Event-triggered Observers-based Output Feedback H∞ Control for Linear Time-invariant Systems with Quantization

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Abstract: This paper is concerned with the H∞ output feedback control for linear time-invariant (LTI) systems with the quantization to realize the operation optimization of a microgrid (MG). Meanwhile, a novel event-triggered mechanism is introduced to reduce the number of control signals. Furthermore, a model of observer which is based on the event-triggered mechanism is proposed to verify the synchronization of LTI systems. Based on this model, a criteria which is derived from linear matrix inequalities (LMIs) is provided such that the system performance can be ensured. Finally, a numerical example is presented to illustrate the effectiveness of the results.

1 INTRODUCTION

With the increase in power demand and the shortage of fossil fuels, the renewable energy (RES) and batteries are combined to achieve power generation. The combination which contains RES and batteries, is called as the microgrid (MG). The MG can realize the flexible application of the distributed power and solve the problem of massive loads. However, the performance of the MG can be disturbed by many factors. Decisive factors for the stable operation of the MG are power outputs, loads and prices. In addition, many factors are uncertain, such as the weather, the peak electricity consumption and random events. These uncertainties can affect the performance of the MG. Thus, the optimal scheduling can't be obtained to achieve the economic operation. According to the literature (A. D. Dominguez-Garcia, 2009), the model of the MG can be transformed into LTI systems (C. A. Desoer, 1968) which are used to study the problem with uncertainties. Thus, LTI systems are used in this paper to analyze the performance of the MG.

Although LTI systems have been introduced to measure the operation state of the MG, the data, which is inside the system, is difficult to be accurately measured due to the complexity in the MG. Thus, the observer is proposed to solve the difficulty of measuring the internal data. Nowadays, observers-based LTI systems have been studied extensively in different fields, such as the digital image processing (J. Alonsomontesinos, 2015) and the electric automatization (G. Bertotti, 1991). Due to the greatly potential effect in analyzing the internal data, some preliminary results have been reported. For instance, the problem of the observers-based circuit is studied by linear matrix inequalities (LMIs) in literature (A. D. Dominguez-Garcia, 2009). After that, many literatures, which study the state estimation by LMIs, have been proposed, such as the state estimation with mixed interval time-varying delays (F. Perez-Gonzalez, 2008), (Z. M. Zhang, 2019), the fault detection and isolate for observers-based linear systems (S.Hajshirmohamadi, 2016) and so on. Consequently, the observer is of great value to investigate the internal working principle.

In actual circuit measurements, the real-time scheduling can not be achieved due to uncertainties. In this paper, an event-triggered mechanism is provided to filter uncertainties in order to obtain appropriate current signals. The event-triggered mechanism is executed when the predefined event occurs. Current signals, which are not satisfied the predefined event, will be filtered. Thus, an effective method is provided to disperse the execution of tasks, namely the event-triggered mechanism only works

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when specific events occur. Despite the event-triggered mechanism has an irreplaceable role in allocating resource, its internal state in the MG is difficult to be measured. Thus, in this paper, the internal state of the MG with the event-triggered control can be measured by the measurement error.

The structure of this paper is as follows. First, the LTI system is presented to describe the MG. Second, the transmission signal is quantized and the observed model which is based on the event-triggered mechanism and the quantification is designed. At last, an example of simulation is given to verify the effectiveness of the proposed method.

The main contributions are emphasized in the following three parts.

(1) The variable parameter in the event-triggered mechanism is changed to compare the triggered probability of the mechanism. Meanwhile, the appropriate parameter is selected to reduce the bandwidth.

(2) A new analytical method which contains the event-triggered mechanism and the LTI system, is presented to analyze the stability of the MG.

(3) The influence of external interference on the performance is reduced by using H∞ index.

2 MODEL DESCRIPTION AND PROBLEM STATEMENT

2.1 Model Description

According to the literature (A. D. Dominguez-Garcia, 2009), the state-space representation of the MG can be indicated as

$$\frac{di(t)}{dt} = -\frac{Y}{L} i(t) + \frac{1}{L} v(t), \tag{1}$$

Where $i(t)$, $v(t)$ represent the current and the voltage respectively, Y indicates the resistor and L is the inductance.

