

Iterated Local Search: Applications and Extensions

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Abstract: Iterated Local Search (ILS) is a conceptually simple and efficient well-known Metaheuristic. The main idea behind ILS is to drive the search not on the full space of all feasible solutions but on the solutions that are returned by some underlying algorithm; typically, local optimal solutions obtained by the application of a local search heuristic. This method has been applied to many different optimization problems having about 10,000 entries in Google Scholar. In this talk, we will review briefly the ILS method emphasizing the extensions of ILS. We will describe three relevant types of extensions: the hybrid ILS approaches combining ILS with other metaheuristics and/or exact methods; the SimILS (Simulation+ILS) to solve Stochastic Combinatorial Optimization Problems; the MoILS to solve Multiobjective Combinatorial Optimization, including multiobjective and stochastic problems. We will discuss the advantages and disadvantages of these extensions and present some applications, including real ones in areas like Marketing, Supply Chain Management, Logistics or Health Care.

1 INTRODUCTION

Optimization problems have been always relevant for businesses and other organizations, to improve the use of the resources and also to reduce the overall system cost. In today's world, with the availability of large amount of data and with the increase complexity of the problems, it is more important than ever to be able to solve efficiently the optimization problems. Optimization problems can appear in all areas of business from marketing to logistics, in public organizations, in health care organizations and even in NGO's entities. Therefore, being able to solve these large-scale complex problems is a relevant and very actual area for researchers and managers. In many cases, within the Optimization area, the best (and only) solution approach to solve these complex problems are the metaheuristics.

In the optimization literature we can find two general types of solution methods: exact methods and, heuristics and metaheuristics. An exact or optimal method in the optimization context refers to an algorithm that computes an optimal solution. A heuristic algorithm (often shortened to heuristic) is a solution method that does not guarantee mathematically an optimal solution, but in general

has a good level of performance in terms of solution quality or convergence, within a very short computational time. Heuristics may be constructive (producing a single solution) or local search (starting from one constructive or random solution and moving iteratively to other nearby solutions) or a combination (constructing one or more solutions and using them to start a local search). A metaheuristic is a framework for producing heuristics, such as simulated annealing, tabu search or genetic algorithms. To develop an heuristic for a particular problem some problem-specific characteristics must be defined, but some other can be general for all problems. The problem-specific may include the definition of a feasible solution, the neighborhood of a solution, rules for changing solutions, and rules for setting certain parameters during the course of execution. In the last years, many metaheuristics have been proposed and successfully applied to solve complex and large-scale problems in many areas (Martí et al. 2018).

In this work, we focus on a metaheuristic known as Iterated Local Search (ILS). ILS is one of the most popular single-solution based metaheuristics. ILS is recognized by many authors as a relatively simple yet efficient framework able to deal with complex optimization problems. In particular it is a very

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successful method when applied to Combinatorial Optimization Problems (COPs), including problems in Logistics, Transportation, Scheduling, Health care, Marketing, etc. The success of the ILS can be explained because it has many of the desirable features of a metaheuristics: accuracy, speed, simplicity and flexibility (Cordeau et al. 2002).

The main objective of this work is to give an accessible description of the underlying principles of ILS and a discussion of basic implementation issues. Next, we present extensions of the application of the ILS: the hybrid ILS approaches combining ILS with other metaheuristics and/or exact methods; the SimILS (Simulation+ILS) to solve Stochastic Combinatorial Optimization Problems; the MoILS/MoSimILS to solve Multiobjective (Stochastic) Combinatorial Optimization. We will discuss the advantages and disadvantages of these extensions and present some applications, including real ones in areas like Marketing, Supply Chain Management, Logistics or Health Care. We will review the main work done by the author in the area of Iterated Local Search.

2 ITERATED LOCAL SEARCH

The Iterated Local Search can be seen as an extension of the well know algorithm know as Local Search (LS). Local Search have been applied extensively to combinatorial optimization problems (COPs). A LS method consists in finding an initial solution, usually using a constructive or random heuristic, and then perform a neighborhood search until a local optimal solution is found. For each specific problem, a neighborhood must be defined. This neighborhood consists usually in a set of solutions that can be obtained from the incumbent solution by performing small modifications, known as moves. For all or almost all COPs, it is relatively simple to obtain an initial solution and define a neighborhood. Therefore, a LS can be defined by using these two concepts.

