

New Method for Initial Alignment of Angular Sway Base based on b-n Solidification Frame

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Abstract: In this paper, for the problem of initial alignment process of strapdown inertial navigation system under the angular sway base solved by the analytical alignment method has a large error. A component is proposed to track the gravity vector at different moments in the navigation frame and the body frame. Take the components of different two moments as the coarse alignment method of the new double vector positioning. The simulation method is applied to verify that this method has the same effect as the traditional analytical alignment method under static base. The result is obviously superior to the traditional analytical alignment under the condition of angular sway base, and has stronger applicability.

1 INTRODUCTION

The initial alignment technique is the first step in the navigation and positioning of Strapdown Inertial Navigation System. Subsequent navigation can be accurate only if the initial alignment is guaranteed to be high precision (Y. Y. Qin, 2009). Generally, the initial alignment can be divided into two processes: coarse alignment and fine alignment (N.S. Reddy and J. Murray, 1991). The coarse alignment is the principle of using the double vector attitude to determine the attitude angle at the initial moment. Fine alignment is the use of Kalman filtering and other techniques to achieve the initial attitude misalignment angle estimation. Many scholars are working on the study of coarse alignment. In the paper (L. Schimelevich, et.al, 1996), using a neural network based on multilayer perceptron in correct the alignment and calibration errors of the inertial measurement unit. Coarse alignment method has been stated in detail in (D. H. Titterton and J. L. Weston, 2004) under the heading of ground alignment methods. Autonomous alignment method mentioned in (O. Tekinalp and M. Ozemre, 2001) also starts with the coarse alignment method. In the paper (H. Y. Zhao, 2011), the accuracy of six analytical coarse alignment modes is compared, and finally the coarse alignment mode with the highest precision is obtained.

The traditional analytical coarse alignment method has better accuracy under static base. However, under the condition of the angular sway base, the angular sway will cause the gyro output error to be large, consequently the alignment accuracy will be affected. Therefore, the research of coarse alignment technology of strapdown inertial navigation system under the condition of angle swaying base is of great significance.

Inspired by the paper (K. S. Yan and Y. L. Liu, 2017; J. Li, Y. Wang, Y. Li and J. Fang, 2018), an initial alignment technique based on b-n solidification frame for angular sway base is proposed. And compare the traditional analytical alignment method by simulation, this method can effectively shield the angular swaying from the initial alignment precision interference, can effectively suppress the influence of the angular sway on the roll angle precision, and greatly improve the accuracy of the initial alignment.

2 THE PROBLEM OF ANALYTICAL COARSE ALIGNMENT

The analytical alignment is mainly obtained by obtaining the component of the earth's rotation angular velocity ω_{ie} and the gravitational

acceleration g in the body frame and the navigation frame (Wang Xinlong, 2013), the alignment matrix is obtained by the conversion relationship between the two frames. The principle is as follows:

$$f_b = C_n^b g_n \tag{1}$$

$$\omega_{ib}^b = C_n^b \omega_{in}^n \tag{2}$$

Where $g_n = [0 \ 0 \ -g]^T$, $\omega_{in}^n = [0 \ \omega_{ie} \cos L \ \omega_{ie} \sin L]^T$.

A new vector can be constructed by the formulas (1) and (2)

$$f_b \times \omega_{ib}^b = C_n^b (g_n \times \omega_{in}^n) \tag{3}$$

According to the above three formulas, the alignment matrix can be found.

$$C_b^n = \begin{bmatrix} g_n \\ \omega_{in}^n \\ g_n \times \omega_{in}^n \end{bmatrix}^{-1} \begin{bmatrix} f_b \\ \omega_{ib}^b \\ f_b \times \omega_{ib}^b \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{4}$$

From the relationship between the alignment matrix and the attitude angle, the attitude angle can be expressed as

$$\theta = \arcsin C_{32} = \arcsin \left(-\frac{f_y^b}{g} \right) \tag{5}$$

$$\gamma = \arctan \left(-\frac{C_{31}}{C_{33}} \right) = \arctan \left(-\frac{f_x^b}{f_z^b} \right) \tag{6}$$

$$\psi = \arctan \left(\frac{C_{12}}{C_{22}} \right) = \arctan \left(\frac{\frac{\sec L}{g \omega_{ie}} (\omega_{ibx}^b f_z^b - \omega_{ibz}^b f_x^b)}{\frac{f_y^b}{g} \tan L + \frac{\omega_{iby}^b}{\omega_{ie}} \sec L} \right) \tag{7}$$

