Finite-Time Synchronization for Neutral-Type Neural Networks with Markovian Switching and Multi-Delays

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Abstract: In this brief, the problem of the finite-time synchronization is considered for neutral-type neural networks (NTNNs) with the Markovian switching and multi-delays. Sufficient conditions are acquired for the finite-time synchronization of NTNNs by Lyapunov stability theory. Then, the adaptive control is designed by a suitable updated law. Finally, a numerical simulation is given to illustrate the effectiveness of the obtained result.

1 INTRODUCTION

During past several years, the stabilization and the synchronization of neural networks (NNs) are studied deeply in many fields (Y. Cao, 2016), such as the robot path planning, and the multi-robot cooperation. In the control system, the model of the the corresponding system is different when there is the disturbance in the system (X. Liu, 2016). According to the transformation relationship of Markovian chain, these different corresponding systems are established as a new system, which is called Markovian switching system. In the NNs, many phenomena can be modeled by the Markovin switching system, such as the packet loss and the time delay. By modeling the Markovian switching system, some detail problems can be ignored and the performance of the system can be studied macroscopically.

The time-varying delay is inescapable in NNs. And NNs often produce the chattering and the instability (X. M. Zhang, 2017). Also, the time-varying delay, which can be called as the neutral delay, may exist in the derivative of the system state (D. Tong, 2017).

On the other hand, it is well known that the finite time synchronization can be obtained in NNs. In reality, NNs would be more economical if the synchronization can be achieved as quickly as possible. In (X. Liu, 2016), the problem of the finite-time synchronization for coupled NNs with a switching approach networks was investigated.

Nowadays, many control methods have been investigated, such as the adaptive control (D. Tong, 2016; R. Guo, 2019), the periodically intermittent control (C. Li, 2007), the discontinuous grid current control (Y. Son, 2017) and the phase current-balance control (J. Han, 2016). The exponential synchronization has been studied by the adaptive control for NNs (W. Zhou, 2012). Furthermore, the parameters of the adaptive control need to be estimated and evolved by some update laws. The exponential stabilization problem for complex systems is investigated by the periodically intermittent control (Z. W. Liu, 2017). Adaptive control is an efficacious control method to achieve synchronization for NNs, and this motivates researchers' interests.

Motivated by the discussions earlier, the adaptive controller is given to solve the finite-time synchronization problem of NTNNs with multi-delays and the Markovian switching. The criteria of the finite synchronization is obtained by constructing a Lyapunov functional and taking the LMI toolbox. Finally, a numerical simulation is provided to prove the effectiveness of the result.

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2 PROBLEM STATEMENTS

Consider the following drive system with multi-delays and the Markovian switching as

$$\begin{aligned} & d[x(t) - A(r_t)x(t - \varsigma_0(t))] \\ & = [-B(r_t)x(t) + E(r_t)h(x(t)) \\ & \quad + \sum_{\beta=1}^{n} F_\beta(r_t)h(x(t-\varsigma_\beta(t)))]d(t), \end{aligned} \tag{1}$$

Where $x(t) \in R^n$ is state vectors. And r_t is a continuous-time Markov process with a transition probability matrix $\amalg = \{\gamma_{ij}\}_{q\times q}$ which is presented as

$$Pr\{r_t + \Delta = j | \ r_t = i\} = \begin{cases} \gamma_{ij}\Delta + \delta(\Delta), & i \neq j, \\ 1 + \gamma_{ij}\Delta + \delta(\Delta), & i = j. \end{cases} \tag{2}$$

Here $\lim_{\Delta\to 0^+} \frac{\delta(\Delta)}{\Delta} = 0$, $\gamma_{ii} = -\sum_{j=1, j\neq i}^{z} \gamma_{ij}$ and $\gamma_{ij} > 0$, $i \neq j$, $i, j \in \varphi$, $\varphi = 1, 2, \cdots, \ell$ are transition rates from mode i to mode j, when the corresponding time t to time $t + \Delta$. $B(r_t) \in R^{n\times n}$ is a diagonal positive constant matrix. $A(r_t) \in R^{n\times n}$, $E(r_t) \in R^{n\times n}$ and $\sum_{\beta=1}^{n} F_\beta(r_t) \in R^{n\times n}$ are matrices associated with connection weight and the delayed connection weight. A continuous activation function $h(x(t)) \in R^n$ satisfies $h(0) = 0$. $\varsigma_{k,t}$ is the time-varying delay which satisfies $0 \leq \varsigma_{k,t} < \varsigma$, and $\dot{\varsigma}_{k,t} \leq \tau < 1$, where τ and ς are given positive constants.