The main issue with such a simple LS method, is that usually the local optimal solution obtained is far away from the overall optimal solution, i.e. the quality of the solutions obtained is not too good. To avoid this issue, we can use a random initial solution and apply to each one a LS approach. This is known as Random Restart approach. This simple random sampling approach can perform poorly, in particularly if the instance size is large. ILS belongs to a category of multi-start metaheuristics that improve the performance of the simple Random Restart by incorporating more sophisticated

procedures (Martí et al. 2013). In this way, ILS tries to avoid the disadvantages of random restart by exploring the solution space using a walk that steps from one local optimal solution to a “nearby” one. To implement this idea, another phase is included in the LS that allows to “restart” the search but try not to “lose” the good properties and components of the solutions already obtained. This component we call it “Perturbation” Phase.

We can know present the Iterated Local Search that consists in four main phases: Generate an Initial Solution, Local Search, Perturbation and Acceptance Criterion.

First of all, an initial solution is constructed (*Generate an Initial Solution*), afterwards a *Local Search* method is applied to obtain a local optimal solution (current solution). Next, a random *Perturbation* phase is applied to obtain a different solution changing some components of the current solution followed by applying again a Local Search method. If the solution obtained passes an *Acceptance Criterion* test, it becomes the next current solution; otherwise, one returns to previous one. In any case, the process is repeated/iterated from the Perturbation phase. The resulting walk is a case of a stochastic search in local optimal solution space of the problem. In Figure 1, we describe the pseudo code of the algorithm.

```

1.  $s_0 = \text{GenerateInitialSolution};$ 
2.  $s^* = \text{LocalSearch}(s_0);$ 
3. Repeat
   3.1.  $s' = \text{Perturbation}(s^*, \text{history});$ 
   3.2.  $s^* = \text{LocalSearch}(s');$ 
   3.3.  $s^* = \text{AcceptanceCriterion}(s', s^*, \text{history});$ 
4. Until TerminationCondition met;
5. Output  $s^*$ . End.

```

Figure 1: Iterated Local Search Method.

As you can see, it is a very simple and elegant method that can be applied to a large number of optimization problems, since for most of these ones an initial solution, local search and perturbation method can be easily defined. In practice, much of the potential complexity of ILS is hidden in the history dependence and in particular in the Perturbation phase design as we will explain later. If no dependence on the past search is used, the walk has no memory: the perturbation and acceptance criterion do not depend on any of the solutions visited previously during the walk, and one accepts or not with a fixed rule. This leads to random walk dynamics on solution space that are “Markovian”. Most of the work using ILS has been of this type, though recent

studies show that incorporating memory enhances performance (Ramalhinho Lourenço et al. 2019).

Next, we explain in detail how to implement an ILS method to solve optimization problems. In each of the following sections, we will present an extension of the ILS and one application.

2.1 Implementation Details

A general description of a metaheuristic can be seen as a methodological approach for designing (problem-specific) heuristics. It is preferable that a metaheuristic to be simple, both conceptually and in practice. But of course, it should also be effective, i.e. it should generate very good solution in short computational time. Most of the heuristics are measured against accuracy, the degree of departure of the obtained solution value from the optimal value, and against speed, the computation time. However, two also important attributes of the metaheuristics are the simplicity and flexibility (Cordeau et al. 2002). The simplicity is related with the number of parameters to be set and facility to be implemented, which are very few in the case of ILS. The flexibility is related with the possibility to accommodate new side constraints and adaptation to similar problems, which is also quite simple to implement in a ILS algorithm.

