Modeling Concept Drift in the Context of Discrete Bayesian Networks

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Abstract: Concept drift is a significant challenge that greatly influences the accuracy and reliability of machine learning models. There is, therefore, a need to detect concept drift in order to ensure the validity of learned models. In this research, we study the issue of concept drift in the context of discrete Bayesian networks. We propose a probabilistic graphical model framework to explicitly detect the presence of concept drift using latent variables. We employ latent variables to model real concept drift and uncertainty drift over time. For modeling real concept drift, we propose to monitor the mean of the distribution of the latent variable over time. For modeling uncertainty drift, we suggest to monitor the change in beliefs of the latent variable over time, i.e., we monitor the maximum value that the probability density function of the distribution takes over time. We implement our proposed framework and present our empirical results using two of the most commonly used Bayesian networks in Bayesian experiments, namely the Burglary-Earthquake Network and the Chest Clinic network.

1 INTRODUCTION

In recent years, machine learning models are increasingly used in many real-world applications. A common challenge for machine learning systems is to model environments wherein data evolves over time, a phenomenon that is commonly known as concept drift (Gama et al., 2014).

Detecting concept drift is crucial and active research in machine learning systems. Concept drift influences the accuracy and reliability of machine learning models. Current approaches to detect concept drift use latent variables (Borchani et al., 2015; Cabañas et al., 2018). Latent variables (a.k.a. unobserved variables) are variables that are not immediately observed but instead they are inferred from different variables that are observed and directly measured. An advantage of concept drift detection techniques that are based on using latent variables is that they tend to estimate the desired effects on the machine learning models more reliably than traditional detection techniques. A large number of observable variables can be aggregated in a model to represent an underlying concept, making it easier to understand the data and detect concept drift over time. However, current efforts for detecting concept drift using latent variables either limited to contentious Bayesian networks (Borchani et al., 2015) or not directly applicable to discrete Bayesian networks (Cabañas et al., 2018). In addition, previous efforts for detecting concept drift using latent variables (Borchani et al., 2015; Cabañas et al., 2018) are limited to naive Bayes classifiers and therefore cannot be used to model concept drift that involves concepts span over multiple variables.

In this paper, we propose a technique for detecting concept drift in the context of discrete Bayesian networks using latent variables. Our technique extends Borchani et al. (Borchani et al., 2015) approach such that it is directly applicable to discrete Bayesian networks. Borchani et al. represent concept drift using unobserved variables in continuous domains, namely in conditional linear Gaussian models. In addition to modeling posterior probability distribution drift, we propose a new method for modeling uncertainty drift.

The main contributions of this paper are as follows. We propose a framework for detecting the presence of concept drift in the context of discrete Bayesian networks using latent variables. Unlike previously proposed approaches (Borchani et al., 2015; Cabañas et al., 2018) which are limited to naive Bayes classifiers, our framework is applicable to general Bayesian network models. We use latent variables to model two types of drifts over time: (1) *Posterior Distribution Drift*, and (2) *Uncertainty Drift*. We develop a modeling technique using latent variables that is able to detect posterior distribution drift. We provide a new method for modeling and detecting con-

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cept drift via modeling uncertainty over time, i.e., the amount of belief that changes over time.

We have implemented our approach and presented our empirical results. Our results indicate that our modeling framework not only is sensitive to changes in both real concept drift and uncertainty drift but also can quickly detect the presence of drifts.

The rest of the paper is organized as follows. In section 2, we present the problem setting. In section 3, we present our framework for detecting concept drift using latent variables in discrete Bayesian networks. In section 4, we extend our modeling framework into higher dimensions. In section 5 we present our empirical results. In section 6, we give an overview of related work. In section 7, we conclude and briefly discuss ongoing work.

2 PROBLEM SETTING

We focus on modeling concept drift in the context of discrete Bayesian networks. In a nonstationary environment, we assume that at each time point t (for $t = 1, 2, \ldots$) data arrives in a batch (a.k.a. *a window*), which is a collection of cases. Let $Batch\ [A_1, \ldots, A_m]$ be the schema of the incoming batch with attributes $A_1, \ldots, A_m$. We assume without loss of generality that the incoming batches have equal sizes, i.e., each batch contains n cases. Let $Batch\ t = \{case_1^t, \ldots, case_n^t\}$ be a collection of cases (a.k.a. *observations* or *findings*) that arrives at time t.

$$\underbrace{case_1^1, case_2^1, \ldots, case_n^1}_{\text{Batch 1, t = 1}}, \underbrace{case_1^2, case_2^2, \ldots, case_n^2}_{\text{Batch 2, t = 2}}, \ldots$$

Each finding, denoted as *case*, is over attributes $A_1, \ldots, A_m$ and of the form $case = \ < A_1 = v_1, \ldots, A_m = v_m >$ (or simply can be written as $case = \ < v_1, \ldots, v_m >$), such that v_k is the value of attribute A_k $(1 \leq k \leq m)$. When a new batch $Batch\ t+1$ arrives at time point $t+1$, the Bayesian network model can simply be updated using Bayes' theorem.

To detect the presence of concept drift between two time points $t = i$ and $t = i+1$, we consider two types of drifts as follows: (1) *Posterior Distribution Drift*, and (2) *Uncertainty Drift*.

(Posterior Distribution Drift; a.k.a. Real Concept Drift): *Posterior distribution drift* occurs when the conditional probability changes on the target variable whereas the input variables remain unchanged (Gama et al., 2014). That is, the value of the posterior probability at time $t = i$, $P_{t_i}(y \mid A)$, is not equal to the value of the posterior probability at time $t = i+1$, $P_{t_{i+1}}(y \mid A)$.

