Investigation of Selection Mechanism of Friction Models in Multibody Systems

Qian Jing[1, a], Ning Mi[1]
[1]School of Mechanical Engineering, Long dong University, Qing Yang 745000, China

Keywords: Friction model, Multibody system, Joints; Dynamics, Adams.

Abstract: In order to research the influence of the different friction models on the frictional characteristics and the dynamic response of the multibody system with different kinds of joints, eleven different kinds of friction models were used in three different specified scenarios. Firstly, each friction model is simply introduced, and its friction characteristics are illustrated. In addition, in order to test the physical properties of these friction models, there are two different scenarios: (i) multibody system with revolute joint; (ii) multibody system with revolute joint and prismatic joint simultaneously. Secondly, when these friction models are applied in the scenarios, the comparison analysis between with friction phenomenon model and without friction phenomenon model is implemented, which is validated by the commercial software ADAMS. Finally, the simulation shows that the type of joint in the multibody system has a significant effect on the selection mechanism of these friction models. Namely, this investigation provides a reference method for choosing the friction model that is the best suitable for the above two different scenarios according to the computational efficiency and position stability.

1 INTRODUCTION

Friction model is a set of mathematical model used to calculate the friction and to explain the mechanism of friction in motion. In general, in order to use differential equations to describe friction phenomena, the friction model can be divided into two types, namely, static friction model and dynamic friction model (Awrejcewicz Jan, Fečkan Michal, Olejnik Pawel, 2005), and the basis of improvement for the static friction model is the Coulomb friction model (Coulomb P.C.A). The Coulomb model states that the direction of friction is opposite to the relative velocity on the contact areas. The magnitude of the friction is independent of the magnitude of the relative velocity, but it is proportional to the magnitude of the normal load. The Coulomb friction model can be regarded as a description of macroscopic friction phenomenon, this is mainly because the effect of dynamic friction is only considered in relative motion. Hence, the Coulomb friction model implied a lot of microscopic phenomena of friction such as a static friction, Stribeck friction, pre-slip, and viscous sliding are not considered. Therefore, when the relative velocity approaches zero, the discontinuity of the friction will inevitably lead to discontinuity in the solution of dynamics in the multibody system, which lead to the result divergent and inaccurate(Armstrong-Hélouvry B., Canudas Dewit, C, 1995). In fact, the change of friction in motion must be a continuous process. It has been proved by a lot of experiments that the magnitude of friction is closely related to the magnitude of velocity when the velocity approaches zero (F. S., A X, Cieszka, et al, 2010). When the relative velocity approaches zero, the reference (F. S., A X, Cieszka, et al, 1990; Berger Ej, 2002) based on a number of experiments pointed out that the magnitude of friction is simultaneously related to the static friction coefficient and the dynamic friction coefficient. If the external tangential force is less than the static friction, the motion is viscous, and if the external tangential force is greater than the static friction force, the motion is sliding. 'Stribeck effect' is a micro-description of the excessive states between viscous and sliding, and it is a great improvement for the Column model in describing the frictional mechanism. The degree of agreement with the experimental data of the friction model can be greatly improved based on the accurate description in viscous and sliding. The discontinuous piecewise function in the Coulomb friction model

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can be originally turned into a continuous function, consequently, the stability of the integrating processes can be increased. Meanwhile, the description of ‘Stribeck effect’ also enhance the computational accuracy of the dynamics in multibody systems. How to describe the transition of viscous and sliding more accurately and solve the problem of self-excited oscillation and bifurcation caused by them has become a hot topic in academic research (Awrejcewicz J, 1998).