The ILS algorithm is composed by four main components: `GenerateInitialSolution`, `LocalSearch`, `Perturbation`, and `AcceptanceCriterion`. A simple implementation of the ILS for a COP can be quite straight-forward to design. The four main components can be defined as follows:

- (i) `GenerateInitialSolution`: a random feasible solution;
- (ii) `LocalSearch`: for most problems a local search algorithm is readily available; Just need to define the Neighborhood structure.
- (iii) `Perturbation`: a random move in a higher-order neighborhood than the one used by the local search algorithm can be surprisingly effective;
- (iv) `AcceptanceCriterion`: a reasonable first guess for the acceptance criterion is to force the cost to decrease, corresponding to a first-improvement descent.

Basic ILS implementations of this type usually lead too much better performance than random restart approaches. The developer can then run this basic ILS to build his/her intuition and try to improve the overall algorithm performance by improving each of the four modules and tuning their interaction.

The next step to improve the performance of the metaheuristic is to introduce some complexity and problem-related properties in the design of the four components of the ILS. Next, we will describe the main issues that it should be considered for the improvement of the four components of the algorithm, and comment on their interaction.

GenerateInitialSolution: Random solutions tend to be of very low quality. Therefore, if very high-quality solutions are to be reached by the local search, then starting from a best possible solution becomes an important issue. One good option is to apply a greedy heuristic. For most of the COPs it exists a greedy heuristic or it can easily define. Greedy solutions are in general of higher quality than random solutions. Other options are also available, like we will explain latter in the hybrid ILS, other metaheuristics or even approximation methods can be used to obtain the initial solution. A good initial solution does not guarantee a good final local optimal solution, but in general it can help to reduce the search time.

LocalSearch: The overall performance of the ILS is highly related with the choice of the local search method, since this is one of the main components of the method. One might think that the better the local search, the better the corresponding ILS but if too much time is spent in the `LocalSearch` phase, it is not really an ILS but more like an extended local search. The equilibrium between the `LocalSearch` phase and the number of iterations is a main issue when designing an ILS. It might be probably more effective to apply a faster and more frequently local search algorithm than a slower and more powerful one (Stützle & Roos 2002). Therefore, the design of the `LocalSearch` phase should be carefully done and the interaction with the other components of the ILS should be carefully considered and studied.

Perturbation: The main goal of this phase is to escape from local optimal solution area by applying perturbations or changes in the current local optimal solution. An important decision is how strong the perturbations should be. If they are too small, one will often fall back to the previous local optimal solution and therefore very few new solutions or a small region of the solution space will be explored. On the other way, if the perturbations are too big, it will be almost like a random search, and we will end up having a random restart type algorithm. The most important issue to take into consideration when designing the perturbation phase is to guarantee that the obtained solution will not be directly undoable by

the local search and but should complement it in some way. The solution obtained by the perturbation should have a relatively different structure from the actual local optimal solution and should also help the search to explore new regions of the solution space. Not necessarily this solution should be of great quality, i.e. small cost in the minimization problem. The design of the Perturbation phase is one of the most important issue when implemented an ILS algorithm. Small perturbation can lead to large computational times and a random restart type algorithm, and large perturbation can lead to jumping from a solution to another without descent to a good local optimal solution. We also recommend that the Perturbation phase should be problem-dependent and they could be complex perturbations as for example the application of exact algorithms to solve subproblems or relaxation models of the optimization problem in consideration (Ramalhinho Lourenço 1995).

Acceptance Criterion: This procedure determines whether is accepted or not the solution just obtained as the new current solution. AcceptanceCriterion has a strong influence on the nature and effectiveness of the search in the solution space. In a way, together with Perturbation, the procedure controls the balance between intensification and diversification of the search. We can consider two strategies: a first improvement type descent which only accepts better solutions; or at the opposite extreme, one can always accept the new solution irrespective of its cost. Many intermediate choices between these two extreme cases are possible, and in particular rather complex acceptance criteria that involve limited amount of directed diversification or intensification are also possible (Stützle & Roos 2002).

After all, to improve the performance of the ILS it should be taken into account the interactions between the four components. Next, we briefly mention some guidelines and suggestions that should be considered:

- The perturbation should not be easily undone by the local search; if the local search has obvious shortcomings, a good perturbation should compensate for them. The idea is that the perturbation modifications should be of different nature of the ones in the neighborhood and change considerably the structure of the solution even at the cost of worst the quality of the solution.
- The iteration between Perturbation and Acceptance Criterion determines the relative balance of intensification and diversification and

should receive a particularly strong attention. Large perturbations are only useful if they can be accepted, and that occurs only if the acceptance criterion is not too biased towards better solutions.