When the pedestal has an angular sway, the output of the accelerometer has almost no effect. However, the output of the accelerometer will be greatly disturbed, and the output of the accelerometer will have a large error. Therefore, the angular sway has less influence on the heading angle

and the pitch angle, and has a greater influence on the roll angle.

3 ANGULAR SWAY BASE COARSE ALIGNMENT NEW METHOD

Under the static base, the traditional analytical coarse alignment can obtain the initial attitude information of the carrier relatively accurately according to the gravity vector g and the earth rotation angular velocity ω_{ie} , which can meet the requirements of fine alignment conditions. However, in actual situations, the Strapdown Inertial Navigation System will be subject to angular sway interference. If the analytical alignment method is also used, it is difficult to meet the precision alignment requirements. According to the principle of double vector positioning, two mutually uncorrelated vectors are determined. Since the gravity vector is constantly changing in the inertial space, the gravity acceleration at different times can be selected to construct the double vector., Thus, the double vector positioning is completed, which is the coarse alignment method of the solidification frame. In this paper, a coarse alignment algorithm based on b-n (body frame and navigation frame) solidification frame is proposed. The principle is as follows: According to the chain rule (L. B. Chang, J. S. Li and S. Y. Chen, 2015)

$$C_b^n(t) = C_{n_0}^{n_t} C_b^n(0) C_{b_t}^{b_0} \tag{8}$$

In the formula, n_0 and b_0 are the inertial frames formed by solidification of the n-frame and b-frame at the initial timing of alignment. $C_{n_t}^{n_0}$ and $C_{b_t}^{b_0}$ respectively describe the attitude change of the n-frame and the b-frame during the $[0, t]$ time in the alignment process, and can be obtained by the following differential equation:

$$\dot{C}_{n_t}^{n_0} = C_{n_t}^{n_0} \omega_{in}^n \times \tag{9}$$

$$\dot{C}_{b_t}^{b_0} = C_{b_t}^{b_0} \omega_{ib}^b \times \tag{10}$$

Where ω_{ib}^b is the angular rate of the carrier itself measured by the gyroscope in the body frame.

$\omega_{in}^n = \omega_{ie}^n + \omega_{en}^n$, Where ω_{ie}^e represents the earth's rotation rate relative to the inertial coordinate system, ω_{en}^n represents the angular rate of the navigation coordinate system relative to the earth coordinate system. In addition, it can be easily observed that the initial conditions of the above differential equation are unit matrices. Therefore, $C_{n_i}^{n_0}$ and $C_{b_i}^{b_0}$ can be obtained by the following equivalent rotation vector method.

ω_{in}^n represents the component of the Earth's rotation angular velocity in the navigation coordinate system, which can be expressed as

$$\omega_{in}^n = \begin{bmatrix} 0 \\ \omega_{ie} \cos L \\ \omega_{ie} \sin L \end{bmatrix} \quad (11)$$

It can be seen that ω_{in}^n is an amount that does not change with time. When the time interval is T, the equivalent rotation vector can be used to find that the equivalent rotation vector can be expressed as

$$\varphi_n = T\omega_{in}^n \quad (12)$$

$$C_{n_i}^{n_{i-1}} = I + \frac{\sin(\|\varphi_n\|)}{\|\varphi_n\|} \varphi_n \times + \frac{1 - \cos(\|\varphi_n\|)}{\|\varphi_n\|^2} (\varphi_n \times)^2 \quad (13)$$

$$C_{n_i}^{n_0} = C_{n_{i-1}}^{n_0} C_{n_i}^{n_{i-1}} \quad (14)$$

And ω_{ib}^b is measured by a gyroscope, which is a quantity that changes with time. When the time interval is T, the equivalent rotation vector can be approximated as