Consider the controlled response system as

$$\begin{aligned} & d[y(t) - A(r_t)y(t - \varsigma_0(t))] \\ & = [-B(r_t)y(t) + E(r_t)h(y(t)) \\ & \quad + \sum_{\beta=1}^{n} F_\beta(r_t)h(y(t-\varsigma_\beta(t))) + U(t)]d(t), \end{aligned} \tag{3}$$

Where $y(t) \in R^n$ is state vectors. The error system is expressed by $e(t) = y(t) - x(t)$. and let $e_{\varsigma_\beta}(t) = e(t) - e(t - \varsigma_\beta(t))$ and $e_{\varsigma_0}(t) = e(t) - e(t - \varsigma_0(t))$. $U(t) \in R^{n\times n}$ is the controller which is designed as follows,

$$\begin{aligned} U(t) = & (\mathrm{diag}\{k_1(t), k_2(t), \ldots, k_n(t)\} - \zeta I - \mu \frac{|\int_{t-\varsigma_0(t)}^{t} e^T(s)Ge(s)ds|^{\frac{1}{2}}}{|e(t) - A^i e_{\varsigma_0}(t)|^2} \\ & - \lambda \frac{sign(e(t) - A^i e_{\varsigma_0}(t))}{|e(t) - A^i e_{\varsigma_0}(t)|})(e(t) - A^i e_{\varsigma_0}(t)), \end{aligned}$$

With the updated law $\dot{k}_j = -\varpi_j[q_i(e_j - A^i e_{\varsigma_0}(t))^2 + \frac{\psi}{\sqrt{\varpi_j}} sign(k_j)]$, where ζ, $\varpi_{j=1,2,\ldots,n} > 0$ and ψ are arbitrary constants, G is a definite matrix. Let $A(r_t) = A^i$, $B(r_t) = B^i$, $E(r_t) = E^i$, $\sum_{\beta=1}^{n} F_\beta(r_t) = \sum_{\beta=1}^{n} F_\beta^i$, the error system is

$$\begin{aligned} & d[e(t) - A^i e_{\varsigma_0}(t)] \\ & = [-B^i e(t) + E^i h(e(t)) \\ & \quad + \sum_{\beta=1}^{n} F_\beta^i)h(e_{\varsigma_\beta}(t)) + U(t)]d(t). \end{aligned} \tag{4}$$

Assumption 1: For neutral-type parameters matrices $A^i (i = 1, 2, \ldots, N)$, there are positive $\kappa_i \in (0,1)$, such that

$\rho(A^i) = \kappa_i \leq \kappa$,

Where $\kappa = \max_{i\in S} \kappa_i$, and $\rho(A^i)$ are the spectral radius of matrix A^i.

Assumption 2: There exist constants z^- and z^+ such that all activation functions of Eqs (1), (3) and (4) satisfy the following conditions,

$$z^- \leq \frac{h(m_1) - h(m_2)}{m_1 - m_2} \leq z^+,$$

Where $m_1, m_2 \in R$, $m_1 \neq m_2$.

Definition 1: Supposed that a continuous and positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\omega V^\alpha(t), \forall t \geq t_0,$$

Where $\omega > 0$ and $0 < \alpha < 1$ are two positive constants. Then, for any $t_0 \geq 0$ and the initial condition $V(t_0) \geq 0$, $V(t)$ satisfies

$$V^{1-\alpha}(t) \leq V^{1-\alpha}(t_0) - \omega(1-\alpha)(t - t_0), \forall t_0 \leq t < T,$$

$V(t) \equiv 0, \forall t \geq T$, where $T = t_0 + V^{1-\alpha}(t_0)/\omega(1-\alpha)$.

