MLD-based Optimal Control Model for Train Traffic Control

Honggang Wang[1, a], Qingru Zou[1, b]
[1]*College of Traffic and Transportation, Chongqing Jiaotong University 400074, China*

Keywords: Train Traffic Control, Mixed Logical Dynamic, Model Predictive Control.

Abstract: Trains are often delayed because of the unexpected events. How to eliminate trains delay is the key function of the train traffic control system. For this problem, an optimal control model is presented in this paper based on the mixed logical dynamic and the model predicative control theory under the situation that trains operation are not disturbed seriously. The process of the train movement is studied and it is pointed out that the train movement is a hybrid dynamic system because the evolution of the train operation status is driven by the continuous part following the Newton's laws of motion and lots of discrete events including the expected events and unexpected ones. Based on the results, the train operation model is built using the mixed logical dynamic theory without considering the junctions and assuming that the disturbances imposed on trains are not serious. Based on the train operation model, the optimal control model for train traffic control is studied using the model predictive control theory where the train operation model is used to predict the future status of trains. Finally, the simulation shows that the model's validity and correctness.

1 INTRODUCTION

Railway transport system is a dynamic system that trains operate in an environment with many uncertain factors such as natural environment and devices fault etc. To guarantee trains safe and provide customers good service, all trains must be monitored and controlled by dispatchers. Generally, this task's implementations include 1) off-line scheduling when all the train arrival times and departure times are calculated before the train starts. The trains behave exactly as they were planned. No unexpected event happens and no new train can appear. 2) on-line scheduling when the scheduling is performed during the train traffic operation. Some trains have variable delays, unexpected events happen, and new train scheduling requests are required and accepted during the operation.

For the on-line scheduling, these uncertain factors usually lead to the train delay even stop in some railway line. Because of the characteristics of the railway transportation that the driver cannot change the train routs at will, these uncertain factors result in conflictions at junctions and trains delay which can be propagated rapidly along the railway line. To ensure trains safety, punctuality and provide better services to passengers, railway transport departments must control all trains. So it is becoming more and more important daily task for the relative railway transport departments to guaranty that all trains behave according to the time table, especially with the improving of trains operation speed and density in recent years. In fact, how to guaranty the safety and punctuality has been an important problem studied in academic circle and railway industry since railway transportation came into being. (Sundaravalli Narayanaswami, Narayan Rangaraj, 2012) defined the train traffic control as scheduling and rescheduling. They defined the scheduling and rescheduling as "an initial time allocation of resources to meet demands in completing a task and rescheduling is a later modification of such resource allocations". We prefer to refer the rescheduling as train traffic control.

In reality, the uncertain factors can be classed into two categories according to its seriousness. One is very serious, for example, a strong earthquake and railway line broken etc, which can force all trains to stop. Another is not serious, for example, heavy rain, strong wind etc, which can result in not all trains stopping but conflictions and train delay. For the first one, dispatchers often use the pre-defined scheme to reduce the economic loss and ensure passengers safe. However, when the second factors happen, dispatchers usually change train route to solve conflicts and determine a new timetable based on the

Wang, H. and Zou, Q.
MLD-based Optimal Control Model for Train Traffic Control.
DOI: 10.5220/0010021800460053
In *Proceedings of the International Symposium on Frontiers of Intelligent Transport System (FITS 2020)*, pages 46-53
ISBN: 978-989-758-465-7