$$\varphi_b = \Delta\theta_1 + \Delta\theta_2 + \frac{2}{3} \Delta\theta_1 \times \Delta\theta_2 \quad (15)$$

$$C_{b_i}^{b_{i-1}} = I + \frac{\sin(\|\varphi_b\|)}{\|\varphi_b\|} \varphi_b \times + \frac{1 - \cos(\|\varphi_b\|)}{\|\varphi_b\|^2} (\varphi_b \times)^2 \quad (16)$$

$$C_{b_i}^{b_0} = C_{b_{i-1}}^{b_0} C_{b_i}^{b_{i-1}} \quad (17)$$

Where ‘ \times ’ represents a cross multiplication operation, and $\Delta\theta_1$ and $\Delta\theta_2$ can be obtained by the following equation

$$\Delta\theta_1 = \int_0^{T/2} \omega_{ib}^b dt \quad (18)$$

$$\Delta\theta_2 = \int_{T/2}^T \omega_{ib}^b dt \quad (19)$$

$C_b^n(0)$ is the transformation matrix of the initial time carrier coordinate system to the navigation coordinate system, and the matrix is a constant value, which can be obtained by the method of double vector positioning. However, the traditional method of double vector positioning is due to the shaking of the pedestal, resulting in a large output error of the gyroscope, so using the tracking gravity vector, that is, the components of the gravity acceleration at different moments in the b-frame and the n-frame at the initial moment are selected as the double vector for the fixed posture. This method can effectively shield the influence of angular sway on the initial alignment accuracy.

According to the definition of the inertial navigation frame, when the earth is in the process of rotation, the component of the gravitational acceleration g in the navigation frame is

$$g_{n_i} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \quad (20)$$

Then transform the component of gravity acceleration g in the navigation frame into the component in the inertial navigation frame, which can be expressed as

$$g_{n_i}^{n_0} = C_{n_i}^{n_0} g_{n_i} \quad (21)$$

The component of the gravitational acceleration g in the body frame can be obtained by an accelerometer

$$f_b = \begin{bmatrix} f_{b_x} \\ f_{b_y} \\ f_{b_z} \end{bmatrix} \quad (22)$$

Then convert it into the component of the inertial body frame to get

$$f_{b_i}^{b_0} = C_{b_i}^{b_0} f_{b_i} \quad (23)$$

According to the force equation, when the linear interference of the carrier is neglected

$$f_{b_0} = C_n^b(0)g_{n_0} \quad (24)$$

For the different moments of the formula (24), there are time t1 and t2

$$f_{b_{n_1}}^{b_0} = C_n^b(0)g_{n_{n_1}}^{n_0} \quad (25)$$

$$f_{b_{n_2}}^{b_0} = C_n^b(0)g_{n_{n_2}}^{n_0} \quad (26)$$

You can get the formula (25) and (26) by multiplying

$$(f_{b_{n_1}}^{b_0} \times f_{b_{n_2}}^{b_0}) = C_n^b(0)(g_{n_{n_1}}^{n_0} \times g_{n_{n_2}}^{n_0}) \quad (27)$$

After transposing the formulas (25), (26), and (27) respectively, the shift items can be obtained.

$$C_n^b(0) = \begin{bmatrix} (g_{n_{n_1}}^{n_0})^T \\ (g_{n_{n_2}}^{n_0})^T \\ (g_{n_{n_1}}^{n_0} \times g_{n_{n_2}}^{n_0})^T \end{bmatrix}^{-1} \begin{bmatrix} (f_{b_{n_1}}^{b_0})^T \\ (f_{b_{n_2}}^{b_0})^T \\ (f_{b_{n_1}}^{b_0} \times f_{b_{n_2}}^{b_0})^T \end{bmatrix} \quad (28)$$

The solution to $C_n^b(0)$ can be solved by the appeal method, however, during the initial alignment of the actual strapdown inertial navigation. Measurement data is obtained through inertial components, it is inevitable that there will be noise interference during the measurement process. In order to effectively block noise without losing information, Integrate $f_{b_{n_1}}^{b_0}$ and $f_{b_{n_2}}^{b_0}$ at time [0, t] respectively.