Lemma 1: Let $\Phi, \Psi \in R^n$, then

$$\Phi^T\Psi+\Psi^T\Phi\le\theta\Phi^T\Phi+\theta^{-1}\Psi^T\Psi,$$

Where $\theta>0$.

Lemma 2: The following inequality

$$(\sum_{i=1}^{n}\Pi^{\sigma_2})^{1/\sigma_2}\le(\sum_{i=1}^{n}\Pi^{\sigma_1})^{1/\sigma_1}$$

Holds for any positive constants $\Pi_1,\Pi_2,\ldots,\Pi_n$ and $0<\sigma_1<\sigma_2$.

3 MAIN RESULTS

Theorem 1: Under Assumption 1-2, NTNNs (3) can be finite-time synchronized with system (1), if there exist positive $\hbar$, q and λ, such that

$$\begin{gathered}-2b+b_0+z^2+\hbar\le 0,\\ \kappa^2+z^2-(1-\tau)\hbar\le 0,\\ nf+\varepsilon-\lambda-\zeta<0\end{gathered}\tag{5}$$

Where $\hbar=\min_{i\in\bar N}(\rho(G))$, $b=\min_{i\in\bar N}(\rho(B^i))$, $b_0=\max_{i\in\bar N}(\rho(B^i))^2$, $\varepsilon=\max_{i\in\bar N}(\rho(E^i))^2$,

$$z=\max(z^+,z^-),$$

$$f=\max_{i\in\varphi}(\rho(F^i_\beta))^2,\beta=1,2,\ldots,n.$$

Proof: Choose a Lyapunov function candidate as

$$V(t,i,e(t))=q_i\,|e(t)-A^ie_{\varsigma_0}(t)|^2+\int_{t-\varsigma_0(t)}^{t}e^T(s)Ge(s)ds+\sum_{j=1}^{n}\frac{1}{\varpi_j}k_j^2.$$

Taking the derivative of $V(t)$ with respect to t along with the trajectories of system (4), one has

$$\begin{aligned}&\dot V(t)\\&=2q_i[e(t)-A^ie_{\varsigma_0}(t)]^T[-B^ie(t)+E^ih(e(t))\\&\quad+\sum_{\beta=1}^{n}F^i_\beta h(e_{\varsigma_\beta}(t))+(\mathrm{diag}\{k_1(t),k_2(t),\ldots,k_n(t)\}\\&\quad-\zeta I-\lambda\frac{sign(e(t)-A^ie_{\varsigma_0}(t))}{|e(t)-A^ie_{\varsigma_0}(t)|}\\&\quad-\mu\frac{\int_{t-\varsigma_0(t)}^{t}e^T(s)Ge(s)ds}{|e(t)-A^ie_{\varsigma_0}(t)|^2}]+e^T(t)Ge(t)\\&\quad-(1-\tau))e^T_{\varsigma_0}(t)Ge_{\varsigma_0}(t)-2\sum_{j=1}^{n}k_{ij}\frac{\psi}{\sqrt{\varpi_{ij}}}sign(k_{ij})\end{aligned}$$

$$\begin{aligned}&\le-2q_i[e(t)-A^ie_{\varsigma_0}(t)]^TB^ie(t)\\&\quad-\lambda q_i\,|e(t)-A^ie_{\varsigma_0}(t)|\\&\quad+2q_i[e(t)-A^ie_{\varsigma_0}(t)]^TE^ih(e(t))\\&\quad-\mu q_i\,|\int_{t-\varsigma_0(t)}^{t}e^T(s)Ge(s)ds|^{\frac12}\\&\quad+q_j\sum_{j=1}^{z}\gamma_{ij}\,|e(t)-A^ie_{\varsigma_0}(t)|^2+e^T(t)Ge(t)\\&\quad+2q_i[e(t)-A^ie_{\varsigma_0}(t)]^T\sum_{\beta=1}^{n}F^i_\beta h(e_{\varsigma_\beta}(t))\\&\quad-2q_i[e(t)-A^ie_{\varsigma_0}(t)]^T[\zeta I_i[e(t)-A^ie_{\varsigma_0}(t)]\\&\quad-(1-\tau))e^T_{\varsigma_0}(t)Ge_{\varsigma_0}(t).\end{aligned}\tag{6}$$