In order to solve the problem of the discontinuity of friction in the Coulomb friction model, the method of replacing the change curve of the friction with a specific function has been used in the static friction model at first when the relative velocity approached zero (Duan Chengwu, Singh Rajendra, 2006). According to the problem of switching state equation in friction model, Karnopp put forward the Karnopp model which create a zero field in a region of relatively low speed (Karnopp D, 1985). On this basis, Leine et al (Leine R. I., Campen D. H. Van, Kraker A. De, et al, 1998) improved the accuracy of numerical calculation and increased the stability of the integral process by introducing the definition of acceleration. Threlfall (Threlfall D. C, 1978) reduced the discontinuity of friction by using a system of equations on the basis of the Coulomb model. Filipe Marques et al (Marques Filipe, Flores Paulo, Pimenta Claro J. C., et al, 2016) improved the Threlfall model at the aspect of coefficient’s improvement and made friction closer to the result of the Coulomb model when the relative velocity approaches zero. In order to obtain the ‘Stribeck effect’, Bengisu and Akay (Bengisu M. T., Akay A, 1994) used two algebraic equations, one of them describes the sliding and another especially describes the ‘Stribeck effect’. Awrejcewicz (Awrejcewicz J, Grzelczyk D, Pyryev Yu, 2009) refined the stick-slip process and proposed a novel friction model which is expressed as four equations. The friction model mentioned above are some static friction model used in high frequency in the dynamic calculation of multibody systems in recent years. In addition, there are many other static friction models, for example, the Wojewoda et al model (Wojewoda J, Stefański A, Wiercigroch M, et al, 2008), the Ambrósio model (Ambrósio Jorge A.C, 2003), the Benson model (Benson David J., Hallquist John O, 1990) used in Multi-body system software COMSOL and the Velocity-based model used in dynamic simulation software ADAMS and so on. Dahl firstly put forward the Dahl model (Dahl P. R, 1968) based on microscopic deformation of bristle, the relative motion regarded as a deformation similar to the spring between contact surfaces in the static stage of the friction was considered for the first time. Based on the assumption of bristle deformation, there are still many other models which also think about the static friction, for instance, the LuGre model (De Wit C. Canudas, Olsson H, Astrom K. J, et al, 1995), the Elasto-plastic model (Dupont P., Armstrong B., Hayward V, 2002), the Stick-slip model (Cha Ho Young, Choi Juhwan, Han Sik Ryu, et al, 2011) and the Gonthier model (Gonthier Yves, Mcphee John, Lange Christian, et al, 2004) and so on. Compared to static friction models, the most of the dynamic friction models can more clearly and effectively reflect the friction characteristics in the movement of the multibody systems, thus, the more accurate results of the dynamic analysis can be obtained. However, the dynamic friction model contains state variables and involves many parameters. How to determine the value of each parameter and choose a more effective step and method of the iteration is very important to solve the problem as the friction phenomena are considered in the process of motion in multibody systems.

The purpose of this study is to analyze the effects of different friction models on the characteristics in friction and the results of kinematics analysis in multibody systems with different kinds of joints. There are eleven common friction models were selected as research objects in this paper, in which the static friction models respectively are the Smooth Coulomb model, the Threlfall model, the Bengisu model, the Karnopp model, the Velocity-based model and the Awrejcewicz model, and the dynamic friction models respectively are the Dahl model, the LuGre model, the Elasto-plastic model, the Stick-slip model and the Gonthier model. Two typical mechanisms which only contain prismatic joints and simultaneously contain prismatic joints and revolute joints respectively are tested, and the result of dynamics is compared with Adams. The influence of friction models on the results of kinematic simulation for different types of multibody systems is illustrated based on the computational efficiency and the stability of the numerical solution of the position. Finally, the optimal selection method of eleven friction models for different types of multibody systems is obtained, which provides a reference for how to more accurately and effectively solve the dynamic analysis when the characteristics in friction need to be considered in the future.