- LocalSearch should be as powerful as possible as but also not too costly in CPU time. Then, given a choice for that module, find a well-adapted perturbation. The equilibrium between LocalSearch and Perturbation is really important to study.
- Finally, define the AcceptanceCriterion procedure so that solution space is sampled adequately.

As a final suggestion, the construction of an ILS algorithm should start by implement a simple version and increase complexity as needed to improve the performance of the algorithm. One way to improve this performance is to optimize each module assuming the others are fixed; this is a “local optimization” approach to the global optimization problem. When performing such optimizations, the interactions between the modules are essential, and for instance the balance between intensification and diversification is very important and remains a challenging problem.

For a more detailed description of the ILS algorithm see the following references: (Lourenço et al. 2010; Lourenço et al. 2003; Ramalhinho Lourenço et al. 2019).

Next, we will describe some history related with the development of the ILS, followed by some applications of ILS to several COPs.

2.2 Some History

The Iterated Local Search algorithm appeared in the literature with several names until it was settled as ILS as it is known today. Some are the following ones:

- Large Step Markov Chains
- Chained Local Optimization
- Iterated Descent
- Iterated Lin-Kernighan
- Local Search with Perturbation
- Iterated Greedy Algorithm
- Iterated Local Search

One of the main and first references to ILS was (Martin et al. 1991) with the name of Large-Step Markov Chains. Before, there were references to similar algorithms that maybe can considered within the structure of ILS, which were: (Baxter 1996), (Baum 1986) and (Johnson 1990). (Lourenço 1993)

extended the ILS method by using an hybrid ILS with other metaheuristics and exact methods and applied it to job-shop scheduling problem. Just a few years later, T. Stützle applied it to flow-shop scheduling problems, (Stützle 1998), and as far as we know this was the first time that the name ILS appeared in the literature. In the following years, many authors contribute to the development of the ILS. Actually, the name Iterated Local Search (ILS) is accepted all over the research community and for a recent and complete survey, the readers are referred to (Ramalinho Lourenço et al. 2019).

We performed a search on google scholar to have an idea of the application of the ILS, and we found out that the application of ILS has been steadily increasing in the last years as shown in Figure 2.

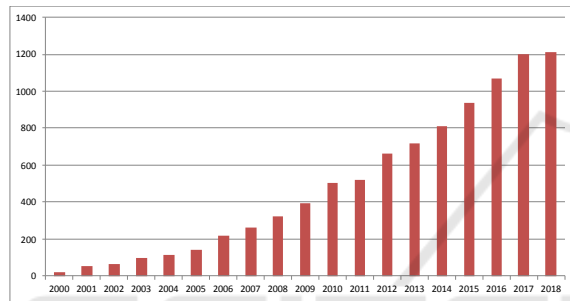


Figure 2: Number of publications in the Google Scholar (search done in January 2019).

2.3 Application 1: The Cut-Clique Problem

The first example of application that we would like to present is the development of an iterated local search heuristics to solve the maximum cut-clique problem published in (Martins et al. 2015). The problem consists in finding the maximum cut-clique of a graph. Given an undirected graph $G=(V,E)$ and a clique C of G , the cut-clique is the set of edges running between C and $V\setminus C$, establishing the cut $(C,V\setminus C)$. The maximum cut-clique in G is to find a clique with the largest number of edges in the neighborhood of the clique, also known as the maximum edge-neighborhood clique. In this work, the authors introduce an Iterated Local Search algorithm to solve the maximum cut-clique problem. They compare the results with the exact methods approach and explore a new application within marketing analysis. They provide an application within the area of analyzing market basket problems in the marketing area providing interesting insides and analysis not presented before.