In Bayesian statistics, Bayes' theorem can be written in a useful form for Bayesian network update and inference as follows: The posterior probability is proportional to the product of the prior probability and the likelihood (Posterior probability $\propto$ Prior probability $\times$ Likelihood (Lynch, 2007)). Having a prior that is conjugate for the likelihood function will make it mathematically convenient to calculate the posterior distribution since the posterior distribution will be from the same family of distribution as the prior (Raiffa and Schlaifer, 1961). For instance, multiplying a beta-distributed prior, $Beta(\alpha, \beta)$, with a binomial-distributed likelihood function, $Binomial(n, \theta)$, yields a beta-distributed posterior distribution, $Beta(q+\alpha, n-q+\beta)$, where n is the total number of cases, and q is the count of successes (Alsuwat et al., 2018).

In what follows, we consider detecting the presence of posterior distribution drift in the context of discrete Bayesian networks with respect to a random variable $\boldsymbol{X}$ that is beta-distributed, which we denote as $\boldsymbol{X} \sim Beta(\alpha, \beta)$. We capture the existence of posterior distribution drift by monitoring the mean of the beta distribution at every time point $t = i$, denoted as μ_i, i.e., the expected value of $\boldsymbol{X}$ at every time point $t = i$, $\boldsymbol{E(X)}$, as follows:

$$\mu_i = \boldsymbol{E(X)} = \frac{q_i + \alpha}{\alpha + n_i + \beta} \tag{1}$$

where n_i and q_i are the total number of cases and the count of successes at time $t = i$, respectively, and hyperparameters α, β are greater than or equal to 1.

(Uncertainty Drift): Measuring the amount of uncertainty in input data is defined as entropy (Shannon, 2001). *Uncertainty drift* is a variable that reflects the change in beliefs over time. That is, for a random variable $\boldsymbol{X}$, the maximum value that a probability density function $f_i(x; \alpha, \beta)$ takes at time $t = i$ is not equal to the maximum value that a probability density function $f_{i+1}(x; \alpha, \beta)$ takes at time $t = i+1$. This kind of drift is mainly caused by the change in the total number of observed cases. It is important to point out that modeling uncertainty drift in the context of Bayesian networks is powerful as it is a sensitive diagnostic for detecting real concept drift.

Herein, we consider detecting the presence of uncertainty drift in the context of discrete Bayesian networks with respect to a random variable $\boldsymbol{X}$ is beta-distributed, $\boldsymbol{X} \sim Beta(\alpha, \beta)$. We capture the existence of uncertainty drift by monitoring the maximum value that the probability density function of the beta distribution takes at every time point $t = i$, which we denote

Table 1: Notations.

Notation	Description
$\boldsymbol{Batch}\ [A_1,...,A_m]$	The schema of incoming batch with attributes $A_1,...,A_m$
$\boldsymbol{Batch\ i}$	A collection of cases that arrives at time i
$\boldsymbol{case^i_j}$	The j^{th} observation of $Batch\ i$
$\boldsymbol{\mu_i}$	The mean of the posterior probability at time i
$\boldsymbol{\psi_i}$	The maximum value that the PDF takes at time i
$\boldsymbol{X}$	A random variable
$\boldsymbol{X} \sim Beta(\alpha,\beta)$	A random variable that is beta-distributed
$\boldsymbol{X} \sim Dir(\alpha_1,\ldots,\alpha_r)$	A random variable that is Dirichlet-distributed

as ψ_i, as follows:

$$\begin{aligned}\psi_i &= \max_{X=x} f_i(x;\alpha,\beta,n_i,q_i)\\ &= f_i(\frac{q_i+\alpha-1}{\alpha+n_i+\beta-2};\alpha,\beta,n_i,q_i)\end{aligned} \tag{2}$$

where n_i and q_i are the total number of cases and the count of successes at time $t = i$, respectively, x is the mode of the beta distribution ($0 \le x \le 1$), and hyperparameters α, β are greater than or equal to 1.

In our setting, we iterate over time steps ($t = 1,2,\ldots$). At each time point $t = i$, we use the incoming batch, *Batch i*, to update the current Bayesian network model. We then use our approaches to detect the existence of model drift. we assume that the distribution of the data does not change inside the batch, i.e., we capture the presence of model drift across time steps ($t = 1,2,\ldots$) and not within the set of observations arrives at a particular time point. If the variations in the values of μ_i and ψ_i are important, we conclude that our Bayesian network model has drifted.
We summarize the notations we use in this paper in Table 1.

3 MODELING CONCEPT DRIFT USING LATENT VARIABLES

In this section, we present a modeling technique for detecting concept drift in discrete Bayesian networks. We explicitly model concept drift using latent variables. To avoid unnecessary complication, we assume that only posterior distribution and uncertainty drift over time, i.e., for each edge $A \to B$ in a Bayesian network model BN_1, we detect the existence of concept drift by monitoring the posterior distribution drift and uncertainty drift of $A \to B$ over time.

Our modeling technique for detecting the presence of concept drift in discrete Bayesian networks is described using plate notation as shown in Figure 1. The fundamental idea of our modeling approach is to add a latent node for each edge $A \to B$ in a given Bayesian network model BN_1. We call this latent node U^t_{AB}. It is important to point out that for each collection of observation j of time t, the unobserved node U^t_{AB} is added as the child of the observed nodes A^t_j and B^t_j.

The latent variable U^t_{AB} captures the posterior drift and the uncertainty drift for each collection of observations j of time t. It is essential to point to the fact that both values of observed variables A^t_j and B^t_j contribute to the drift of the latent variable U^t_{AB} as follows:
(**Posterior Distribution Drift**): In our modeling technique presented in Figure 1, the posterior distribution drift of the latent variable U^t_{AB} that is monitored at each time point $t = i$ is as follows:

$$\begin{aligned}\mu_i &= P_{t_i}(U^t_{AB} \mid A^t, B^t)\\ &= \frac{q_i+\alpha_u}{\alpha_u+n_i+\beta_u}\end{aligned}$$

where n_i and q_i are the total number of cases and the count of successes at time $t = i$, respectively, and hyperparameters α_u, β_u are greater than or equal to 1.