2 STRUCTURE OF THE INVESTIGATION

This paper is aimed to present and compare several friction models that can be used in multibody systems containing different kinds of joints. In order to obtain the frictional characteristics at low-speed motion and the accurate solution of the dynamics in a multibody system, two aspects of research are mainly done in this paper. In section 3 and section 4, six kinds of static and five kinds of dynamic friction models commonly used in a lot of previous literature were concluded respectively and their algebraic equations were briefly introduced. The comparison of the ability for describing the four kinds of friction phenomena is previewed in section 5. In section 6, three types of mechanical systems that only include prismatic joints, only include revolute joints and simultaneously include prismatic joints and revolute joints are selected as the research objects. In section 7, the influence of the different friction models on the friction characteristics and computational dynamics in multibody systems with different kinds of joints is analyzed according to the computational efficiency and the stability of simulation results, and a reference method is provided for choosing the friction model that is the best suitable for three different scenarios mentioned in the preceding section in the end.

3 COMPARISON BETWEEN FRICTION MODELS

This study takes into account the number of design parameters, the difficulty of parameter selection and the calculation efficiency of the friction model. Six static friction models and five dynamic friction models are selected for a brief introduction, and the mathematical equations of friction are listed. The ability of the description of friction phenomena is a very important evidence in estimating the computational accuracy of the friction (Gonthier Yves, Mcphee John, Lange Christian, et al, 2004), and the expression of friction model need to be consistent with actual conditions, which depends on the number of friction phenomena that can be accurately described. However, it is impossible to take all of the influence factors of friction into account. This paper focuses on four kinds of friction phenomena, namely, the dynamic friction, the static friction, the "Stribeck effect" and the pre-sliding, See Table 1 for contrastive details.

Table 1. Phenomena of friction models.

Name	Dynamic Friction	Static Friction	Stribeck	Pre-sliding
Smooth	🔴	🔵	🔵	🔵
Threlfall	🔴	🔵	🔵	🔵
Bengisu	🔴	🔴	🔴	🔵
Karnopp	🔴	🔴	🔵	🔵
Velocity-based	🔴	🔴	🔵	🔵
Awrejcewicz	🔴	🔴	🔵	🔵
Dahl	🔴	🔵	🔵	🔴
LuGre	🔴	🔴	🔴	🔴
Elasto-plastic	🔴	🔴	🔴	🔴
Stick-slip	🔴	🔴	🔴	🔵
Gonthier	🔴	🔴	🔴	🔴

Where 🔴 means it can be described, 🔵 means it can't be described. It can be seen from Table 2 that the dynamic friction model compared with the static friction model generally reflects the pre-sliding due to the consideration of the average deformation of the bristle in the static friction. In addition, when the four kinds of friction phenomena mentioned above can be all observed it is necessary to investigate other friction phenomena for the actual requirement and select an appropriate friction model according to the efficiency of calculation and the complexity of parameters used in the friction model.

4 NUMERICAL EXAMPLES AND COMPARISON

The advantages and drawbacks of the proposed friction model have briefly introduced above, and the specific calculation process is summarized. The purpose of this study is to analyze the influence of different friction models on the frictional characteristics and the results of the kinematic analysis in the multibody systems with the different kinds of joints in the case of dry friction. The mechanisms are the Rabinowicz case, the single pendulum, and the single pendulum box respectively. The dynamic simulation of them is carried out and the results of the analysis are compared with ADAMS.

4.1 Model with Prismatic Joints

The Rabinowitz case composed of sliders and springs is a single degree of freedom (DOF) model, and it is often used to study the viscous and sliding of friction phenomena in the dynamic test of multibody systems. After a lot of research and continuous improvement (Marques Filipe, Flores

Paulo, Pimenta Claro J. C., et al, 2016), the simplified model structure is shown in Fig. 1.

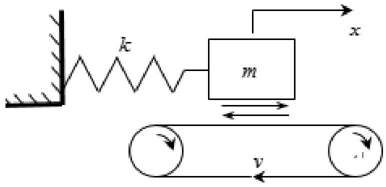

Fig 1. Diagram of the mechanism.