3 HYBRID ILS

The ILS can be extended by using other metaheuristics in some of the phases of the ILS, as for example in the `GenerateInitialSolution` or `LocalSearch` phases, or even use exact methods to solve sub- or relaxation problems in the `Perturbation` phase.

Any local search-based metaheuristic approach like tabu search, variable neighborhood search, GRASP or simulated annealing may be used in the `LocalSearch` phase and often the performance of the algorithm increases by using a more complex method in this phase. This type of combination can be included in a larger area known as Hybrid Metaheuristics, (Blum et al. 2011). One of the first application of hybrid ILS can be found in (Lourenço & Zwijnenburg 1996), that uses a tabu search as the embedded heuristic in an Iterated Local Search algorithm to solve the job-shop scheduling problem.

Another extension that leads to very good results is the use of exact method to solve sub- or relaxed problems during the `Perturbation` phase. Many optimization problems are difficult to solve, but some relaxation or small instances of these problems can be solved exactly very efficiently. So, designing a `Perturbation` phase by applying an exact method to solve simple instances of the optimization problem leads to solutions with structures or properties that are not easily undo in the local search phase. This combination of exact methods and ILS is known as MathILS and can be included in the larger area of Metaheuristics (Talbi 2013; Dumitrescu & Stützle 2003; Puchinger & Raidl 2005). One of the first references in the literature of a MathILS can be found on (Ramalinho Lourenço 1995) where, in the `Perturbation` phase, an exact method is applied to solve an one-machine scheduling problem representing a relaxation of a job-shop scheduling problem.

3.1 Application 2: Distribution Problem

Distribution planning is an important activity for many companies in the area of retailing, production etc. The cost associated with the delivery and distribution of goods can be an important component of the final cost of the products. The work (Coelho et al. 2016) presents a study on a Vehicle Routing Problem (VRP) variant inspired on a real case of a large distribution company. In particular, they consider a VRP with a heterogeneous fleet of vehicles that are allowed to perform multiple trips. The

problem also includes docking constraints in which some vehicles are unable to serve some particular customers, and a realistic objective function with vehicles' fixed and distance-based costs and a cost per customer visited. They present an Iterated Local Search based algorithm, that combines ILS, Greedy Randomized Adaptive Search Procedure (GRASP) and Variable Neighborhood Descent (VND) procedures. The method was tested with real instances, and it was able to obtain competitive and realistic solutions that improved the company solutions and led to significant savings in distribution costs.

4 SIMULATIONS AND ILS

A natural extension of the Iterated Local Search metaheuristic is to be able to study problems with some non-deterministic or stochastic data. In real-life most of the problems present some kind of uncertainty; therefore, in these cases it is common to simplify the mathematical model assuming deterministic data to be able to solve the problem. But this assumption turns the model into a less accurate one that does not reflect the stochastic nature of the real-life problem. In order to overcome this issue, the work (Grasas et al. 2014) proposed a framework that combines ILS and Simulation to enable the resolution of Stochastic Optimization Problems, known as SimILS. This method is included in the area of SimHeuristics (Juan et al. 2015). The proposed algorithm is an easy-to-implement simheuristic able to deal with stochastic COPs in a natural way. The integration between simulation and the ILS algorithms must be done carefully in order to avoid incurring in prohibitive computational times due to the simulation component.

The general SimILS framework is presented in Figure 3 and as described, it integrates simulation at some specific steps, resulting in a simulation-optimization procedure capable of dealing with stochastic COPs.

In the SimILS, the simulations are inserted after the application of the LocalSearch phase to evaluate the current local optimal solution in the simulation environment. These simulations take the current solution and a parameter indicating whether the simulation should be run for a long or a short time, and then obtains the corresponding simulated objective function (cost), along with other relevant statistics or Key Performance Indicators (KPI) to evaluate the solution in terms of stochastic environment. Also, these statistics or KPI can be used

later to improve the search by updating the LocalSearch or Perturbation phases. A long simulation component is also inserted at the end of the ILS process to reevaluate the final solution in a realistic environment.

```

1.  $s_0$ =GenerateInitialSolution;
2.  $s^*$ =LocalSearch( $s_0$ );
3. ( $s^*$ , cost, statistics)=Simulation( $s^*$ , long);
4. Repeat
  4.1.  $s$ =Perturbation( $s^*$ , history);
  4.2.  $s^*$ =LocalSearch( $s$ );
  4.3. ( $s^*$ , cost, statistics)=Simulation( $s^*$ , short);
       $s^*$ =AcceptanceCriterion( $s^*$ ,  $s^*$ , history);
5. Until TerminationCondition met;
6. ( $s^*$ , cost, statistics)=Simulation( $s^*$ , long);
7. Output  $s^*$ . End.