original timetable according to real situations. The aim is to restore the train operation order and ensure the trains safety and punctuality finally. For the adjustment of the train timetable problem, (Jih-Wen Sheu, Wei-Song Lin, 2012) developed and evaluated the Adaptive Optimal Control (AOC) method and the evaluation shows that the AOC method is able to find a near-optimal solution rapidly and accurately. For conflicts and train delays considerably in time and space during trains operation, in order to realistically forecast and minimize delay propagation, (Andrea D'Ariano and Marco Pranzo, 2008) decompose a long time horizon into tractable intervals and use the ROMA dispatching system to pro-actively detect and globally solve conflicts on each time interval. (Joaquín Rodriguez, 2007) presented a constraint programming model for the routing and scheduling of trains running through a junction. The model can be integrated into a decision support system used by operators who make decisions to change train routes or orders to avoid conflicts and delays and has been applied to a set of problem instances. Preliminary results show that the solution identified by the model yields a significant improvement in performance within an acceptable computation time. (Tiberiu Letia, Mihai Hulea and Radu Miron, 2008) presented a distributed method to schedule new trains where the paths containing the block sections from one station to another are dynamically allocated without leading to deadlocks. In addition, some researchers made lots of studies for the seriously disturbed railway transportation (Francesco Corman, Andrea D'Ariano, Ingo A. Hansen etc, 2011). In order to save energy, the reference (Shigeto HIRAGURI, YujiHIRAO, IkuoWATANABE etc. 2004) presents a train control method based on trains movement and data communication between railway sub-systems, which can decrease some unexpected deceleration or stop between stations.

The paper mainly focuses on the problem how to restore the train operation order and reduce trains delay. It is organized as: the Mixed Logic Dynamic (MLD) is described briefly in section 2; In section 3, the reason that train traffic control is a dynamic system is given, and the MLD model for train traffic control is given, which is used to describe the train dynamic movement; In section 4, the optimal control model for train traffic control is established based on the model predictive control theory; In section 5, a simulation result is given which shows the validity of the model presented in this paper. In the last part, we will point out the further studies.

2 MIXED LOGIC DYNAMIC SYSTEM

In industries, there exist many systems having the same characters. The systems are comprised of two parts, the continuous part following the physical laws and the discrete part. The two parts work together and drive these systems evolution. These systems are referred to Hybrid System (HS) or Hybrid Dynamic System (HDS) which is attracting many researchers interests (Y.Bavafa-Toosi, Christoph Blendinger, Volker Mehrmann etc, 2008, M.J. Dorfman, J. Medanic, 2004). In past, the continuous part and the discrete part were studied separately. For example, the discrete one is described using Petri Net, Automata and Finite State Machine etc, meanwhile the continuous part using difference equation. However it is difficult to analysis the HS performance if these two interactive parts are concerned separately. In order to take the two parts into account as a whole system, (Alberto Bemporad, Manfred Morari, 1999) presented a framework for modeling and controlling systems described by interdependent physical laws, logic rules, and operating constraints. This framework was denoted as mixed logical dynamical (MLD) systems. Since the presence of the MLD concept, the theory and its application were studied by many researchers (Martin W. Braun and Joanna Shear, 2010, Kazuaki Hirana, Tatsuya Suzuki and Shigeru Okuma, 2002, Akira Kojima and Go Tanaka, 2006). Here are given the procedures of modeling a system in short, the detailed information on MLD theory can be found in conference (Alberto Bemporad, Manfred Morari, 1999) .

Step 1: Building the system's mathematical model following physical law under different situation through analyzing the system.

Step 2: Building the logical proposition according to the qualitative knowledge and constraints existed in the model and transferring the logical proposition into linear inequalities by introducing some logical variables.

Step 3: Introducing some auxiliary variables according to the relations between discrete events and continuous variables and getting other inequalities.

Through combining the results from step 2 and the results from step 3, a MLD model can be established. Generally, the formulation of MLD model has the style as below.

$$\begin{cases} x(t+1) = Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \\ y(t) = Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \\ E_2 \delta(t) + E_3 z(t) \le E_1 u(t) + E_4 x(t) + E_5 \end{cases}$$

Where $x(t)$ is the state variable, $y(t)$ the output variable, $u(t)$ the input control variable, $\delta(t)$ the logical variable, $z(t)$ the auxiliary continuous variable and A, $B_i (i = 1, 2, 3)$, C, $D_i (i = 1, 2, 3)$, $E_i (i = 1, 2, 3, 4, 5)$ are the constant coefficient matrixes.

3 MLD MODEL FOR TRAIN TRAFFIC CONTROL

3.1 Analysis of the Train Traffic Control

The train motion must follow the physical law, $s(t) = s(t_0) + v(t_0)(t - t_0) + a(t - t_0)^2 / 2$, $s(t)$ stands for the train position at time t, $v(t)$ the train velocity at time t, a is the acceleration. However, train operation process must obey other rules which are referred as discrete events, for example, trains must decelerate when approaching a station or catching up a strong cross wind, trains must change their velocities when receiving dispatch information from dispatchers. Obviously, the status evolution of trains is driven by two parts, one is the continuous part following physical law, another one is the discrete events. So the train traffic control system is a typical hybrid dynamic system.