The belt rotates at a constant speed v in the Rabinowicz case, and the block moves under the combined action of friction and the tension of spring. When the tangential force namely the spring tension is less than the static friction, the slider is static. At this time, it should be in the stage of pre-sliding and static friction. When the tension force of the spring is greater than the static friction, the friction decreases with the increase of the relative velocity, that is "Stribeck effect". Meanwhile, the block begins to be in sliding until the tension force of the spring is less than the static friction again, and the process of motion begins to cycle. The parameters of each component in the mechanism are shown in Table 2, and the reference of the parameters involved in each model is shown in Table 3. The curves of the relative displacement, the relative velocity, the relative acceleration and the friction with time are respectively drawn in Fig. 2~ Fig. 5.

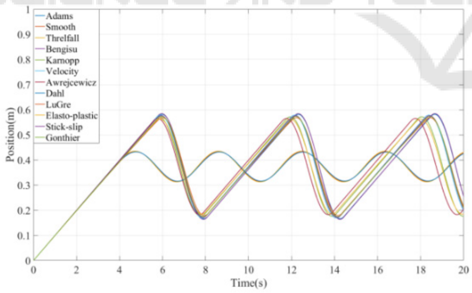

Fig 2. Relative displacement of the body.

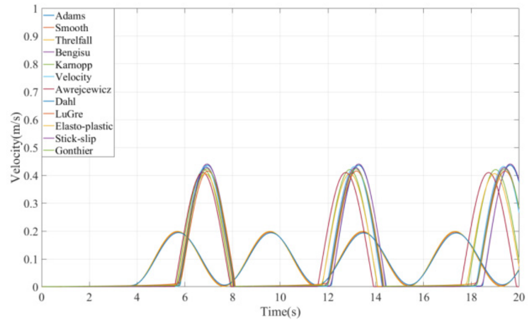

Fig 3. Relative velocity of the body.

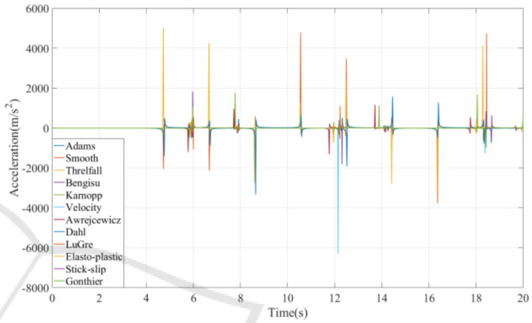

Fig 4. Relative acceleration of the body.

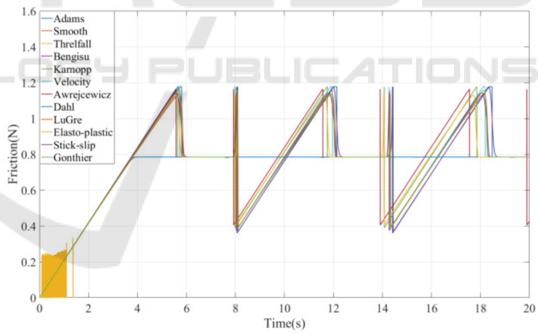

Fig 5. Friction of the body.

Table 2. Rabinowicz model parameters.

Name	Value	Name	Value
Mass (m)	0.8 kg	Step size(Δt)	10^{-6}s
Belt velocity(v)	0.1m/s	Time step(t)	20s
Stiffness coefficient(k)	2.1N/m	Integral method	Runge-Kutta

Table 3 Friction model parameters for Rabinowicz case

Name	Symbol	Value	Name	Symbol	Value
Dynamic friction coefficient	u_d	0.1	Damping coefficient	σ_1	190Ns/m
Static friction coefficient	u_s	0.15	Adhesion coefficient	σ_2	0 Ns/m
Velocity error	v_d	10^{-3}m/s	Breakaway displacement	z_{ba}	10^{-7}m
Stribeck velocity	v_{str}	10^{-3}m/s	Maximum deformation	z_{max}	10^{-6}m
Stiffness coefficient	σ_0	10^5N/m	Dwell-time constant	τ_{dw}	0.1