```

Figure 3: General SimILS Method.

The "simplicity" of the ILS combined with Simulation makes this method a good alternative to solve very complex and large-scale Stochastic Combinatorial Optimization Problems. Some applications can be found in: (Pagès-Bernaus et al. 2017; De Armas et al. 2017; Quintero-Araujo et al. 2017; Juan et al. 2011; Juan et al. 2014; Juan et al. 2013).

4.1 Application 3: Designing a Supply Chain

The design of a supply chain is a relevant problem for many industries and in particular for the retailing one. Many references can be found in Supply Chain Design or Location Theory. But very few studies consider the e-commerce within the supply chain design. E-commerce activities present characteristics that are different from those related to traditional retailing business. The work of (Pagès-Bernaus et al. 2017) presents two facility-location models to represent supply chains in e-commerce. The proposed optimization models consider stochastic demands as well as a restricted number of regular suppliers per customer. They also propose two solving methodologies; the first one is a two-stage Stochastic Programming Approach that solve a Deterministic Equivalent Model using CPLEX. However, this approach has several limitations to solve large-scale instances due to the significant computational effort required. They then propose a SimILS algorithm that is able to solve large-scale instances in short computing times. The proposed models and algorithms are illustrated and tested using a set of

benchmark instances. Results show that better designs can be obtained with the proposed models and solving methodologies, and the SimILS is able to solve efficiently large-scale instances.

5 MULTIOBJECTIVE ILS

The last extensions of ILS that we would like to mention are the Multi-Objective Iterated Local Search (MoILS) for (deterministic) multiobjective problem and the Multi-objective Simulated Iterated Local Search (MoSimILS) for multiobjective stochastic problems.

In the case of the application of metaheuristics to multiobjective (deterministic) problems, the objective is to find the approximation of the Pareto frontier and this method is included in the area of Multiobjective Metaheuristics (Gandibleux et al. 2004).

As proposed in (Ehr Gott & Gandibleux 2000), the main idea of multiobjective local search methods is to consider a weighted sum of the objective functions to search for an approximation of the nondominated frontier. This local aggregation of the objectives produces the effect to focus the search on a part of the nondominated frontier. Then the weights are updated and the search is repeated to approximate completely the nondominated frontier.

As the algorithms are extensions of Iterated Local Search, all phases of the ILS and SimILS have to be defined when a specific optimization problem has to be solved. In the multiobjective case, the ILS will output a pool of approximate Pareto Optimal solutions. It is important to incorporate the multiobjective and stochastic aspects in the definition of the different phases of the extended ILS algorithms. For example, the weights in the aggregation of the objectives can help to drive the search in a direction or other, and the statistics obtained by the simulation can help to define that weights. This is still an ongoing work, and in the near future it is expected to have more applications of these methods.

5.1 Application 4: Home Health Care

Home Health Care (HHC) service consists of assistance provided by medical personal, such as nurses, physical therapists and home care aides, to people with special needs, for example old adults, chronically ill or disabled people. The main criteria to evaluate an HHC service solution are basically the following ones: the service quality and the service

cost. The quality of life perceived by patients who stays home is higher than if they stay at the hospital. Also, a patient in a hospital has a high cost to the community. Therefore, the benefit of the Home Health Care service is the significant decrease in the hospitalization rate that leads to a cost reduction in the whole health system for one side, and the perception of a better quality from the patient on the other side.

During the last decade the Health Care service industries experienced significant growth in many European countries due to the governmental pressure to reduce healthcare costs, the demographic changes and the development of new services and technologies. Therefore, a set of new optimization problems arise that need to be solved inefficiently (Rais & Viana 2010; Oberscheider & Hirsch 2016).