(**Uncertainty Drift**:) In our modeling technique shown in Figure 1, to capture the uncertainty drift of the latent variable U^t_{AB} over time, we monitor the maximum value that a probability density function $f_i(x;\alpha_u,\beta_u)$ of the latent variable takes at each time point $t = i$ as follows:

$$\begin{aligned}\psi_i &= \max_{X=x} f_i(x;\alpha_u,\beta_u,n_i,q_i)\\ &= f_i(\frac{q_i+\alpha_u-1}{\alpha_i+n_i+\beta_u-2};\alpha_u,\beta_u,n_i,q_i)\end{aligned}$$

where n_i and q_i are the total number of cases and the count of successes at time $t = i$, respectively, x is the mode of the beta distribution ($0 \le x \le 1$), and hyperparameters α_u, β_u are greater than or equal to 1.

It is important to emphasize that our modeling technique, at each time point $t = i$, receives j observations where $j = 1$ to n. These observations are used to update the Bayesian network model. The latent variable U^t_{AB} is then used to capture the presence of

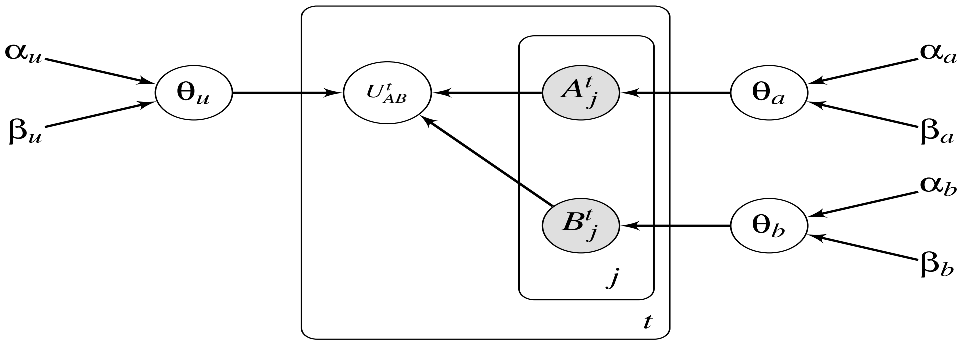
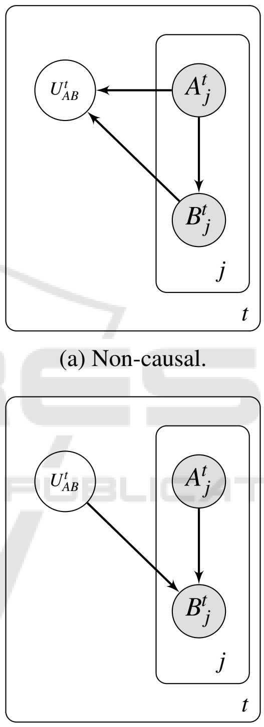


Figure 1: Modeling concept drift with latent variables in discrete Bayesian networks. A^t_j and B^t_j are observed nodes. U^t_{AB} is a latent (unobserved) node. θ_a, θ_b, and θ_u are model parameters. α_a, β_a, α_b, β_b, α_u, and β_u are model hyperparameters.

posterior drift (i.e., drift in the value of μ_i) and uncertainty drift (i.e., drift in the value of ψ_i). If the values of μ_i and ψ_i vary significantly, we conclude that our Bayesian network model has drifted.

The a priori expected values of concept and uncertainty drifts can be expressed via the prior distribution for the latent node U^t_{AB}. We use hyperparameters α_u and β_u to express the prior knowledge that we may have about concept and uncertainty drifts at a particular time point.

An important point to be made concerning the development of our modeling technique for detecting concept drift (presented in Figure 1) is that it contains no causal interpretation. We do not place any causal assumption on the interaction between the observed variables and the latent variable. Despite the fact that it is mathematically feasible to build causal and non-causal modeling techniques (as shown in Figure 2) to detect the presence of concept drift, it is not necessary to consider causal effects between variables as these effects are not the main focus of our modeling approach. For this reason, we tolerate that the interpretation of our modeling approach of concept drift is merely statistical, i.e., associational.



(a) Non-causal.

(b) Causal.

Figure 2: Options for building a modeling approach for detecting concept drift.

4 GENERALIZATION OF OUR FRAMEWORK INTO HIGHER DIMENSIONS

To expand our modeling framework for variables with more than two states, we can use the Dirichlet distribution, which is a continuous multivariate probability distribution. In Bayesian statistics, Dirichlet distribution, which is denoted as $Dir(\alpha_1,\ldots,\alpha_r)$, is parameterized by r hyperparameters $\alpha_1,\ldots,\alpha_r$ such that α_i $(1 \le i \le r)$ is integer and $\alpha_i \ge 1$ (Neapolitan et al., 2004). This distribution is the generalization of the beta distribution for $r > 2$, i.e., beta is a special case when $r = 2$. A Dirichlet distributed prior is conjugate for the likelihood function that is multinomial distributed. That is, multiplying a Dirichlet-distributed prior, $Dir(\alpha_1,\ldots,\alpha_r)$, with a multinomial-distributed likelihood function, $Multi(w_1\ ,\ldots,w_r;c_1,\ldots,c_r)$, yields a Dirichlet-distributed posterior distribution, $Dir(\alpha_1 + c_1,\ldots,\alpha_r + c_r)$, where $\alpha_1,\ldots,\alpha_r$ are Dirichlet distribution hyperparameters, $w_1,\ldots,w_r$ are Dirichlet distributed random variables, and $c_1,\ldots,c_r$ are the number of occurrences of each category.