From Fig.2~ Fig.5 it can be seen that when the Coulomb model, the Threlfall model, and the Dahl model is adopted for the Rabinowicz case, the dynamic characteristics and the friction of slider in the mechanism with only prismatic joints are very similar, and the most of friction phenomena cannot be found except the dynamic friction. Nevertheless, the other models show the obvious process of the viscous and sliding. The differences of characteristics of the motion in the Rabinowicz case with different friction models gradually increase with time, which is mainly caused by the difference of parameters contained in each friction model and the accumulated error generated by the iterative process. The integral adopts the ode15s that are applicable to the dynamic friction model for the Runge-Kutta method and the absolute error is 10-8. Table 5 lists the calculation time used for each model. In order to select the friction model that satisfies the requirements of the frictional characteristics in actual conditions, the stability of the positional solution of each model is calculated by the equation (1), and the friction model that can meet the specific phenomenon in friction is sorted by the efficiency (t) and stability (s). The results are shown in Table 5, in which J stands for the static friction, S stands for the "stribeck effect", and Y stands for the pre-sliding.

$$s_i = \sqrt{\frac{\sum\left(x_i - \bar{x}_i\right)^2}{n_i - 1}}\left(i = 1,2,3,\ldots,11\right) \tag{1}$$

Where xi represents the solution of position, $\bar{x}_i$ represents the average value of the position, ni represents the number of solutions and i is the number of friction models.

Table 4. The time and position stability of friction models.

Name	*T*(*s*)	*S*($\times 10^2$)
Smooth	2.6988	12.7889
Threlfall	2.7909	11.6026
Bengisu	2.6242	13.8757
Karnopp	2.3851	13.9528
Velocity-based	2.1976	13.8141
Awrejcewicz	2.3762	12.5478
Dahl	9.6875	13.4439
LuGre	11.3336	15.0088
Elasto-plastic	>100	14.5011
Stick-slip	10.4979	13.3829
Gonthier	16.7579	14.6707

Table 5. Comparison of selection order for various friction phenomena.

Name	J		S		Y		J+S		J+S+Y	
	t	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*
Smooth										
Threlfall										
Bengisu	④	④	①	②			①	②		
Karnopp	③	⑤								
Velocity	①	③								
Awrejcewicz	②	①								
Dahl					①	①				
LuGre	⑥	⑧	③	⑤	②	④			①	③
Elasto-plastic	⑧	⑥	⑤	③	④	②			③	①
Stick-slip	⑤	②	②	①			②	①		
Gonthier	⑦	⑦	④	④	③	③			②	②

When the static friction phenomenon is only required to be observed in the Rabinowicz case, it can be seen from Table 6 that if the efficiency of calculation is firstly considered the Velocity-based model should be selected, and if the stability is a priority the Awrejcewicz model should be chosen first. When the "stribeck effect" only needs to be observed in practice, the Bengisu model should be chosen first for computational efficiency but the Stick-slip model for stability. The Dahl model can be selected directly when the pre-sliding is only considered in actual conditions. Similarly, when the static friction and the "stribeck effect" need to be observed at the same time, the first choice is the Bengisu model for the computational efficiency, and for the stability of position the first choice is the Stick-slip model. Finally, if three friction phenomena mentioned above all need to be described at the same time the LuGre model should be selected when the efficiency of calculation is considered firstly, but the Elasto-plastic model should be chosen in consideration of the data stability.

4.2 Model with Prismatic and Revolute Joints

Summarizing the structural features of the two cases before, the single pendulum box both with revolute joints and prismatic joints is considered as the research object. The effects of different friction models on the motion characteristics of the two components in the single pendulum box are studied. The schematic diagram of the mechanism is shown in Fig. 6, and the parameters of each component in the single pendulum box are shown in Table 10. The pendulum hangs on the midcourt line on the top of the box, the initial angle is 30° and the distance to the ground from the body center of mass is h. The free swing of the single pendulum drives the box to slide left and right and finally comes to rest. Considering the frictional force of the prismatic and revolute joints at the same time, the box appears to be the viscous and sliding as its velocity approaches zero. See Fig.6 ~ Fig.12 for its characteristic curves in motion.