Considering the disturbance and the output in the MG, let $x(t) = i(t) \in R^n$ is the state vector, $u(t) = v(t) \in R^m$ is control input vector and $A = -\frac{Y}{L}$, $B = \frac{1}{L}$ are known constant matrices with appropriate dimensions. Then, the following equation can be obtained as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \\ y(t) = Cx(t), \end{cases} \tag{2}$$

Where $y(t) \in R^q$ is the output, $\omega(t) \in L_2[0,\infty)$ is the disturbance, C and B_ω are known constant matrices with appropriate dimensions.

2.2 Event-Triggered Mechanism

The event-triggered mechanism is proposed as

$$\begin{aligned} t_{k+1}h = t_k h + \min_i \{ & ih \mid (y(t_k h) - y(t_k h + ih))^T \\ & \times H(y(t_k h) - y(t_k h + ih)) \\ & \le \sigma y^T(t_k h) H y(t_k h) \}, \end{aligned} \tag{3}$$

Where $i \in Z^+$, $h \in R^+$, $t_k h$ is the sampling time with $k \in Z^+$, $y(t_k h)$ is the triggered time, H is a positive definite matrix and $\sigma \in [0,1)$ is a given scalar.

In the whole model frame, the sensor receives the state vector from the LTI system as $x(t)$ and exports the output vector $y(t)$. Then, $y(t)$ is sampled as $y(kh)$ by periodical sampling. In every sampling period, the event-triggered mechanism judges that the output $y(kh)$ satisfies the mechanism (3) or not. If the mechanism (3) holds, the date $y(t_k h)$ is immediately transformed into the controller with the time delay τ_k, $\tau_k \in (0, \bar{\tau}]$ and $\bar{\tau} \ge 0$.

However, the introduction of τ_k may lead to that the signals have different orders of arrival to the controller. Inspired by [12]-[13], the time intervals are elaborated in the following content.

$$\begin{aligned} [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}) = & [t_k h + \tau_k, t_k h + h + \bar{\tau}) \\ & \cup \{\cup_{i=1}^{d_M - 1} [t_k h + ih + \bar{\tau}, \\ & \quad t_k h + ih + h + \bar{\tau})\} \\ & \cup [t_k h + d_M h + \bar{\tau}, t_{k+1}h + \tau_{k+1}), \end{aligned} \tag{4}$$

Where $d_M \in Z^+$ and $d_M = i - 1$.

During the range $[t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, $\tau(t)$ is defined as

$$\tau(t)=\begin{cases} t-t_k h, & t\in[t_k h+\tau_k, t_k h+h+\overline{\tau}) \\ t-t_k h-ih, & t\in\{\cup_{i=1}^{d_M-1}[t_k h+ih+\overline{\tau}, \\ & t_k h+ih+h+\overline{\tau})\} \\ t-t_k h-d_M h. & t\in[t_k h+d_M h+\overline{\tau}, t_{k+1}h+\tau_{k+1}) \end{cases} \quad (5)$$

Here we define that $\overline{\tau}+h=\tau_m$, where τ_m is the upper delay bound of $\tau(t)$. Meanwhile, the error state $e_k(t)$ between $y(t_k h)$ and $y(t_k h+ih)$ can be expressed as

$$e_k(t)=\begin{cases} 0, & t\in[t_k h+\tau_k, t_k h+h+\overline{\tau}) \\ y(t_k h)-y(t_k h+ih), & t\in\{\cup_{i=1}^{d_M-1}[t_k h+ih+\overline{\tau}, \\ & t_k h+ih+h+\overline{\tau})\} \\ y(t_k h)-y(t_k h+d_M h). & t\in[t_k h+d_M h+\overline{\tau}, \\ & t_{k+1}h+\tau_{k+1}) \end{cases} \quad (6)$$

According to (4)-(6), the event-triggered mechanism (3) can be indicated as

$$e_k^T(t)He_k(t)\le\sigma y^T(t-\tau(t))Hy(t-\tau(t)), \quad (7)$$

Where $t\in\left[t_k h+\tau_k, t_{k+1}h+\tau_{k+1}\right)$.

2.3 Event-Triggered Quantized Control Problem

The controller of event-triggered H∞ with quantization in this paper is designed as

$$u(t)=Ky(t) \quad (8)$$

Where K is a controller gain matrix with appropriate dimensions.

