hedge compared to NONE. Hence, the reductions in the misclassification percentages by using the $MF^{(3)}$-$MF^{(\frac{1}{3})}$, $MF^{(4)}$-$MF^{(\frac{1}{4})}$, NONE-$MF^{(\frac{1}{3})}$, and NONE-$MF^{(\frac{1}{4})}$ hedges were not accompanied by any significant increase in the number of rules.

We conclude that combining MME or CAIM with an appropriate asymmetric hedge, such as NONE-$MF^{(\frac{1}{4})}$, when the hedge was applied to the side of the direction of the skew, led to a significant improvement in accuracy over the original NEFCLASS classifier.

4.2 Experiments using Real-world Data

In this section, we assess the performance of these classifiers using real-world data. The EMG and WDBC datasets were used for training and testing the best performed modified classifiers, NEF-MME-NONE-$MF^{(\frac{1}{4})}$ and NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$.

4.2.1 Experiments on the EMG Dataset

Table 6 depicts the misclassification percentages (as median± IQR) and the number of rules (as median± IQR). Table 7 gives the test results.

- **Comparison of Misclassification Percentages:** As shown in Table 7, the test revealed that the misclassification percentages significantly decreased by applying the combination of the MME discretization method and the NONE-$MF^{(\frac{1}{4})}$ asymmetric hedge.
- **Comparison of Number of Rules:** Additionally, the test showed that the higher accuracy of NEF-MME-NONE-$MF^{(\frac{1}{4})}$ was achieved with a significantly lower number of rules compared to that of NEF-ORG.

4.2.2 Experiments on the WDBC Dataset

Table 8 depicts the misclassification percentages (as median± IQR) and the number of rules (as median± IQR). Table 9 reports the results of the M-W-W test.

- **Comparison of Misclassification Percentages:** as shown in Table 9, the test indicated a significant decrease in the misclassification percentages obtained by NEF-MME-NONE-$MF^{(\frac{1}{4})}$ compared to NEF-ORG.
- **Comparison of Number of Rules:** the test did not identify a significant decrease in the number of rules obtained by applying an asymmetric hedge.

We conclude that the accuracy of the NEFCLASS classifiers, when trained by the EMG and WDBC datasets, was significantly improved by the combination of MME discretization method with the NONE-$MF^{(\frac{1}{4})}$ hedge. Also, in the case of EMG data, the higher accuracy of NEF-MME-NONE-$MF^{(\frac{1}{4})}$ was achieved with considerably lower number of rules.

However, it is notable that the application of the $MF^{(4)}$-$MF^{(\frac{1}{4})}$ hedge did not show a significant positive effect on the accuracy. The presence of features with negative and zero skewed distribution in the EMG and WDBC datas might be the cause of a lower accuracy for the NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$ classifier of which applies a hedge to the both sides of a membership function. Therefore, we suggest choosing an appropriate hedge based on the amount and direction of skew in data.

5 CONCLUSIONS

Devising the asymmetric hedges with an appropriate dilation hedge significantly improved the accuracy of the NEFCLASS classifiers. Combining MME with an appropriate asymmetric hedge, such as NONE-$MF^{(\frac{1}{4})}$, when the hedge was applied to the side of the direction of the skew, led to a significant improvement in accuracy over the original NEFCLASS classifier. This study revealed that if the shape of the membership function resembles the skewness in the data, then it minimizes the effect of bias within the data. Hence, this improves the accuracy of the classifier. Therefore, we suggest using asymmetric hedges to express the information distribution in the domain of fuzzy logic. Higher classification accuracy can be achieved using a proper choice of a hedge. We suggest that it is fruitful to consider choosing an appropriate hedge based on the skew amount and direction.

This study demonstrates that applying asymmetric hedges to the membership functions not only resulted in improving classification accuracy but also led to the building of a more robust classifier when trained by skewed data. This improvement of accuracy was achieved without increasing the number of rules.

Employment of asymmetric hedges is not limited to the NEFCLASS neuro-fuzzy classifier; it can be applied to any application where fuzzy logic is used.

Table 6: Misclassification percentages (Median ± IQR) and number of rules (Median ± IQR) obtained from applying asymmetric hedges using the EMG dataset.

Classifier	Misclassification Percentage	Number Of rules
NEF-ORG	54.18 ± 28.00	149.00 ± 4.00
NEF-MME-NONE-$MF^{(\frac{1}{4})}$	33.00 ± 35.00	88.00 ± 7.00
NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	52.00 ± 28.00	167.00 ± 14.00

Table 7: Results of one-tailed M-W-W to compare the results with and without using asymmetric hedges for the EMG dataset.

Classifier	Misclassification Percentage	Number Of rules
NEF-ORG *vs.* NEF-MME-NONE-$MF^{(\frac{1}{4})}$	***	**.01**
NEF-ORG *vs.* NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	.29	.95

*** significant at 95% confidence ($p < .05$)

Table 8: Misclassification percentages (Median ± IQR) and the number of rules (Median ± IQR) obtained from applying asymmetric hedges using the WDBC dataset.

Classifier	Misclassification Percentage	Number Of rules
NEF-ORG	17.39 ±5.00	77.00 ± 89.00
NEF-MME-NONE-$MF^{(\frac{1}{4})}$	12.50 ± 5.00	157.00 ± 7.00
NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	17.80 ± 8.75	80.00 ± 10.00

Table 9: Results of one-tailed M-W-W to compare the results with and without using asymmetric hedges for the WDBC dataset.

Classifier	Misclassification Percentage	Number Of rules
NEF-ORG *vs.* NEF-MME-NONE-$MF^{(\frac{1}{4})}$	***	.98
NEF-ORG *vs.* NEF-MME-$MF^{(4)}$-$MF^{(\frac{1}{4})}$	.19	.40

*** significant at 95% confidence ($p < .05$)

This method can be applied in areas such as healthcare, security, and finance where datasets are skewed. In this work, we examined positively skewed data. However, this approach can be extended and modified to address negative skewness.

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