$$V_{b_0} = C_n^b(0)V_{n_0} \quad (29)$$

In the same way, take two different moments t1 and t2, then

$$V_{b_{n_1}} = C_n^b(0)V_{n_{n_1}} \quad (30)$$

$$V_{b_{n_2}} = C_n^b(0)V_{n_{n_2}} \quad (31)$$

$$V_{b_{n_1}} \times V_{b_{n_2}} = C_n^b(0)(V_{n_{n_1}} \times V_{n_{n_2}}) \quad (32)$$

So matrix A can be solved by (33)

$$C_n^b(0) = \begin{bmatrix} (V_{n_{n_1}})^T \\ (V_{n_{n_2}})^T \\ (V_{n_{n_1}} \times V_{n_{n_2}})^T \end{bmatrix}^{-1} \begin{bmatrix} (V_{b_{n_1}})^T \\ (V_{b_{n_2}})^T \\ (V_{b_{n_1}} \times V_{b_{n_2}})^T \end{bmatrix} \quad (33)$$

V_{b_i} And V_{n_i} can be accurately obtained by the formulas (34) and (35)

$$V_{b_i} = \int_0^t f_{b_i}^{b_0} dt = \int_0^t C_{b_i}^{b_0} f_{b_i} dt \quad (34)$$

$$V_{n_i} = \int_0^t f_{n_i}^{n_0} dt = \int_0^t C_{n_i}^{n_0} f_{n_i} dt \quad (35)$$

And then can be converted to a recursive formula (Y. X. Wu and X. F. Pan, 2011)

$$V_{b_i} = V_{b_{i-1}} + C_{b_i}^{b_0} \left[\Delta v_1 + \Delta v_2 + \frac{1}{2}(\Delta \theta_1 + \Delta \theta_2) \times (\Delta v_1 + \Delta v_2) + \frac{2}{3}(\Delta \theta_1 \times \Delta v_2 + \Delta v_1 \times \Delta \theta_2) \right] \quad (36)$$

$$V_{n_i} = V_{n_{i-1}} + C_{n_i}^{n_0} \left(TI + \frac{T^2}{2} \omega_{in}^n \times \right) g_{n_i} \quad (37)$$

3 SIMULATION VERIFICATION AND RESULT ANALYSIS

In order to verify this new method, this paper uses Matlab to simulate this, and the simulation conditions are set to: the position of the strapdown inertial navigation system is 118° east longitude, 32.2° north latitude; T=0.01s; the total simulation time is 2000s; the attitude angle of static base is $\Psi=2^\circ$, $\theta=3^\circ$, $\gamma=4^\circ$. The constant drift of the gyroscope is $1 \times 10^{-3}g$, the random drift is $5 \times 10^{-4}g$, the constant value drift of the accelerometer is $0.02^\circ/h$, and the random drift is $0.01^\circ/h$.

The angular sway causes the attitude angle to appear periodically:

$$\begin{aligned} \psi &= 2^\circ \cos\left(\frac{2\pi}{7}t + \frac{\pi}{3}\right) \\ \theta &= 3^\circ \cos\left(\frac{2\pi}{5}t + \frac{\pi}{4}\right) \\ \gamma &= 4^\circ \cos\left(\frac{2\pi}{6}t + \frac{\pi}{7}\right) \end{aligned} \quad (38)$$

The simulation results under static pedestal conditions are shown in Fig.1, it can be seen that the traditional analytical alignment under the static pedestal condition is basically consistent with the initial alignment technique based on the b-n solidification frame proposed herein. The theoretical attitude angles are the heading angle $\theta=3^\circ$, the pitch angle $\gamma=4^\circ$, and the roll angle $\Psi=2^\circ$. Due to the static pedestal conditions, the traditional analytical alignment can be better adapted. However, the proposed method is very close to the theoretical value under static pedestal conditions, indicating that the method can also have good alignment effect under static pedestal conditions. However, due to the constant value drift and random drift of the inertial element itself, the result fluctuates around the theoretical result. The new method proposed in this paper is very close to the theoretical value under static base conditions, and the fluctuation is small, it shows that the method can also have good alignment effect under static base conditions. The results show that the analytical coarse alignment method has a maximum deviation of the yaw angle of 0.0086° , a maximum deviation of the pitch angle of 0.0092° , and a maximum deviation of the roll angle of 0.1715° . The b-n solidification frame coarse alignment algorithm has a maximum deviation of the heading angle of 0.007° , a maximum deviation of the yaw angle of 0.008° , and a maximum deviation of the roll angle of 0.132° .