In reality, the events imposed on trains can be classed as two categories according to their seriousness. One is serious, which forces trains to stop for safety. Another is not serious, which only forces trains to decelerate for safety. For the first one, it’s difficult to build a mathematic model to describe the train traffic control. Actually, what dispatchers can do is to use predefined scheme to ensure the train safety when the first one happens. For the second, dispatchers can adjust the timetable to restore the train operation order according to the real time situation. Only the second events are considered in this paper.

Under the normal situation, according to the train operation plan, each train leaves a station, accelerates its speed to the maximum speed, decelerates its speed when approaching the next station and stops or goes through the station. Each train repeats these procedures until it arrives at the terminal station. For this situation, dispatchers only monitor all trains and hardly control all trains. Under the abnormal situation, i.e. train operation process is affected by some events such as cross winds, conflictions at junctions etc, trains must decelerate to some speed for safety and then result in train delay. For this situation, dispatchers must take measures to restore train operation order and ensure the train safety. It is the main task for dispatchers to make a new timetable (or adjustment plan) to minimize or eliminate the train delay in short time as soon as possible.

In addition, the discrete events imposed on trains can be classified as two types: the predictive one and the un-predictive one. The predictive events are the events whose emerging time can be predictive or calculated in advance such as trains entering station event, trains leaving station events and dispatching events etc. On the contrary, the un-predictive events are the events whose emerging time cannot be predictive or calculated in advance such as equipment fault, strong wind and heavy rain etc.

Based on this analysis, we can draw a conclusion that the essence of train traffic control is to minimize or eliminate train delay resulting from the un-predictive events through the predictive events.

3.2 MLD Model for Train Traffic Control

(1) The Status Space Equation

According to the physical law, the status space equation for train i is

$$\begin{cases} v_i(k+1) = v_i(k) + a_i(k)T \\ s_i(k+1) = s_i(k) + v_i(k)T + a_i(k)T^2 / 2 \end{cases} \tag{1}$$

Where T is the sampling time interval, $a_i(k)$ the acceleration of train i at time kT, $v_i(k)$ the velocity of train i at time kT, $S_i(k)$ the position of train i at time kT.

The equation (1) can describe the train operation without any disturbances, but it is not suitable for train operation with disturbances. Let $r_i(k) \in \{0,1\}$ stand for the event imposed on the train i at time kT. $r_i(k) = 1$ means that the event happens, which forces the train i to slow down its speed to a security speed v_g. Using the logical variable $r_i(k)$, the status space equation (1) can be re-written as

$$\begin{cases} v_i(k+1) = (v_i(k) + a_i(k)T)(1 - r_i(k)) + r_i(k)v_g \\ s_i(k+1) = s_i(k) + v_i(k)T + a_i(k)T^2 / 2 \end{cases} \quad (2)$$

The equation (2) can describe the train operation with disturbances, however it does not include the information on train entering station, train leaving station and dispatching event.

Let $\delta^i_{N1}(k)$, $\delta^i_{N2}(k)$ and $\delta^i_{N3}(k)$ stand for the entering station event, the leaving station event and the running event at a constant speed separately, where $\delta^i_{Nj}(k) \in \{0,1\}$ $j = 1,2,3$. $\delta^i_{N1}(k) = 1$ means that the entering station event happens at time k, otherwise the event doesn't happen at time k. The $\delta^i_{N2}(k)$ and $\delta^i_{N3}(k)$ have the same meaning as the $\delta^i_{N1}(k)$. Let $\delta^i_{A1}(k)$, $\delta^i_{A2}(k)$ and $\delta^i_{A3}(k)$ stand for the accelerating command event, the decelerating command event and the uniform command event separately, where $\delta^i_{Aj}(k) \in \{0,1\}$ $j = 1,2,3$. $\delta^i_{A1}(k) = 1$ means that dispatchers send an accelerating command to the train i at time k, otherwise means that dispatchers don't send an accelerating command to the train i at time k. The $\delta^i_{A2}(k)$ and $\delta^i_{A3}(k)$ are similar to the $\delta^i_{A1}(k)$. The acceleration value of train i at time k $a_i(k)$ is described by these logic variables as below:

$$a_i(k) = \begin{cases} a & \delta^i_{N1}(k) + \delta^i_{A1}(k) = 1 \\ 0 & \delta^i_{N3}(k) + \delta^i_{A3}(k) = 1 \\ -a & \delta^i_{N2}(k) + \delta^i_{A2}(k) = 1 \end{cases} \quad (3)$$