We focus on detecting the presence of posterior distribution drift in the context of discrete Bayesian networks with respect to a random variable $\boldsymbol{X} = [\boldsymbol{X_1,...,X_r}]$ that is Dirichlet-distributed, which we denote as $\boldsymbol{X} \sim Dir(\alpha_1, \ldots, \alpha_r)$. We capture the existence of posterior distribution drift by monitoring the mean of the Dirichlet distribution at every time point $t = i$, denoted as μ_i, i.e., the expected value of $\boldsymbol{X_j}$ at every time point $t = i$, $\boldsymbol{E(X_j)}$, as follows:

$$\begin{aligned}\mu_i &= \boldsymbol{E(X_j)} \\ &= \frac{\alpha_j + c_j}{\alpha_{all}}\end{aligned}$$

where $\alpha_{all} = \sum_{s=1}^{r} \alpha_s + c_s$ and c_j is the number of occurrences of $\boldsymbol{X_j}$.

In addition to detecting the posterior drift, we consider detecting the presence of uncertainty drift in the context of discrete Bayesian networks with respect to a random variable $\boldsymbol{X}$ is Dirichlet-distributed as described above. We capture the existence of uncertainty drift by monitoring the maximum value of $\boldsymbol{X_j}$ that the probability density function of the Dirichlet distribution takes at every time point $t = i$, which we denote as ψ_i, as follows:

$$\begin{aligned}\psi_i &= \max_{X_j = x} f_i(x; \alpha, \beta, \alpha_{all}, c_j) \\ &= f_i(\frac{\alpha_j + c_j - 1}{\alpha_{all} - r}; \alpha, \beta, \alpha_{all}, c_j)\end{aligned}$$

where $\frac{\alpha_j + c_j - 1}{\alpha_{all} - r}$ is the mode of the Dirichlet distribution.

5 EMPIRICAL RESULTS

We have implemented our modeling framework and tested our approach using two of the most commonly used example networks in Bayesian experiments, Burglary-Earthquake Network and Chest Clinic network.

5.1 Burglary-Earthquake Network

The Burglary-Earthquake Network was created by Pearl (Pearl, 2014) and is a commonly used example in Bayesian networks. As shown in Figure 3, the Burglary-Earthquake Network is a fictitious network that could be used to model an alarm system in a house. The network consists of five nodes and four edges. The nodes are as follows: (1) Node **B** shows if there is a burglary, (2) Node **E** shows whether there is an earthquake, (3) Node **A** shows if the alarm goes off, (4) Node **M** shows if Mary calls, and (5) Node **J** shows if John calls. The causal relations between the nodes in this network is expressed by directed edges. For instance, the edge $B \rightarrow A$ means that burglary may cause the alarm to be activated and so on. We refer the readers to (Pearl, 2014) for a full description of this network.

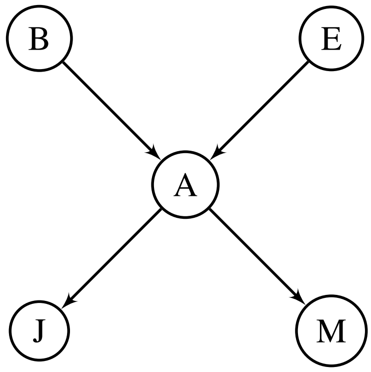


Figure 3: The original Burglary-Earthquake Network.

We apply our approach for detecting the presence of concept drift in discrete domains over time to the Burglary-Earthquake Network. To set up our experiment, we have implemented this network using *Hugin^{TM} Research 8.4*. *Hugin^{TM} case generator* (Madsen et al., 2005; Olesen et al., 1992) is then used to generate 15 simulated datasets of $1,000$ cases each. These datasets are named $Batch$ 1 through $Batch$ 15. During the simulation process of some datasets, the posterior probabilities are changed in order to simulate the existence of concept drift as follows: (1) The edge $B \rightarrow A$: (i) the posterior probabilities, $P(A = F \mid B = F)$ and $P(A = T \mid B = F)$, are changed during the simulation process of datasets $Batch$ 3, $Batch$ 7, and $Batch$ 12. (ii) the posterior probabilities, $P(A = T \mid B = F)$ and $P(A = T \mid B = T)$, are changed during the simulation process of the dataset $Batch$ 3. (2) The edge $E \rightarrow A$: the posterior probabilities, $P(A = F \mid E = F)$ and $P(A = T \mid E = F)$, are changed during the simulation process of the dataset $Batch$ 4. (3) The edge $A \rightarrow J$: the posterior probabilities, $P(J = F \mid A = T)$, is changed during the simulation process of the dataset $Batch$ 7. (4) The edge $A \rightarrow M$: the posterior probabilities, $P(M = F \mid A = F)$ and $P(M = T \mid A = T)$, are changed during the simulation process of the dataset $Batch$ 7. In our experiment, we assume that at each time point t ($t = 1, \ldots, 15$), we receive $Batch$ t which has j instances (we set $j = 1,000$ cases).

To implement our framework, we added a latent node for each edge in the Burglary-Earthquake Network. That is, we added latent nodes U^t_{BA}, U^t_{EA}, U^t_{AJ}, and U^t_{AM} to detect the presence of real concept drift and uncertainty drift for the edges $B \rightarrow A$, $E \rightarrow A$,