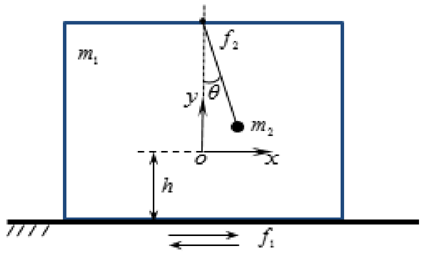

Fig 6. Simple diagram of simple pendulum box.

Table 6. Simple pendulum box parameters.

Name	Value	Name	Value
Box mass(m_1)	6 kg	Initial angular (θ)	30°
Box moment(I_1)	0.1 kg.m^2	Initial position (pendulum)	[0,0.4]
The height of center of mass(h)	0.2m	Pendulum moment(I_2)	1.8 kg.m^2
Initial position (Box)	[0,0]	Step size(Δt)	10^{-6}s
Pendulum mass(m_2)	20 kg	Time step(t)	15s
Rod length(L)	0.3m	Integral method	Runge-Kutta

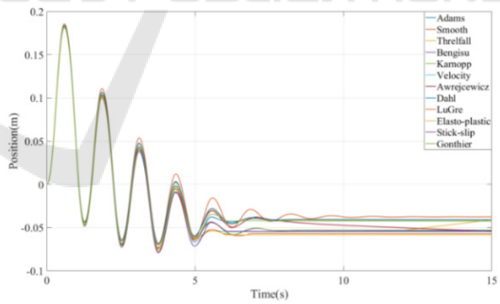

Fig 7. Relative displacement for the box.

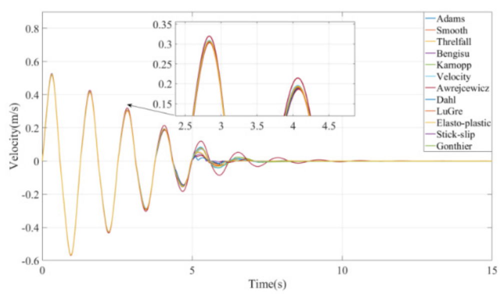

Fig 8. Relative velocity for the box.

Table 7. The friction model parameters for Simple pendulum box.

Simple pendulum			Box		
Name	Symbol	Value	Name	Symbol	Value
Dynamic friction coefficient	u_d	0.002	Dynamic friction coefficient	u_d	0.02
Static friction coefficient	u_s	0.003	Static friction coefficient	u_s	0.03
Velocity error	v_d	10^{-2}m/s	Velocity error	v_d	0.06m/s
	v_s	10^{-3}m/s		v_s	0.0005
	v_t	10^{-3}m/s		v_t	10^{-3}m/s
Stribeck velocity	v_{str}	10^{-3}m/s	Stribeck velocity	v_{str}	10^{-3}m/s
	v_{str}	10^{-5}m/s(Gon)		v_{str}	10^{-5}m/s(Gon)
Stiffness coefficient	σ_0	10^4N/m	Stiffness coefficient	σ_0	10^4N/m
Damping coefficient	σ_1	2Ns/m	Damping coefficient	σ_1	2Ns/m
Adhesion coefficient	σ_2	0 Ns/m	Adhesion coefficient	σ_2	0 Ns/m
Breakaway displacement	z_{ba}	10^{-7}m	Breakaway displacement	z_{ba}	10^{-7}m
Maximum deformation	z_{max}	10^{-7}m	Maximum deformation	z_{max}	10^{-7}m
Dwell-time constant	τ_{dw}	0.01	Dwell-time constant	τ_{dw}	0.01

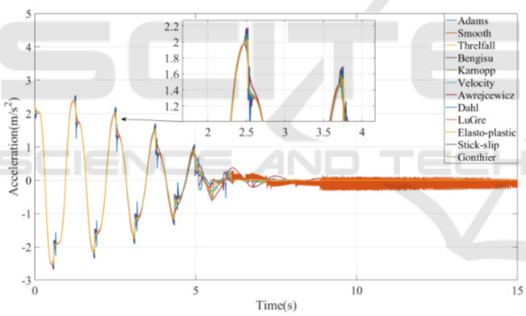

Fig 9. Relative acceleration for the box.