Where the values of $\delta^i_{Nj}(k)$ ($j = 1,2,3$) are determined by the onboard train control equipment and track side signal equipment. The values of $\delta^i_{Aj}(k)$ ($j = 1,2,3$) are determined by dispatchers according to the real time situation. The variable a ($a \geq 0$) stands for the acceleration performance of trains. $a_i(k) = a$ means that train i is accelerating, $a_i(k) = 0$ means that train i is running at a constant speed and $a_i(k) = -a$ means that train i is decelerating.

Generally, the predictive event $\delta^i_{Nj}(k)$ and the dispatch event $\delta^i_{Aj}(k)$ do not happen simultaneously because a dispatcher cannot send an accelerating command when he knows that the train is accelerating. So, these logic variables $\delta^i_{Nj}(k)$ and $\delta^i_{Aj}(k)$ must satisfy the below constraint.

$$\sum_{j=1}^{3} (\delta^i_{Nj}(k) + \delta^i_{Aj}(k)) = 1 \quad (4)$$

According to the equation (4), the equation (3) can be formulated as below.

$$a_i(k) = \left(\delta^i_{N1}(k) + \delta^i_{A1}(k)\right)a - \left(\delta^i_{N2}(k) + \delta^i_{A2}(k)\right)a \quad (5)$$

We substitute $a_i(k)$ in status space (2) with equation (5) and get the status space equation as below.

$$\begin{cases} v_i(k+1) = \{v_i(k) + [(\delta^i_{N1}(k) + \delta^i_{A1}(k)) - (\delta^i_{N2}(k) + \delta^i_{A2}(k))]aT\} \\ \qquad \times (1 - r_i(k)) + r_i(k)v_g \\ s_i(k+1) = s_i(k) + v_i(k)T + \\ \qquad [(\delta^i_{N1}(k) + \delta^i_{A1}(k)) - (\delta^i_{N2}(k) + \delta^i_{A2}(k))]aT^2 / 2 \end{cases} \quad (6)$$

(2) Constraints

According to the railway transport rules, the departure time for a passenger train must be less than the planned departure time, i.e. if train i is at the station r, the departure time kT must satisfy

$$kT \geq t_{ir}, if \left|s_i(k) - st_r\right| < \varepsilon \ and \ \delta^i_{N1} + \delta^i_{A1} = 1 \quad (7)$$

Where t_{ir} denotes the departure time of train i at station r, st_r denotes the position of station r in the railway line, ε is a minimal positive real.

The condition (7) implies

$$\left|s_i(k) - st_r\right| < \varepsilon \wedge \delta^i_{N1} + \delta^i_{A1} = 1 \rightarrow kT \geq t_{ir} \quad (8)$$

According to the MLD modeling theory, it can be transferred to a inequality constraint form.

Let τ_{ir} be the minimal dwell time for train i at station r, then the real time interval for train i at station r must satisfy

$$kT - k_{ira}T \geq \tau_{ir} \tag{9}$$

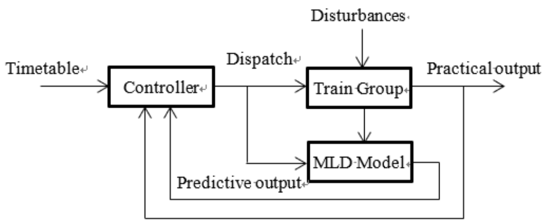

Figure 1: Architecture of Optimization Control for Train Traffic Control.

Where $k_{ira}T$ is the real arriving time for train i at station r.