It follows from Assumption 1 that

$$\begin{aligned}&-2q_i[e(t)-A^ie_{\varsigma_0}(t)]^TB^ie(t)\\&=-2q_ie^T(t)B^ie(t)+2q_ie^T_{\varsigma_0}[A^i]^TB^ie(t)\\&\le-2q_ib\,|e(t)|^2+q_ie^T_{\varsigma_0}(t)[A^i]^TA^ie_{\varsigma_0}(t)\\&\quad+q_ie^T(t)[B^i]^TB^ie(t)\\&\le-q_i(2b-b_0)\,|e(t)|^2+q_i\kappa^2\,|e_{\varsigma_0}(t)|^2\\&\le-q_i(2b-b_0)\,|e(t)|^2+q_i\kappa^2\,|e_{\varsigma}(t)|^2,\end{aligned}\tag{7}$$

Where $|e_\varsigma(t)|=\max_{\beta\in(0,1,..,n)}|e^T_{\varsigma_\beta}|$.

Using Assumption 2 and Lemma 1, one can obtain

$$\begin{aligned}&2q_i[e(t)-A^ie_{\varsigma_0}(t)]^TE^ih(e(t))\\&\le q_i[e(t)-A^ie_{\varsigma_0}(t)]^T[E^i]^TE^i[e(t)-A^ie_{\varsigma_0}(t)]\\&\quad+q_ih^T(e(t))h(e(t))\\&\le q_i\varepsilon\,|e(t)-A^ie_{\varsigma}(t)|^2+qz^2\,|e(t)|^2.\end{aligned}\tag{8}$$

From Assumption 2, one has

$$2q_i[e(t)-A^i e_{\varsigma_0}(t)]^T \sum_{\beta=1}^{n} F_\beta^i h(e_{\varsigma_\beta}(t)) \le nq_i d\,|e(t)-A^i e_{\varsigma}(t)|^2 + qz^2\,|e_{\varsigma}(t)|^2, \tag{9}$$

$$e^T(t)Ge(t)-(1-\tau)e_{\varsigma_0}^T Ge_{\varsigma_0} \le \hbar\,|e(t)|^2 + (1-\tau)\hbar\,|e_{\varsigma}(t)|^2 . \tag{10}$$

Then, one can obtain that

$$-2\sum_{j=1}^{n} k_{ij}\frac{\psi}{\sqrt{\varpi_{ij}}} sign(k_{ij}) = -2\sum_{j=1}^{n}\frac{\psi}{\sqrt{\varpi_{ij}}}|k_{ij}| \le -2\psi(\sum_{j=1}^{n}\frac{1}{\varpi_{ij}}k_j^2)^{1/2}. \tag{11}$$

It can be deduced from inequalities (6)-(11) and Theorem 1 that

$$\dot{V}(t) \le -2\psi(\sum_{j=1}^{n}\frac{1}{\varpi_{ij}}k_j^2)^{1/2} - \lambda q_i\,|e(t)-A^i e_{\varsigma_0}| - \mu q_i\,|\int_{t-\varsigma_0(t)}^{t} e^T(s)Ge(s)ds|^{\frac{1}{2}} . \tag{12}$$

By Lemma 2, we have

$$\begin{aligned}&\dot{V}(t)\\ &\le -\partial(\sum_{j=1}^{n}\frac{1}{\varpi_{ij}}k_j^2)^{1/2} - \partial\sqrt{q_i}\,|e(t)-A^i e_{\varsigma_0}|\\ &\quad -\partial\,|\int_{t-\varsigma_0(t)}^{t} e^T(s)Ge(s)ds|^{\frac{1}{2}}\\ &\le -\partial(q_i\,|e(t)-A^i e_{\varsigma_0}|^2 + \sum_{j=1}^{n}\frac{1}{\varpi_{ij}}k_j^2\\ &\quad +\int_{t-\varsigma_0(t)}^{t} e^T(s)Ge(s)ds)^{\frac{1}{2}},\end{aligned}$$