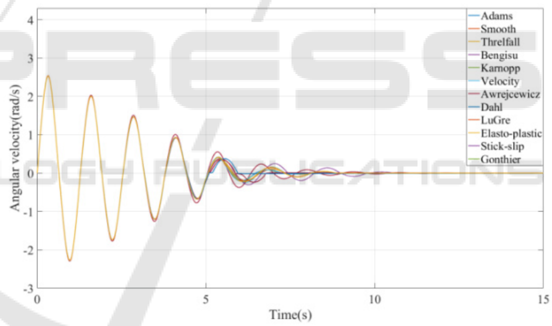

Fig 11. Relative angular velocity for the pendulum.

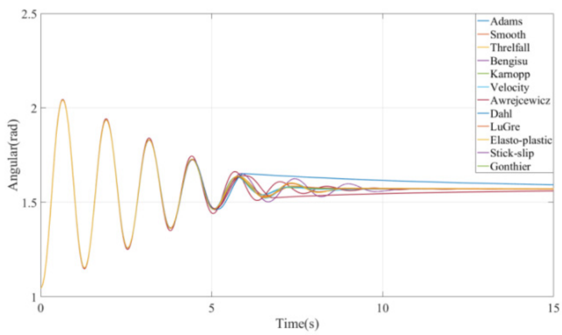

Fig 10. Relative angular for the pendulum.

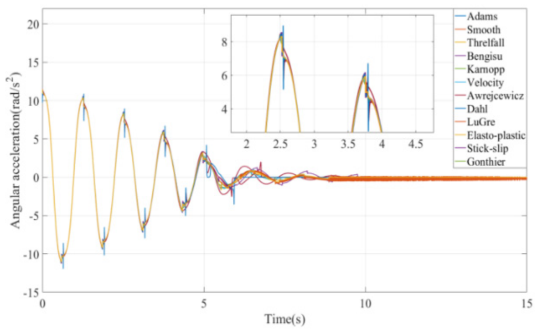

Fig 12. Relative angular acceleration for the pendulum.

Table 8. The time and position stability of friction models.

Name	$T(s)$	$S(\times10^2)$
Smooth	3.5109	5.8842
Threlfall	4.3273	6.4755
Bengisu	5.8561	6.7982
Karnopp	6.5503	7.0905
Velocity-based	8.3748	3.7149
Awrejcewicz	6.1151	7.1542
Dahl	45.3545	6.8376
LuGre	76.8279	6.2963
Elasto-plastic	229.0278	3.3126
Stick-slip	99.3741	1.4088
Gonthier	129.9982	5.3765

Table 9. Comparison of selection order for various friction phenomena.

Name	J		S		Y		J+S		J+S+Y	
	t	*s*	*t*	*s*	*t*	*s*	*t*	*s*	*t*	*s*
Smooth										
Threlfall										
Bengisu	①	⑥	①	⑤			①	②		
Karnopp	③	⑦								
Velocity	④	③								
Awrejcewicz	②	⑧								
Dahl					①	④				
LuGre	⑤	⑤	②	④	②	③			①	③
Elasto-plastic	⑧	②	⑤	②	④	①			③	①
Stick-slip	⑥	①	③	①			②	①		
Gonthier	⑦	④	④	③	③	②			②	②