When the simulation condition is angular sway base, the traditional analytical alignment method based on double vector positioning is affected by angular sloshing, which will cause a large error in the output of the gyroscope. It can be seen from the relationship between the attitude angle and the alignment matrix that the yaw angle and pitch angle accuracy are mainly related to the accelerometer output accuracy, and the roll angle accuracy is not only related to the accelerometer, but also affected by the gyroscope output accuracy. The angular sway mainly affects the output accuracy of the gyroscope, therefore, the accuracy of the traditional analytical alignment roll angle is greatly affected, while the heading angle and the pitch angle are less affected. It can be seen from Fig.2 that the alignment method based on the b-n solidification frame proposed in this paper is compared with the conventional analytical alignment method. Under the condition that the alignment accuracy of the heading angle and the pitch angle are kept constant, the influence of the angular sway on the accuracy of the roll angle is greatly reduced, thereby improving the initial alignment accuracy. It can be seen that the two coarse alignment methods have a similar yaw angle and pitch angle to the ideal result. However, the traditional analytical alignment roll angle fluctuates between -89.81° and 89.99° , with the new method, the roll angle fluctuates between -2.37° and 2.28° . This new method has a roll angle that is closer to ideal value.

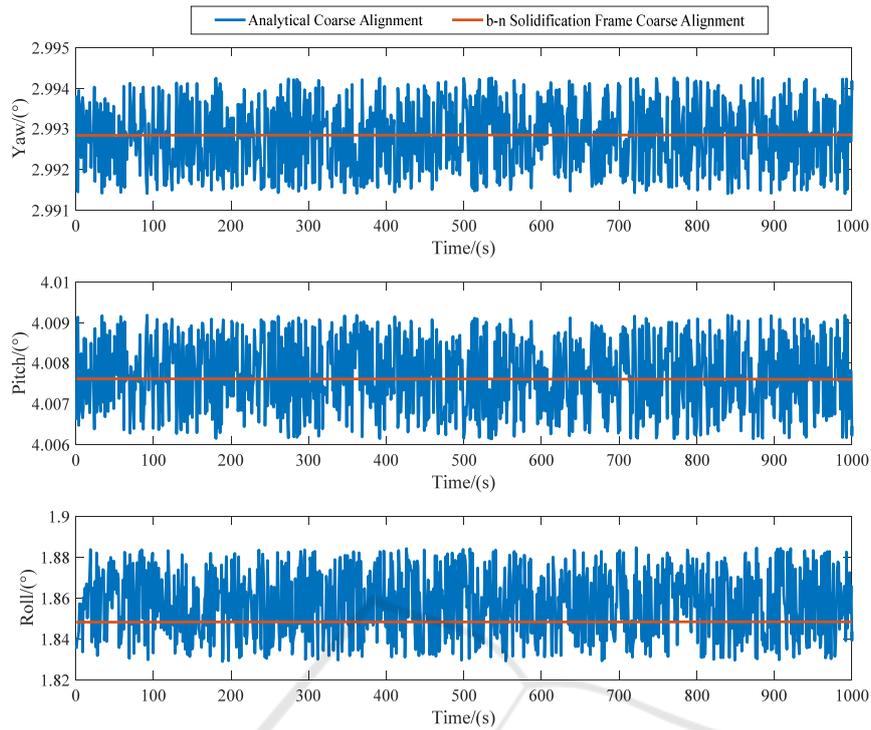


Figure 1. Static base two coarse alignment method results.