Where $\partial = \min_{i\in\bar{N}}\{2\psi,\ \lambda\sqrt{q_i},\ \mu q_i\}$, one can easily get $\dot{V}(t) \le -\partial[V(t)]^{1/2}$. According to Lemma 3, we know that $E\{V(T)\}=0$ as $E\{T\}=t_0+\sqrt{V(t_0)}$. Since $V(t)\ge 0$, it can derived that $E\{\|e(t)\|\}=0$ when $t \ge T$. The proof is completed.

Remark 1: An appropriate ∂ can be easily find by the LMI of Theorem 1. For instance, let $q=1.75,\ \psi=0.5,\ \mu=0.2$, then $\partial=0.35$. Then, the conditions in Theorem 1 can also be easily checked.

4 NUMERICAL EXAMPLES

One example is presented to indicate the effectiveness of our results. Consider a time-delayed NTNNs (1) and its response system (3) with following network parameters,

$$A^1=\begin{bmatrix}0.3 & 0\\ 0 & 0.3\end{bmatrix},\ A^2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},$$
$$B^1=\begin{bmatrix}2.3 & 0\\ 0 & 2.7\end{bmatrix},\ B^2=\begin{bmatrix}3.3 & 0\\ 0 & 2.8\end{bmatrix},$$
$$E^1=\begin{bmatrix}-0.3 & -1.6\\ 0.1 & 1.6\end{bmatrix},\ E^2=\begin{bmatrix}0.5 & 1\\ 0 & 2.8\end{bmatrix},$$

$$F^{11}=\begin{bmatrix}-0.3 & -1,5\\ 0 & 0.7\end{bmatrix},\ F^{12}=\begin{bmatrix}-1.8 & 0\\ 1.7 & 2.3\end{bmatrix},$$
$$F^{21}=\begin{bmatrix}-0.3 & -1.6\\ 0.1 & 1.6\end{bmatrix},\ F^{22}=\begin{bmatrix}0.5 & 1\\ 0 & 2.8\end{bmatrix},$$

$$\amalg=\begin{bmatrix}-3 & 3\\ 2 & -2\end{bmatrix},\ h(e(t))=0.3\tanh(e(t)),$$
$$\tau=0.12,\ z=0.2,\ \zeta=0.43.$$

Initial values of system (4) are set to be $e(0)=[0.2,3.7]^T,\ k(0)=[0.11,0.18]^T$. The dynamic curve e of NTNNs without the controller is not synchronization. In the example, the following results can be obtained by solving the LMI (5) based on the Matlab toolbox, and it obtains that $\hbar=0.57,\ q=2.17,\ \lambda=1.52$.

The dynamic curve of the NTNNs (4) with the controller is illustrated by FIG. 1. And the dynamic curve of the adaptive gain $k(t)$ is given in FIG. 2. Thus, one can see that the zero solution of NTNNs (4) can be got via the adaptive control.

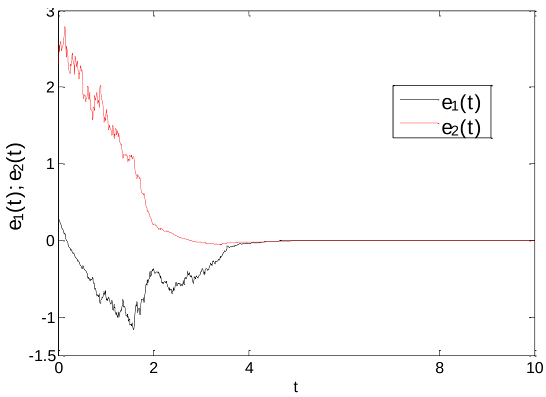

Figure 1. Dynamic curve of NTNNs $e(t)$ with control.