The LTI system (2) is indicated as

$$\begin{cases} \dot{x}(t)=Ax(t)+BKy(t-\tau(t))+BKe_k(t)+B_\omega\omega(t) \\ y(t)=Cx(t), \end{cases} \quad (9)$$

In this paper, the quantizer $q_i(y_i)=[q_1(y_1), q_2(y_2), .., q_n(y_n)]^T$ assumed to be symmetric is introduced in this paper, where $q_i(-y_i)=-q_i(y_i)$. For each $q_i(y_i)$, the set of quantized levels S has the following form $S=\{\pm v_i, v_i=\rho^i v_0, v_o>0, i\in Z\}\cup\{0\}$, where $0<\rho<1$.

The quantizer $q_i(y_i)$ is defined as

$$q_i(y_i)=\begin{cases} u_i, & \text{if } \frac{1}{1+\delta_{qi}}v_i<y_i<\frac{1}{1-\delta_{qi}}v_i, y_i>0, \\ 0, & \text{if } y_i=0, \\ -q_i(-y_i), & \text{if } y_i<0, \end{cases} \quad (10)$$

Where $\delta_{qi}=\frac{1-\rho}{1+\rho}$ $(0<\rho<1)$ and the ρ is the quantization density.

Based on the literature [8], a sector bound condition is proposed by

$$q_i(y)=(I+\Delta)y, \quad (11)$$

Where $\Delta=\mathrm{diag}\{\Delta_1, \Delta_2, \ldots, \Delta_\mathrm{n}\}$ and $\Delta\in[-\delta_{qi}, \delta_{qi}]$.

Thus, the quantized triggered output $y(t_k h)$ can be indicated as

$$\tilde{y}(t_k h)=(I+\Delta)y(t_k h). \quad (12)$$

According to the literature [9], the observer can be constructed as

$$\begin{cases} \dot{\hat{x}}(t)=A\hat{x}(t)+M(\hat{y}(t)-\tilde{y}(t)), \\ \hat{y}(t)=C\hat{x}(t), \end{cases} \quad (13)$$

Where $\hat{x}(t)$ is estimated state vector, $\hat{y}(t)$ is estimated output vector, and M is the observer gain matrix.

Combining (9), (12) and (13), the error system can be confirmed as

$$\begin{aligned} \dot{e}(t)=&(A+MC)e(t)+(M\Delta+BK)y(t-\tau(t)) \\ &+(M\Delta+BK)e_k(t)+B_\omega\omega(t), \end{aligned} \quad (14)$$

Where $e(t)=x(t)-\hat{x}(t)$.

Definition 1. The system (14) with an H∞ disturbance attenuation level $\gamma>0$ needs to satisfy the following two conditions to be asymptotic stability. The system (14) is asymptotically stable with $\omega(t)=0$. And under the zero initial condition, (14) with any nonzero $\omega(t)\in L_2[0,\infty)$ should satisfy $\|y(t)_2\|\le\gamma^2\|\omega(t)_2\|$.

Lemma 1. (A. Seuret, 2013) (Wirtinger inequality) for the given matrix G, the inequality can be obtained when $x\in[c, d]$

$$\int_c^d \dot{x}(q)^T G\dot{x}(q)dq \geq \frac{1}{d-c}(x(d)-x(c))^T G(x(d)-x(c)) + \frac{3}{d-c}\Omega^T G\Omega,$$

Where $\Omega = x(c)+x(d)-\frac{2}{d-c}\int_c^d x(q)dq.$

3 MAIN RESULT

In this section, we consider the observers-based quantized H∞ control of LTI systems under the event-triggered mechanism. Based on LMIs and the Lyapunov function, sufficient conditions are given for the error system (14) to be asymptotic stability with H∞ performance level γ.

3.1 Theorem

For given parameters $\sigma \in (0,1]$, the error system (14) is asymptotic stability if there exist matrices $P>0, Q>0, R>0$ and $H>0$ satisfying the following inequality:

$$\Gamma = \begin{bmatrix} \Theta_{11} & -\frac{2}{\tau_m}R & 0 & \frac{6}{\tau_m}R & 0 & \Theta_{16} & \Theta_{17} & PB_\omega \\ * & -\frac{8}{\tau_m}R & -\frac{2}{\tau_m}R & \frac{6}{\tau_m}R & \frac{6}{\tau_m}R & 0 & 0 & 0 \\ * & * & -Q-\frac{4}{\tau_m}R & 0 & \frac{6}{\tau_m}R & 0 & 0 & 0 \\ * & * & * & -\frac{12}{\tau_m}R & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{12}{\tau_m}R & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & \Theta_{67} & \Theta_{68} \\ * & * & * & * & * & * & \Theta_{77} & \Theta_{78} \\ * & * & * & * & * & * & * & \Theta_{88} \end{bmatrix} < 0 \quad (15)$$