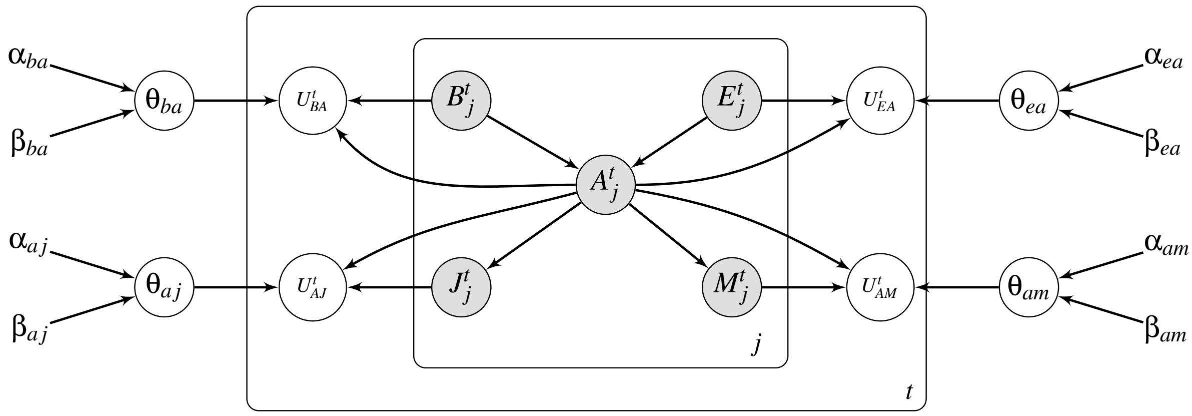


Figure 4: Our proposed framework for modeling concept drift with latent variables in the Burglary-Earthquake Network.

$A \rightarrow J$, and $A \rightarrow M$, respectively, as shown in Figure 4. We assume that we have no prior knowledge about concept drift. That is, we assume that all hyperparameters of the latent variables, $\alpha(.)$ and $\beta(.)$, are equal to 1.

The results of using our framework to detect the presence of real concept drift and uncertainty drift are summarized in Table 2 and Table 3, respectively. Note that values shown in bold in Table 2 and Table 3 indicate the presence of drift. Our framework succeeded in detecting the existence of real concept drift and uncertainty drift. We observe that a change in the posterior probability and the uncertainty is reflected by a variation in the evolution of the corresponding latent variable. For instance, we observe drifts in the posterior probabilities and the uncertainties of the latent variable U^t_{BA}, namely when $U = u \mid B = F,\ A = F$ and $U = u \mid B = F,\ A = T$, at time points 3, 7, and 12. We also observe that the posterior and the uncertainty of the latent variable U^t_{BA} drift at time point 3 namely when $U = u \mid B = T,\ A = F$ and $U = u \mid B = T,\ A = T$.

We observe that our framework is sensitive to changes in the underlying distribution of data that newly incoming batches may cause. That is, if the number of observations in the newly incoming *Batch t* at time t is less than the expected number of observations, then the framework shall report a drop in the posterior and the uncertainty at time t and vice versa. For instance, for the edge $B \rightarrow A$, namely when $U = u \mid B = F,\ A = F$, our framework captured a drop in the posterior and uncertainty drifts in the incoming batch at time point $t = 3$, *Batch* 3. This drop is due to that fact that the number of observed cases in *Batch* 3 was less than the expected number of cases. It should be noted that after each drift, the values of the posterior and uncertainty will be smoothly re-increasing/re-decreasing attempting to recover from the drift. It is also important to point out that if the number of cases in the newly incoming batch is as expected, our framework concludes that there is no drift to anticipate, and thus no action needs to be taken. Explanations of the other experiments for other edges trivially follow the explanation of the edge $B \rightarrow A$.

All in all, we have shown that our framework that is based on using latent variables to detect the presence of concept drift is effective and sensitive to changes in the underlying distribution of data in non-stationary environments over time. Our framework was successfully able to detect the existence of both real concept drift and uncertainty drift. Our new proposed approach for capturing uncertainty drift is sensitive and useful as it can ensure the occurrence of real concept drift.

5.2 Chest Clinic Network

The Chest Clinic network, a.k.a. the Visit to Asia network, was created by Lauritzen and Spielgelhalter (Lauritzen and Spiegelhalter, 1988) and is widely used in Bayesian network experiments. This network is a simple, fictitious medical network which could be employed in a medical facility to diagnose patients as shown in Figure 5. The Chest Clinic network consists of eight nodes, which represent random variables, and eight edges, which indicate the causal relations between the nodes. A complete description of this medical Bayesian network model is as follows (Lauritzen and Spiegelhalter, 1988):

> Shortness-of-breath (dyspnoea) may be due to tuberculosis, lung cancer, or bronchitis, or none of them, or more than one of them. A recent visit to Asia increases the chances of tuberculosis, while smoking is known to be a risk factor for both lung cancer and bronchitis. The results of a single chest X-ray do not discriminate between lung cancer and tuberculosis, as neither does the presence or absence of dyspnoea.

Table 2: Results of using our framework to detect the presence of real concept drift in the Burglary-Earthquake Network.

(a) The result of using the latent variable U^t_{BA} to detect the presence of real concept drift for the edge $B \rightarrow A$.

Posterior of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{B=F,A=F}$	0.98	0.98	**0.94**	0.95	0.96	0.96	**0.94**	0.95	0.96	0.96	0.96	**0.94**	0.95	0.95	0.96
$U^t_{B=F,A=T}$	0.006	0.006	**0.04**	0.032	0.027	0.024	**0.04**	0.032	0.029	0.027	0.026	**0.04**	0.032	0.031	0.029
$U^t_{B=T,A=F}$	0.0005	0.0005	**0.003**	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
$U^t_{B=T,A=T}$	0.01	0.01	**0.02**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(b) The result of using the latent variable U^t_{EA} to detect the presence of real concept drift for the edge $E \rightarrow A$.

Posterior of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{E=F,A=F}$	0.96	0.96	0.96	**0.92**	0.93	0.93	0.94	0.94	0.94	0.95	0.95	0.95	0.95	0.95	0.95
$U^t_{E=F,A=T}$	0.01	0.01	0.01	**0.06**	0.05	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02
$U^t_{E=T,A=F}$	0.016	0.015	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013
$U^t_{E=T,A=T}$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(c) The result of using the latent variable U^t_{AJ} to detect the presence of real concept drift for the edge $A \rightarrow J$.

Posterior of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=F,J=F}$	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
$U^t_{A=F,J=T}$	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$U^t_{A=T,J=F}$	0.002	0.002	0.002	0.002	0.002	0.002	**0.005**	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.003
$U^t_{A=T,J=T}$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(d) The result of using the latent variable U^t_{AM} to detect the presence of real concept drift for the edge $A \rightarrow M$.