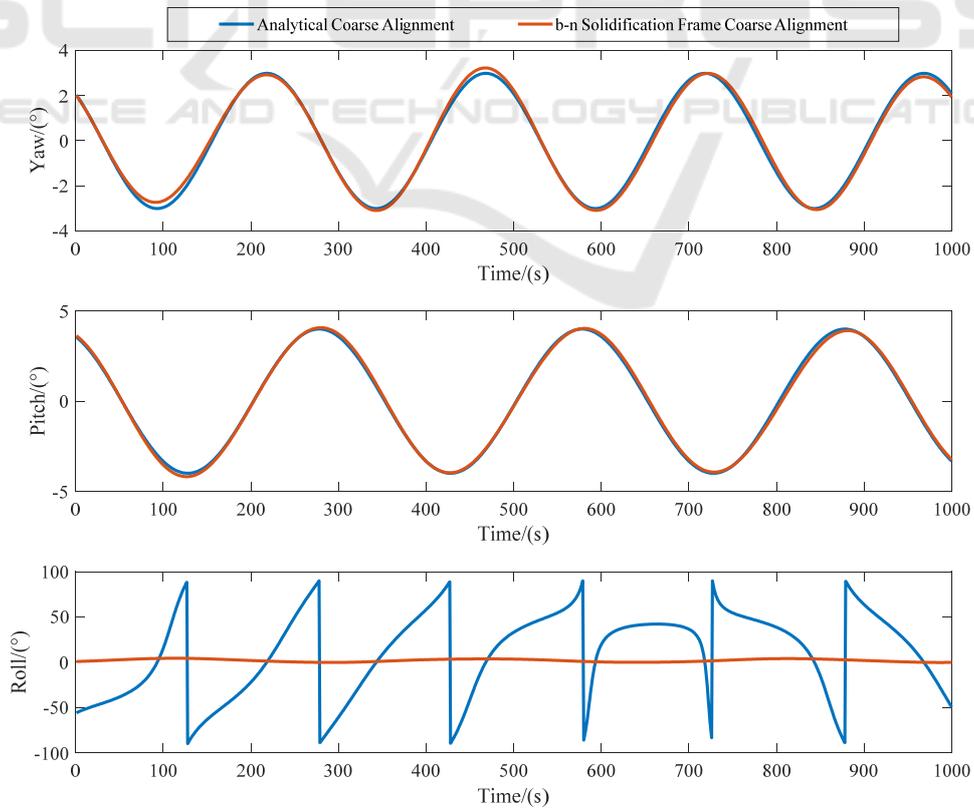


Figure 2. Angular sway base two coarse alignment method results.

The two alignment processes are completed by introducing a fine alignment method of Kalman filtering. The scheme is shown in Table 1.

Table 1. Two alignment schemes.

Alignment Scheme 1	b-n Solidification Frame Coarse Alignment (2min) + Kalman Filter Fine Alignment (5min)
Alignment Scheme 2	Analytical Coarse Alignment (2min) + Kalman Filter Fine Alignment (5min)

The simulation results are shown in Fig.3, 4, regardless of the static base or the angular sway base, the b-n solidification frame and Kalman filter method has little difference between the eastward misalignment angle and the northward misalignment angle with the analytical coarse alignment and Kalman filter method, however, the two methods have large deviations from the heavenward

misalignment angle. Under static base, the method of analyzing coarse alignment and Kalman filter fine alignment, the eastward misalignment angle is 0.0007", the northward misalignment angle is 0.0009", and the heavenward misalignment angle is 0.067". The method of b-n solidification frame coarse alignment and Kalman filter fine alignment, the three misalignment angles are almost 0". Under the condition of angular sway base, the method of analysing coarse alignment and the Kalman filter fine alignment, the eastward misalignment angle is 0.017", the northward misalignment angle is -0.0103", and the heavenward misalignment angle is 1.914"; The b-n solidification frame coarse alignment and the Kalman filter fine alignment method, the eastward misalignment angle is -0.0039", the northward misalignment angle is -0.002", and the heavenward misalignment angle is 0.1392".