Where

$$\Theta_{11} = P(A+MC)+(A+MC)^T P+Q + \tau_m (A+MC)^T R(A+MC) - \frac{4}{\tau_m}R,$$

$$\Theta_{16} = P(M\Delta+BK)+\tau_m(A+MC)^T RM\Delta,$$

$$\Theta_{17} = P(M\Delta+BK)+\tau_m(A+MC)^T RM\Delta,$$

$$\Theta_{66} = \sigma H+(M\Delta+BK)^T R(M\Delta+BK),$$

$$\Theta_{67} = (M\Delta+BK)^T R(M\Delta+BK),$$

$$\Theta_{68} = \tau_m (M+BK)^T RB_\omega,$$

$$\Theta_{77} = (M\Delta+BK)^T R(M\Delta+BK)-H,$$

$$\Theta_{78} = (M\Delta+BK)^T R(M\Delta+BK)-H,$$

$$\Theta_{88} = -\gamma^2 I+\tau_m B_\omega^T RB_\omega,$$

Then, the error system (14) is asymptotic stability with the H∞ inhibition of index γ.

3.2 Proof

Construct the following Lyapunov function as

$$V(t) = e(t)^T Pe(t) + \int_{t-\tau_m}^t e^T(s)Qe(s)ds + \int_{t-\tau_m}^t \int_s^t \dot{e}^T(v)R\dot{e}(v)dvds. \quad (16)$$

By using Lemma 1, one has

$$\begin{aligned} \dot{V}(t) \leq\ & 2e^T P\dot{e}(t)+e^T(t)Qe(t)-e^T(t-\tau_m) \\ & \times Qe(t-\tau_m)+\tau_m \dot{e}^T(t)R\dot{e}(t) \\ & -\frac{1}{\tau(t)}(e(t)-e(t-\tau(t))^T R(e(t)-e(t-\tau(t)) \\ & -\frac{3}{\tau(t)}\Omega_1^T R\Omega_1 - \frac{1}{\tau(t)}(e(t-\tau(t))-e(t-\tau_m))^T \\ & \times R(e(t-\tau(t))-e(t-\tau_m)) - \frac{3}{\tau(t)}\Omega_2^T R\Omega_2 \\ & + e_k^T(t)He_k(t) - \sigma y^T(t-\tau(t))Hy(t-\tau(t)), \end{aligned} \quad (17)$$

Then, under the zero initial condition, we have

$$\begin{aligned} & \int_0^T [y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)]dt \\ & = \sum_{v=0}^{k-1}\{\int_{t_v h+\tau_v}^{t_{v+1}h+\tau_{v+1}}[y^T(t)y(t)+\dot{V}(t) \\ & \quad -\gamma^2\omega^T(t)\omega(t)]dt\} - V(t) \\ & \leq \sum_{v=0}^{k-1}\{\int_{t_v h+\tau_v}^{t_{v+1}h+\tau_{v+1}}[y^T(t)y(t)+\dot{V}(t) \\ & \quad -\gamma^2\omega^T(t)\omega(t)]dt\}, \end{aligned} \quad (18)$$

Where $v \in Z^+$.

By using the Lyapunov-Krasovskii functional candidate in (16), it follows

$$\int_0^T [y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)]dt \leq \eta^T(t)\Gamma\eta(t) \quad (19)$$

Where

$$\beta = [e^T(t) \quad e^T(t-\tau(t)) \quad e^T(t-\tau_m) \quad \frac{1}{\tau(t)}\int_{t-\tau(t)}^t e^T(s)$$

$$\frac{1}{\tau_m-\tau(t)}\int_{t-\tau_m}^{t-\tau(t)} e^T(s)ds \quad e_k^T(t) \quad e^T(t) \quad \omega(t)]^T.$$

Furthermore, let $T\to\infty$, the following result can be obtained

$$\dot{V}(t)\le\gamma^2\omega^T(t)\omega(t)-y^T(t)y(t)+\eta^T(t)\Gamma\eta(t). \qquad (20)$$

The Theorem has been proved by the statement (16)-(20). The error system (14) is proved to be asymptotic stability under the H∞ control with the inhibition of level γ. The proof is completed.