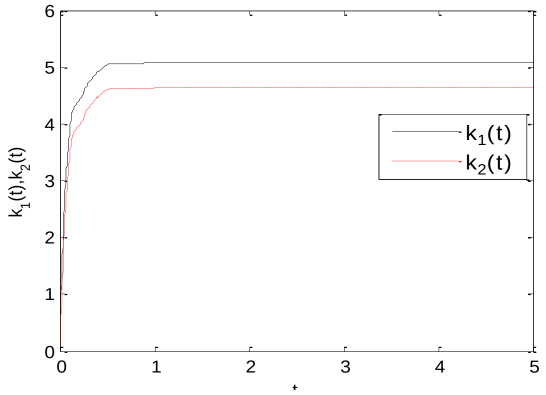

Figure 2 Dynamic curve of adaptive control gain $k(t)$.

5 CONCLUSION

The finite-time synchronization problem for NTNNs is considered by using the adaptive control with mutil-delays and the Markovian switching. Sufficient synchronization conditions for the coexistence of the neutral item, time-varying delays, the Markovian switching in NTNNs with the adaptive control are given to solve the difficulty of the mathematical complexity.

REFERENCES

C. Li, G. Feng, and X. Liao, 2007. "Stabilization of nonlinear systems via periodically intermittent control," *IEEE Trans.Circuits Syst*. Express Briefs, vol. 54, no. 11, pp. 1019–1023.

D. Tong and Q. Chen, 2017. "Delay and its time-derivative dependent model reduction for neutral-type control system," *Circuits Syst. Signal Process*, vol. 36, no. 6, pp. 2542–2557.

D. Tong, L. Zhang, W. Zhou, J. Zhou, and Y. Xu, 2016. "Asymptotical synchronization for delayed stochastic neural networks with uncertainty via adaptive control," *Int. J. Control Autom. Syst*., vol. 14, no. 3, pp. 706–712.

J. Han and J.-H. Song, 2016. "Phase current-balance control using DC-link current sensor for multiphase converters with discontinuous current mode considered," *IEEE Trans. Ind.Electron*., vol. 63, no. 7, pp. 4020–4030.

R. Guo, Z. Zhang, C. Lin, Y. Chu, and Y. Li, 2019. "Finite time state estimation of complex-valued BAM neutral-type neural networks with time-varying delays," *Int. J. Control Autom. Syst.*, vol. 17, no. 3, pp. 801–809.

W. Zhou, D. Tong, Y. Gao, C. Ji, and H. Su, 2012. "Mode and delay-dependent adaptive exponential synchronization in pth moment for stochastic delayed neural networks with Markovian switching," *IEEE Trans. Neural Netw. Learn. Syst*., vol. 23, no. 4, pp. 662–668.

X. Liu, H. Su, and M. Z. Chen, 2016. "A switching approach to designing finite-time synchronization controllers of coupled neural networks," *IEEE Trans. Neural Netw*. Learn. Syst., vol. 27, no. 2, pp. 471–482.

X. M. Zhang, Q. L. Han, Z. Wang, and B.-L. Zhang, 2017. "Neuronal state estimation for neural networks with two additive time-varying delay components," *IEEE Trans. Cybern*., vol. 47, no. 10, pp. 3184–3194.

Y. Cao, S. Wen, M. Z. Chen, T. Huang, and Z. Zeng, 2016. "New results on anti-synchronization of switched neural networks with time-varying delays and lag signals," *Neural Netw*., vol. 81, pp. 52–58.

Y. Son and J.-I. Ha, 2017. "Discontinuous grid current control of motor drive system with single-phase diode rectifier and small DC-link capacitor," *IEEE Trans. Power Electron*., vol. 32, no. 2, pp. 1324–1334.

Z. W. Liu, X. Yu, Z.-H. Guan, B. Hu, and C. Li, 2017. "Pulse modulated intermittent control in consensus of multiagent systems," *IEEE Trans. Syst. Man Cybern. Syst*., vol. 47, no. 5, pp. 783–793.