Posterior of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=F,M=F}$	0.97	0.97	0.97	0.97	0.97	0.97	**0.95**	0.95	0.96	0.96	0.96	0.96	0.96	0.96	0.96
$U^t_{A=F,M=T}$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$U^t_{A=T,M=F}$	0.0059	0.0055	0.0053	0.0052	0.0053	0.0054	0.0055	0.0054	0.0054	0.0054	0.0056	0.0058	0.0059	0.0062	0.0063
$U^t_{A=T,M=T}$	0.012	0.011	0.011	0.011	0.011	0.010	**0.031**	0.029	0.027	0.025	0.024	0.023	0.022	0.021	0.020

Table 3: Results of using our framework to detect the presence of uncertainty drift in the Burglary-Earthquake Network.

(a) The result of using the latent variable U^t_{BA} to detect the presence of uncertainty drift for the edge $B \rightarrow A$.

Uncertainty of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{B=F,A=F}$	100.12	141.89	**92.23**	117.43	139.34	160.91	**149.16**	165.48	181.69	196.20	210.32	**200.34**	213.09	225.98	238.56
$U^t_{B=F,A=T}$	176.08	239.53	**110.58**	143.84	173.97	203.90	**179.13**	200.81	223.02	243.46	262.87	**238.04**	255.17	271.71	288.88
$U^t_{B=T,A=F}$	368.79	736.67	**419.45**	559.04	659.50	791.25	876.52	1001.63	1075.17	1100.22	1210.16	1272.71	1332.53	1434.97	1489.15
$U^t_{B=T,A=T}$	120.16	174.42	**172.15**	209.05	239.86	269.18	270.64	293.38	315.84	335.86	354.83	371.80	389.11	408.88	425.84

(b) The result of using the latent variable U^t_{EA} to detect the presence of uncertainty drift for the edge $E \rightarrow A$.

Uncertainty of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{E=F,A=F}$	65.90	93.85	118.30	**93.13**	110.12	125.49	139.30	152.34	165.06	177.75	189.38	200.50	211.33	222.22	232.59
$U^t_{E=F,A=T}$	110.77	160.13	203.07	**109.76**	133.27	156.12	177.12	197.56	217.20	236.51	256.01	274.14	291.73	310.17	327.28
$U^t_{E=T,A=F}$	103.32	148.89	192.56	224.58	252.62	272.56	291.30	310.42	327.11	348.07	361.79	379.68	396.79	413.21	429.03
$U^t_{E=T,A=T}$	125.87	174.43	212.23	241.51	270.10	295.95	319.71	339.92	362.59	385.71	407.63	425.18	443.61	461.32	479.88

(c) The result of using the latent variable U^t_{AJ} to detect the presence of uncertainty drift for the edge $A \rightarrow J$.

Uncertainty of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=F,J=F}$	50.10	70.86	86.79	99.88	111.89	123.16	132.71	142.09	150.68	158.79	166.51	173.99	181.16	188.05	195.24
$U^t_{A=F,J=T}$	57.31	81.07	98.99	113.62	127.27	139.17	153.76	164.29	174.01	183.21	191.80	200.35	208.54	215.99	223.75
$U^t_{A=T,J=F}$	271.21	391.32	482.51	559.05	626.30	687.01	**459.35**	511.28	560.94	608.57	647.27	684.19	726.51	774.78	822.20
$U^t_{A=T,J=T}$	103.32	146.44	181.48	210.82	236.56	267.54	294.44	313.32	329.78	345.52	364.22	379.68	393.45	408.88	425.84

(d) The result of using the latent variable U^t_{AM} to detect the presence of uncertainty drift for the edge $A \rightarrow M$.

Uncertainty of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=F,M=F}$	77.67	111.99	136.36	157.73	176.53	194.16	**156.79**	173.49	187.71	200.94	213.50	225.03	236.74	246.86	257.27
$U^t_{A=F,M=T}$	120.16	178.65	215.57	247.11	280.35	305.22	330.48	360.53	380.07	394.85	412.89	428.49	445.26	459.79	473.95
$U^t_{A=T,M=F}$	176.08	250.97	308.18	356.32	391.04	423.24	453.32	487.13	518.79	543.58	562.72	577.34	592.19	603.58	615.41
$U^t_{A=T,M=T}$	120.16	170.49	209.04	244.26	272.56	305.22	**190.01**	212.59	232.98	253.50	272.82	291.47	310.56	327.51	345.56

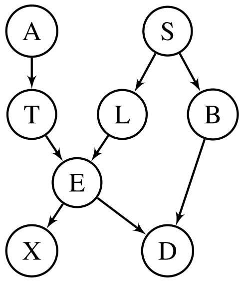


Figure 5: The original Chest Clinic network.

We apply our approach for detecting the presence of concept drift in discrete domains to the Chest Clinic network. To avoid unnecessary computations, we use our framework to detect the presence of concept drift of the weakest edge in the Chest Clinic network. Using Alsuwat et al.'s link strength measure, the edge from $A \rightarrow T$ is the weakest edge in this network (Alsuwat et al., 2019). Therefore, we employ our framework to detect the existence of concept and uncertainty drifts of the edge $A \rightarrow T$.

To set up our experiment, we have implemented this network using *Hugin*^{TM} *Research 8.4*. *Hugin*^{TM} *case generator* (Madsen et al., 2005; Olesen et al., 1992) is then used to generate 15 simulated datasets of $2,000$ cases each. These datasets are named *Batch* 1 through *Batch* 15. To simulate the presence of concept drift, we change the posterior probabilities during the simulation process as follows: (1) the posterior probability $P(T = no \mid A = no)$ is changed during the simulation process of datasets *Batch* 4 and *Batch* 11. (2) the posterior probability $P(T = yes \mid A = no)$ is changed during the simulation process of dataset *Batch* 4. (3) the posterior probability $P(T = no \mid A = yes)$ is changed during the simulation process of dataset *Batch* 11. (4) the posterior probability $P(T = yes \mid A = yes)$ is changed during the simulation process of datasets *Batch* 2 and *Batch* 10. In this experiment, we assume that at each time point t ($t = 1, \ldots, 15$), our framework receives *Batch* t which has j observations (j is set at $2,000$ cases).