4 OPTIMAL CONTROL MODEL FOR TRAIN TRAFFIC CONTROL

The above model based on MLD theory can be used to describe the process of train traffic control, involving the trains running process, some uncertain factors and the dispatcher's activities. However, this model cannot be used to optimize the train traffic control problem. To get an optimal or sub-optimal schedule solution, the Model Predictive Control (MPC) is used. The architecture is illustrated as Fig.1

The Train Group module stands for the practical trains, MLD module is the model for train traffic control which is built above and the Controller is an optimization controller. The Train Group module outputs trains real trajectories, and the MLD module trains future trajectories in some time interval. The real trajectories and the future trajectories are fed backed to the Controller module. The Controller module will make an optimal solution or sub-optimal solution according to the time table, the real information from the Train Group module and the predictive information from the MLD module.

4.1 Performance of the Optimization

There are many performances of optimization for train traffic control, such as to minimize traffic delay, to maximize traffic system throughput to fulfill train timing requirements and to guaranty system safety. The minimization traffic delay is used in this paper.

Let t_{aij} stands for the real arriving time of train i at station j, t_{dij} the real departure time of train i at station j; Let t^*_{aij} and t^*_{dij} stands for the planned arriving time and the planned departure time respectively. The performance can be described as below:

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{(t_{aij}-t^*_{aij})^2+(t_{dij}-t^*_{dij})^2}$$

4.2 Optimal Control Model for Train Traffic Control

Based on the performance and the MLD model, the optimal control model for train traffic control is described as below:

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{(t_{aij}-t^*_{aij})^2+(t_{dij}-t^*_{dij})^2}$$

$$s.t.\begin{cases} v_i(k+1)=\{v_i(k)+[(\delta^i_{N1}(k)+\delta^i_{A1}(k))-(\delta^i_{N2}(k)+\delta^i_{A2}(k))]aT\} \\ \qquad \times(1\text{-}r_i(k))+r_i(k)v_g \\ s_i(k+1)=s_i(k)+v_i(k)T+[(\delta^i_{N1}(k)+\delta^i_{A1}(k))-(\delta^i_{N2}(k)+ \\ \qquad \delta^i_{A2}(k))]aT^2/2 \end{cases}$$

$$kT\geq t_{ir}, \quad if \left|s_i(k)-st_r\right|<\varepsilon \; and \; \delta^i_{N1}+\delta^i_{A1}=1$$

$$\left|s_i(k)-st_r\right|<\varepsilon\wedge\delta^i_{N1}+\delta^i_{A1}=1\rightarrow kT\geq t_{ir}$$

$$kT-k_{ira}T\geq\tau_{ir}$$

5 SIMULATION

Taking a dispatch section including six stations and 5sections as an example, the railway line as below:

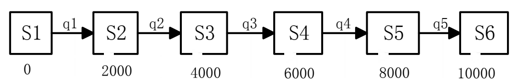

Figure 2: Diagram of simulation railway line.

Where $\mathrm{Si}(\mathrm{i}=1,2,\ldots,6)$ Stands for stations, the number (unit: meter) under the station is the station's position in the railway line, and $\mathrm{qi}(\mathrm{i}=1,2,\ldots,5)$ stands for the corresponding section.

The parameters about trains are listed in the Table 1.

Table 1: Parameters of trains.

Parameters	Value
Traction Acceleration a	$2(m/s^2)$
Breaking Acceleration a	$-2(m/s^2)$
Normal speed in section v	$40(m/s)$
limit speed in section v_M	$50(m/s)$
Safe speed v_g	$20(m/s)$
The length of station l	$50(m)$

The minimal dwell time at station S2, S3, S4 and S5 are listed in the Table 2 (Unit: second)

Table 2: The minimal dwell time at station.

Train ID	S2	S3	S4	S5
H2	15	15	0	20
H4	15	15	0	20
H6	25	15	25	15
H8	10	15	15	15
H10	10	20	20	15
H12	15	15	15	20

The train operation plan is as figure 3.

During simulation, the Branch and Bound method is used to solve the optimal control model. At last, the adjusting process is illustrated as Fig 4 and the Fig.5 shows the real train operation situation.