The single pendulum box is a kind of mechanism in which the sliding block is driven to back and forth by the weight component of the pendulum. In the whole process of moving, the friction model is coupled with the mechanical system, which is because the friction in the prismatic joints and the friction in revolute joints are both considered. From Fig. 7 to Fig. 9, it can be seen that the static friction model and the dynamic friction model have little influence on the relative position and velocity of the box, but the acceleration has an obvious error and appears big fluctuation. See Fig.10 to Fig.12, it is found that the relative angle, the relative angular velocity and relative angular acceleration of the single pendulum in the selection of the LuGre model and the Elasto-plastic model have obvious errors at the end of the motion. The same as two cases above, the best choice of each friction model for the specified friction phenomenon can be obtained by sorting the efficiency of calculation and the stability of position. The following conclusions can be obtained by comparing the select method of the three cases:

(1) For the single pendulum box, it is different from the previous two cases when the static friction phenomenon is only required to be observed in actual conditions. If the efficiency of calculation is taken first, the Bengisu model should be selected; if the stability is taken first, the Stick-slip model should be taken.

(2) When the actual conditions only need to observe the "stribeck effect", the selection of the friction model is the same as the Rabinowicz case. When the pre-sliding is only needed to be considered, the Dahl model was selected owing to the computational efficiency has little influence on the variation of the kinds of the joint in the mechanism and remained the highest effect all the time. When the stability of position is considered first, the select method of the single pendulum box is the same as the Rabinowicz case, namely, the Elasto-plastic model should be selected.

(3) When the static friction and the "stribeck effect" need to be observed at the same time or the three kinds of friction phenomena mentioned in the previous section need to be observed simultaneously, the change of the prismatic joints and the revolute joints in the mechanism has no influence on the selection of friction model. Considering the difficulty of parameter selection, it is generally preferred the LuGre model in actual conditions.

5 CONCLUSION

In this paper, six static friction models and five dynamic friction models are briefly reviewed for the problem of dynamic performance affected by the different friction models. Two kinds of mechanisms including a model with prismatic joints and model both with prismatic joints and revolute joints were tested, the dynamic simulation of the three cases was conducted and the change curve was drawn. The numerical solution is compared with ADAMS, and the analysis shows that:

(1) According to the special requirements of friction phenomena in practical application, the selected order of friction models discussed in this paper is different when the multibody system includes different joints, especially in considering a certain friction phenomenon. Only fewer friction models can be selected when need to describe more friction phenomena. This is a very important reason to limit the selected order of friction model when the multibody system includes different kinds of joints, such as only including prismatic joint or including revolute joint and prismatic joint simultaneously.

(2) Compared to the static friction models, the dynamic friction models own favorable continuity when the multibody system contains different kinds of joints. It is more important that the dynamic friction models can better depict the nonlinear behavior such as the pre-sliding, the "Stribeck effect", the static friction and the viscous- sliding.

(3) Regarding the multibody system only includes prismatic joints, the effect of different kinds of friction modes on its dynamic response is not obvious. When the multibody system simultaneously includes prismatic joints and revolute joint, the effect of dynamic friction model on the acceleration is significant.

(4) Due to the dynamic friction model involves a lot of parameters and has a significant influence on the multibody system with revolute joint, in order to improve the computational accuracy and the stability of calculated results, hence, the dynamic friction model should avoid being selected in the multibody system with revolute joint. However, the LuGre model is the best choice when the more friction phenomena need to be studied.

In order to eliminate the adverse factors caused by the friction to improve the dynamic performance of the mechanical system, the effects of different friction models on the characteristics in the motion of the multibody systems with prismatic joints and revolute joints are considered in this study. According to the computational efficiency and the stability of different friction models in different mechanisms, the optimal friction model with different kinds of joints in multibody systems is obtained. Different multibody systems select different friction models according to the actual conditions, the computational efficiency and the stability of simulation results. The friction model with different kinds of joints is a very important factor for the results of the dynamic calculation. The qualitative analysis of different friction models in the dynamic characteristics of the mechanisms with different kinds of joints provides an important theoretical basis for the following study of dynamics in multibody systems with clearance and collision.

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