To implement our framework, we added a latent node for the weakest edge in the Chest Clinic network. That is, we added the latent node U^t_{AT} to detect the presence of real concept drift and uncertainty drift for the edges $A \rightarrow T$ as shown in Figure 6. We assume that we have no prior knowledge about concept drift, i.e., we assume that the hyperparameters of the latent variable U^t_{AT}, α_{at} and β_{at}, are equal to 1.

The results of applying our framework to detect the existence of real concept drift and uncertainty drift of the weakest edge in the Chest Clinic network are summarized in Table 4 and Table 5, respectively. Note that values shown in bold in Tables 4 and 5 indicate

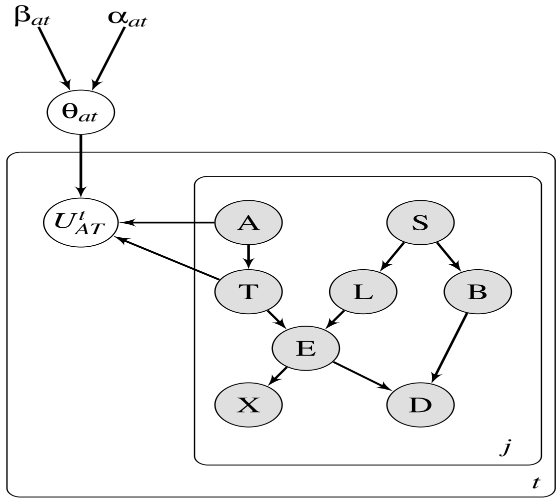


Figure 6: Our proposed framework for modeling concept drift of the weakest edge in the Chest Clinic network using a latent variable.

the presence of drift. Our framework was successfully able to detect the presence of real concept drift and uncertainty drift. We observe that a change in the posterior probability and the uncertainty is reflected by a variation in the evolution of the latent variable U^t_{AT}. For example, we observe drifts in the posterior probabilities and the uncertainties of the latent variable U^t_{AT} as follows: (1) when $U = u \mid A = no,\ T = no$, the posterior and the uncertainty drift at time points 4 and 11, (2) when $U = u \mid A = yes,\ T = no$, the posterior and the uncertainty drift at time point 4, (3) when $U = u \mid A = no,\ T = yes$, the posterior and the uncertainty drift at time point 11, and (4) when $U = u \mid A = yes,\ T = yes$, the posterior and the uncertainty drift at time points 2 and 10.

We observe that our framework is sensitive to changes in the underlying distribution of incoming data. Moreover, our framework is able to quickly detect the existence of drifts. Another important observation is that receiving more observations that belong to the cell with the highest test statistics value will reflect a higher variation of the evolution of the corresponding latent variable and thus will reflect a drift in the posterior and the uncertainty. Overall, we have shown that our framework that is based on using latent variables to model concept drift in nonstationary environments is efficient to detect posterior and uncertainty drifts of the weakest edge in a given Bayesian network model.

Table 4: Results of using our framework to detect the presence of real concept drift of the weakest edge in the Chest Clinic network.

Posterior of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=no,T=no}$	0.98	0.98	0.98	**0.96**	0.96	0.97	0.97	0.97	0.97	0.97	**0.95**	0.96	0.96	0.96	0.96
$U^t_{A=yes,T=no}$	0.008	0.008	0.008	**0.02**	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$U^t_{A=no,T=yes}$	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	**0.02**	0.02	0.02	0.02	0.02
$U^t_{A=yes,T=yes}$	0.0009	**0.002**	0.001	0.001	0.001	0.001	0.0009	0.0009	0.0009	**0.001**	0.001	0.001	0.001	0.001	0.001

Table 5: Results of using our framework to detect the presence of uncertainty drift of the weakest edge in the Chest Clinic network.

Uncertainty of	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$U^t_{A=no,T=no}$	137.74	184.65	228.18	**197.55**	231.22	262.59	290.81	317.06	340.94	361.32	**298.71**	318.90	338.57	357.81	376.66
$U^t_{A=yes,T=no}$	205.74	287.02	346.34	**236.70**	282.12	324.22	363.55	400.53	434.71	468.68	501.12	530.61	558.87	587.62	615.37
$U^t_{A=no,T=yes}$	183.30	263.10	326.76	374.87	419.77	462.42	499.52	534.05	564.76	595.52	**374.68**	402.07	429.08	455.05	480.74
$U^t_{A=yes,T=yes}$	736.31	**527.74**	751.41	955.80	1144.48	1373.21	1539.98	1696.67	1844.71	**1591.53**	1716.84	1838.06	1955.52	2069.48	2180.19

6 RELATED WORK

In this section, we will give a brief overview of concept drift, concept drift classification, and concept drift detection methods.

Concept Drift Overview: Applications are increasingly critically dependent on concept schemes for the semantic interoperability of their data (Wang et al., 2010). As data evolves over time, real-time data analytics are undermined as the models built to foster this learning becomes obsolete (Žliobaitė et al., 2016). In machine learning, concept drift is a nonstationary learning problem that develops over time, often because the training and data application mismatch in real life scenarios (Moreno-Torres et al., 2012; Gama et al., 2014). Therefore, concept drift is associated with a greater probability for prediction inaccuracies due to misalignment driven by changes in the statistical properties of the target variable. Most real-world applications confront some form and degree of shift, which renders this topic highly relevant to the existing and emerging machine learning community (Moreno-Torres et al., 2012). Concept drift thus plays a key role in machine learning and predictive analytics optimization, as adequately accounting for this phenomenon strengthens the overall integrity, utility, and functionality of the machine learning model. Recent surveys on concept drift can be found in (Iwashita and Papa, 2019; Gama et al., 2014).