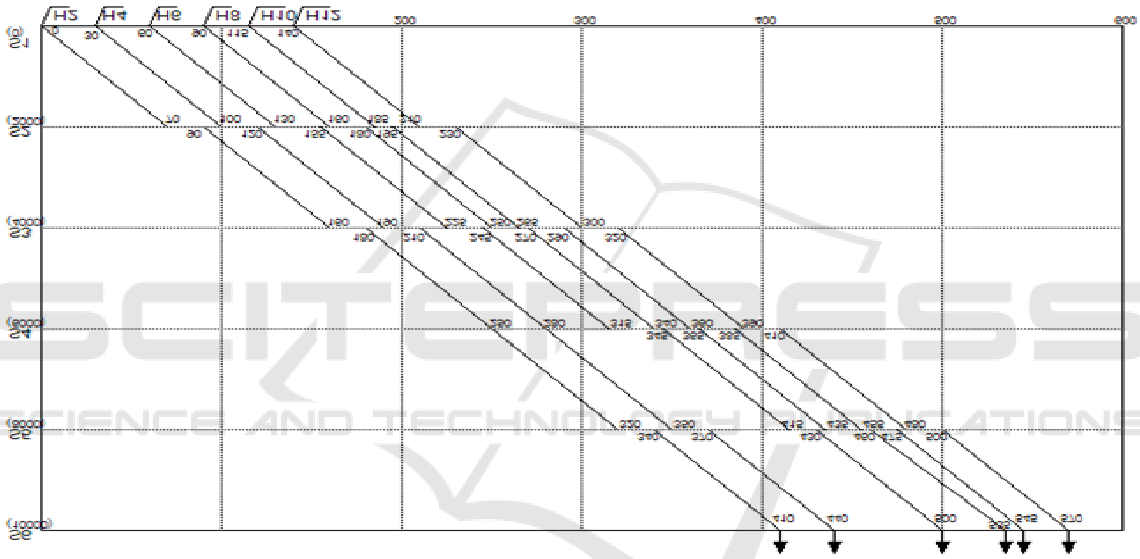

Figure 3: Diagram of train operation plan.

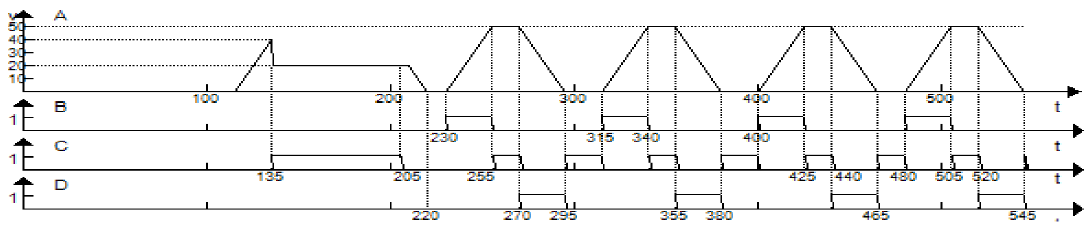

Figure 4: Diagram of adjusting process.