4 NUMERICAL EXAMPLE

In this section, a numerical example is presented to prove the effectiveness of the results. Considering the following parameters

$$A=\begin{bmatrix}0.2 & -0.17\\ -0.17 & 0.2\end{bmatrix}, B=\begin{bmatrix}2.5 & -2\\ -0.6 & 1.7\end{bmatrix},$$

$$C=\begin{bmatrix}2.2 & 1.1\\ -2 & 1.1\end{bmatrix}, K=\begin{bmatrix}10.155 & -5.217\\ 6.175 & 40.568\end{bmatrix}.$$

Assuming that $B_\omega=2, M=2I, h=1$ and the initial $\sigma=0.2$, $u(t)=\begin{bmatrix}\sin(2t)-0.2\\ \sin(t)\end{bmatrix}$, the H∞ performance index $\gamma=2$.

Then, the following data can be obtained by LMIs as

$$P=\begin{bmatrix}-0.556 & 0.4537\\ 0.4537 & -0.2961\end{bmatrix}, Q=\begin{bmatrix}-6.837 & 2.7\\ 2.7 & -1.29\end{bmatrix},$$

$$R=\begin{bmatrix}0.29 & 0.046\\ 0.046 & 0.2736\end{bmatrix}, H=\begin{bmatrix}160.6 & -375.5\\ 300.14 & 307.54\end{bmatrix}.$$

What's more, the triggered intervals of the system (14) based on the event-triggered mechanism (3) is described in Figure 1.

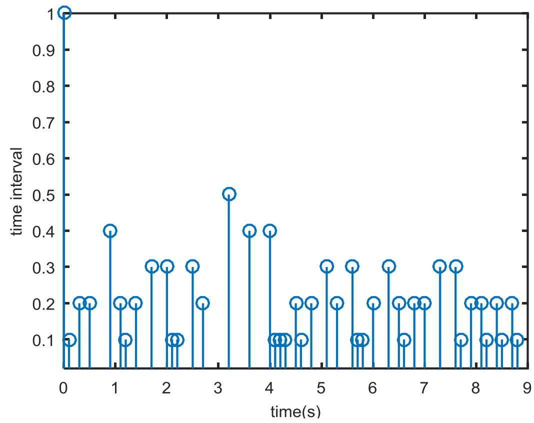

Figure 1. The relationship between triggered intervals and time.

By the different value of σ, the upper bound of the time delay τ_m changed. TABLE 1 is given to reveal the relationship between the two factors and show the minimum of H∞ performance index.

TABLE 1 shows that the increase of σ can decrease the upper bound of the time delay, so a suitable numerical value can be chosen within the appropriate limits to minimize the delay. This utility reduced network bandwidth and decrease the pressure on network transmission.

Table 1. The relationship between the parameters.

σ	0.1	0.2	0.3	0.4
τ_m	0.1122	0.1001	0.0962	0.0324
γ_{min}	0.5305	0.7311	3.0322	3.7421

Furthermore, the relationship between state vector $x(t)$ and release instants t can be obtained in the Figure 2.

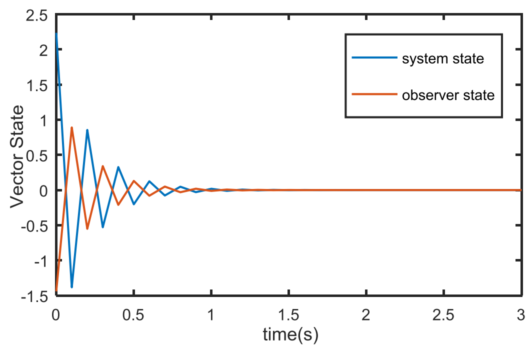

Figure 2. State responses of the system with the event triggered mechanism.

5 CONCLUSIONS

The problem of observers-based event-triggered H∞ control has been addressed in this paper. By considering the event-triggered mechanism and the quantization, a new observers-based system is proposed in this paper. Based on this system, we derived H∞ performance criterion that guarantees the LTI system is stable with H∞ performance index γ. Periodical sampling and output feedback controller is also used to complete the design of the system. The effectiveness of the proposed method has been demonstrated by a numerical example.

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