Concept Drift Classification: In contemporary scientific literature, several research has been proposed to characterize types of concept drift (Webb et al., 2016; Gama et al., 2014; Iwashita and Papa, 2019). Webb et al. (Webb et al., 2016) categorized types of concept drift based on (i) Drift subject, which indicates what aspects of the joint probability drifts over a period of time, (ii) Drift frequency, which shows how often concept drift happens during a particular time, (iii) Drift transition, which indicates the means wherein the process of changing from one concept to another occurs, (iv) Drift reoccurrence, which shows whether or not the occurring concept drift has previously appeared, and (v) Drift magnitude, which points out the degree of drift between two time points.

Drift subject is mathematically defined as a change in the joint probability between two time points t_0 and t_1 as follows: $P_{t_0}(X,y) \neq P_{t_1}(X,y)$, where X is the input variables and y is the target variable (Gama et al., 2014). Drift subject is divided into two types (Gama et al., 2014): (1) real concept drift, and (2) virtual concept drift. Real concept drift occurs when the conditional probability changes on the target variable y whereas the input variables X remain unchanged, i.e., the posterior probability changes between two time points t_0 and t_1 as follows: $P_{t_0}(y \mid X) \neq P_{t_1}(y \mid X)$. Virtual concept drift occurs when the prior distribution changes between two time points t_0 and t_1 while the posterior probability remains unchanged (Tsymbal, 2004; Widmer and Kubat, 1996), i.e., $P_{t_0}(X) \neq P_{t_1}(X)$. Real concept drift is the most important aspect in the category of drift subject since changes in real concept drift will degrade the accuracy of the machine learning model and thus require an update of the model (Kelly et al., 1999). Therefore, the discussion of this paper is related to the notion of real concept drift which we refer to as concept drift.

Concept Drift Detection: One of the challenging tasks in the context of concept drift is to rapidly detect concept drift and provide a practical measure of drift magnitude. A variety of concept drift detection methods have been recently developed. Gama et al. (Gama et al., 2014) categorized such methods into four general groups as follows: (1) methods based on sequential analysis (members of this group include the Cumulative Sum (CUSUM) and the Page-Hinkley (PH) (Page, 1954)), (2) methods based on statistical process control (members of this group include the Drift Detection Method (DDM) (Gama et al., 2004), the Early Drift Detection Method (EDDM) (Baena-Garcıa et al., 2006), and the Exponentially Weighted Moving Average (EWMA) (Ross et al., 2012)), (3) methods based on contextual approaches (a member of this group includes the Splice system (Harries et al., 1998)), and (4) methods based on Monitoring distributions on two different time-windows (members of this group include the Adaptive sliding Window (ADWIN) (Bifet and Gavalda, 2007), the Adaptive Cumulative Windows Model (ACWM) (Sebastião et al., 2017), and SEED Drift Detector (SEED) (Huang et al., 2014)).

The contribution of this work belongs to the last one of the four groups. Methods based on monitoring distributions on two different time-windows are techniques that use statistical tests to compare the distributions of a fixed reference window on the previous data and a sliding window on the most recent data (Gama et al., 2014). Kifer et al. were first to propose comparing two detection window distributions in relation to data streams (Kifer et al., 2004). The team's presented algorithms assessed samples taken from two probability distributions to identify key differences in the distributions. Another example of such methods was proposed by (Gama et al., 2006) is the VFDTc system, which is an algorithm for mining in nonstationary environments with the ability to detect and adapt to concept drift. The VFDTc system is used in concept drift resolution through ongoing monitoring of observed differences between two class-distributions, including evaluation of: 1) class-distribution when a node was a leaf, and 2) weighted sum of class-distributions in the node's leaf-descendants (Gama et al., 2006).

Other more recent concept drift detection methods based on monitoring distributions on two different time-windows were proposed in (Borchani et al., 2015) and (Cabañas et al., 2018). In this work, we study concept drift detection via comparing distributions on two different time-windows. We aim to use latent variables to model and detect concept drift in the context of discrete Bayesian networks. Borchani et al. proposed a modeling technique with conditional linear Gaussian (CLG) that used latent variables to detect concept drift (Borchani et al., 2015). Their model is applicable to continuous Bayesian networks and was applied to continuous domains. Cabanas et al. proposed a method for detecting concept drift in discrete streaming data (Cabañas et al., 2018). Their proposed preprocessing algorithm transferred discrete data into continuous data before applying Borchani at el. model to detect concept drift. However, Cabanas et al.'s technique is susceptible to data loss and results in increased processing overhead when used in incremental learning domains.

7 CONCLUSION AND FUTURE WORK

Detecting changes in the underlying distribution of incoming data, a.k.a. concept drift detection, is a vital and active research area in machine learning systems. In this paper, we studied the presence of concept drift in the context of discrete Bayesian networks in non-stationary environments. We have proposed a framework for modeling concept drift using latent variables in discrete Bayesian networks. Our modeling technique using latent variables is capable of detecting real concept drift and uncertainty drift over time. We have applied our framework for detecting the presence of concept drift in discrete domains over time to the Burglary-Earthquake Network and the Chest Clinic Network, which are the most widely used networks in Bayesian experiments. Our results indicate that our framework is not only sensitive to changes in the underlying distribution of incoming data but also can easily detect the real concept drift and uncertainty drift over time. Our ongoing work extends these results to find explanations for the changes of the models. Such explanations will improve our understating of the evolution of the concept drift. This indeed may permit to distinguish malicious attacks from natural model shift. In addition, we aim to acquire an authentic dataset for further experiments and compare our approach with other approaches that model concept drift using latent variables.

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