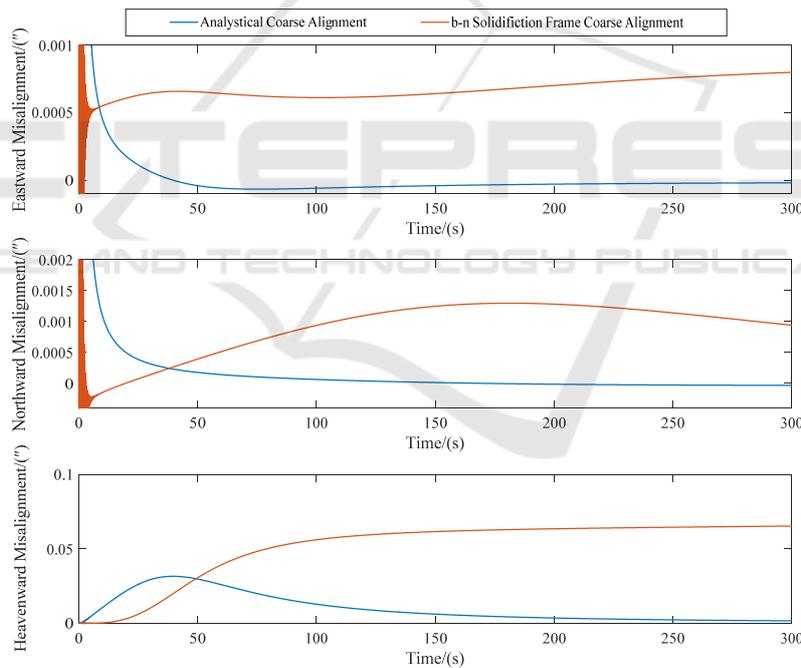


Figure 3. Static base two alignment scheme results.

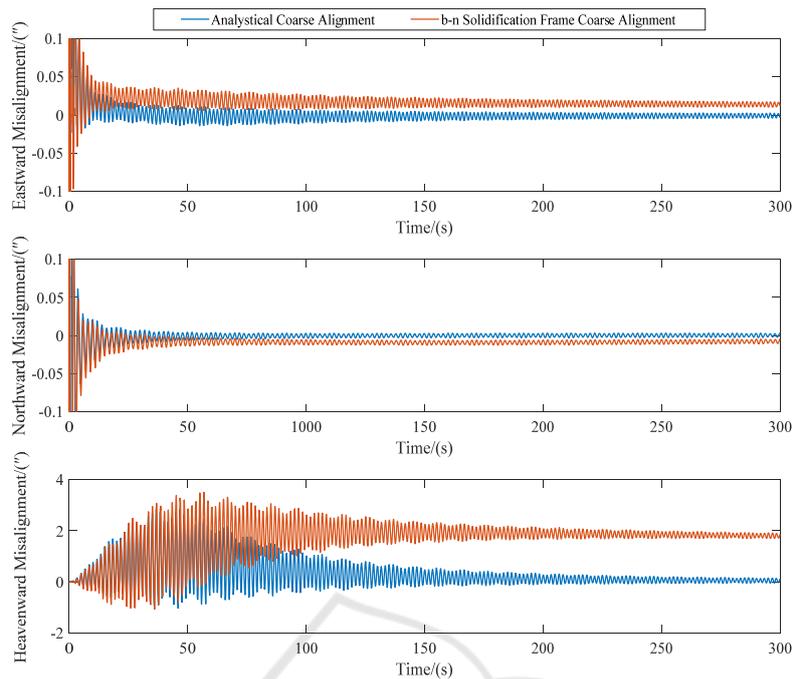


Figure 4. Angular sway base two alignment scheme results.

4 CONCLUSION

In this paper, a method of coarse alignment of strapdown inertial navigation based on b-n solidification frame is proposed. This method is obtained by the method of double vector attitude determination by the components of gravity vector in navigation frame and body frame at different moments. The attitude angle at the initial moment. The method can effectively shield the interference of the angular sway on the initial alignment, especially the disturbance of the roll angle. Combined with the Kalman filter fine alignment method, the simulation results show that the method can reduce the eastward misalignment angle by 77.1%, reduce the northward misalignment angle by 80.9%, and reduce the heavenward misalignment angle by 92.7%. In general, the method has high alignment accuracy and is highly applicable.

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