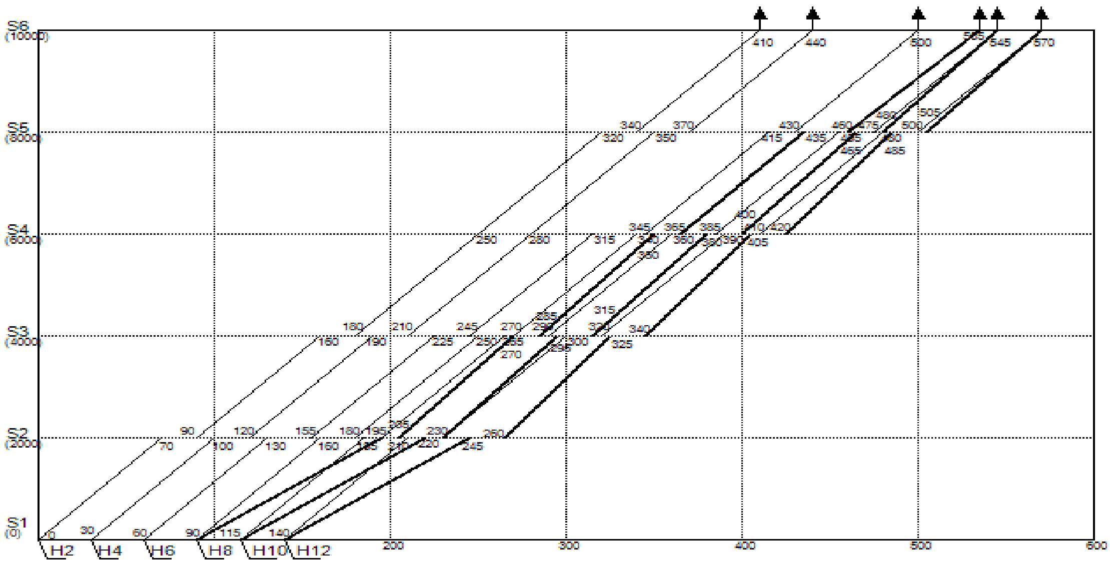

Figure 5: Diagram of train adjustment results.

6 CONCLUSION

The essence of the train traffic control is to eliminate or reduce the trains delays resulting from some unexpected factors under the train safe, to ensure trains behave according to train operation plan. Based on this idea, the MLD model and optimal control model are built in this paper taking in to account some constraints such as dwell time, safe distance etc. Finally, taking a dispatch section including six stations and five sections as an example, the models are verified on computer and the results shows it's validity and correctness.

REFERENCES

Akira Kojima and Go Tanaka, 2006. A Modeling of pedestrian dynamics in crows based on MLD system approach, *Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems*, 634-637.

Alberto Bemporad, Manfred Morari, 1999. Control of systems integrating logic, dynamics, and constraints, *Automatica*, 35, 407-427.

Andrea D'Ariano and Marco Pranzo, 2008. An Advanced Real-Time Train Dispatching System for Minimizing the Propagation of Delays in a Dispatching Area Under Severe Disturbances. *Networks and Spatial Economics*, 9(1): pp.63-84.

Francesco Corman, Andrea D'Ariano, Ingo A. Hansen etc, 2011. Dispatching trains during seriously disrupted traffic situations, *2011 International Conference on Networking, Sensing and Control*, pp.323-328.

Jih-Wen Sheu, Wei-Song Lin, 2012. Adaptive Optimal Control for Designing Automatic Train Regulation for Metro Line. *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, 20(5):1319-1327.

Joaquín Rodriguez, 2007. A constraint programming model for real-time train scheduling at junctions. *Transportation Research Part B: Methodological*, 41(2):231-245.

Martin W. Braun and Joanna Shear, 2010. A Mixed Logical Dynamic Model Predictive Control Approach for Handling Industrially Relevant Transportation Constraints, *6th annual IEEE Conference on Automation Science and Engineering, pp:966-971*.

Kazuaki Hirana, Tatsuya Suzukiz and Shigeru Okuma, 2002. Optimal motion planning for peg-insertion task based on Mixed Logical Dynamical System framework, *Proceedings of the 41st SICE Annual Conference*, Vol.2, 1165 – 1170.

M.J. Dorfman, J. Medanic, 2004. Scheduling trains on a railway network using a discrete event model of railway traffic, *Transportation Research Part B: Methodological,* 38(1):81-98.

Shigeto HIRAGURI, YujiHIRAO, IkuoWATANABE etc. 2004. Advanced Train and Traffic Control Based on Prediction of Train Movement, *JSME International Journal Series C*, 27(4):523-528.

Sundaravalli Narayanaswami, Narayan Rangaraj, 2012. Scheduling and Rescheduling of Railway Operations: A Review and Expository Analysis, *Technology Operation Management*, 2(2): 102-122

Tiberiu Letia, Mihai Hulea and Radu Miron, 2008.. Distributed Scheduling for Real-Time Railway Traffic Control, *Proceedings of the International Multi-*

conference on Computer Science and Information Technology, pp.675-689.

Y.Bavafa-Toosi, Christoph Blendinger, Volker Mehrmann etc, 2008. A New Methodology for Modeling, Analysis, Synthesis, and Simulation of Time-Optimal Train Traffic in Large Networks, IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING, 5(1): 43-52.