Home Health Care Problem (HHCP) consist in defining the medical assistance route considering all human resources constraints, working time limit and all medical constraints. The main objectives are to minimize the costs, the maximization of loyalty between the medical personal and patient and, the balance of workload between the medical personal. The service time and transportation time is stochastic, so the problem turns out to be a multiobjective stochastic problem. (Galvani & Ramalhinho 2019) studied this problem and propose a MoSimILS to solve it. They also present the application of the method to solve a realistic problem in Italy.

6 CONCLUSIONS

ILS has many of the most desirable features of a metaheuristic: it is simple, easy to implement, robust, and highly effective. The essential idea of ILS lies in focusing the search not on the full space of solutions but on a smaller subspace defined by the solutions that are locally optimal for a given optimization objective function. The efficiency of this method depends on the design of the four main components of the method: GenerateInitialSolution, LocalSearch, Perturbation, and AcceptanceCriterion. However, even with the simplest implementations of these parts, ILS can obtain quite good solutions. And, if more complex components are designed, the algorithm becomes very competitive or even a state of the art one for many optimization problems. The modular nature of the Iterated Local Search leads to short development times and gives ILS an edge over more complex metaheuristics in the world of industrial applications. We present briefly some

applications to exemplify the potential of the ILS metaheuristic.

Finally, we also review some of the most relevant extensions of the ILS that have been developed or are under studied to be applied to many other problems, including stochastic and multiobjective problems. Notice that most of the real-life problems present these two characteristics. There are still too many questions that need more research in this last mention extensions and there are definitely worth to study. For example, the design of the MathILS is still an open problem for many optimization problems. Also, the study of MoILS and MoSimILS is still relatively new and it requires a deeper study and more applications.

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REFERENCES

- De Armas, J. et al., 2017. Solving the deterministic and stochastic uncapacitated facility location problem: From a heuristic to a simheuristic. *Journal of the Operational Research Society*, 68(10), pp.1161–1176.
- Baum, E.B., 1986. Towards practical “neural” computation for combinatorial optimization problems. In J. Denker, ed. *Neural Networks for Computing, AIP conference proceedings*.
- Baxter, J., 1996. Local optima avoidance in depot location. *Journal of the Operational Research Society*, 32, pp.125–133.
- Blum, C. et al., 2011. Hybrid metaheuristics in combinatorial optimization: A survey. *Applied Soft Computing*, 11(6), pp.4135–4151. Available at: <http://linkinghub.elsevier.com/retrieve/pii/S1568494611000962> [Accessed February 21, 2014].
- Coelho, V.N.N. et al., 2016. An ILS-based algorithm to solve a large-scale real heterogeneous fleet VRP with multi-trips and docking constraints. *European Journal of Operational Research*, 250(2), pp.367–376. Available at: <http://dx.doi.org/10.1016/j.ejor.2015.09.047>.
- Cordeau, J.F. et al., 2002. A guide to vehicle routing heuristics. *Journal of the Operational Research Society*, 53, pp.512–522.
- Dumitrescu, I. & Stützle, T., 2003. A survey of methods that combine local search and exact algorithms. *Applications of Evolutionary Computation*, (i), pp.211–223.
- Ehrgott, M. & Gandibleux, X., 2000. A survey and annotated bibliography of multiobjective combinatorial optimization. *OR Spectrum*, 22(4), pp.425–460. Available at: <http://link.springer.com/10.1007/s002910000046>.
- Galvani, M. & Ramalhinho, H., 2019. *An Iterated Local Search approach to solve multi-objective and stochastic Home Health Care Problem*, Barcelona, Spain.
- Gandibleux, X. et al., 2004. *Metaheuristics for Multiobjective Optimisation* Lecture No. Springer, ed.,
- Grasas, A., Juan, A.A.A.A. & Lourenço, H.R.H.R., 2014. SimILS: a simulation-based extension of the iterated local search metaheuristic for stochastic combinatorial optimization. *Journal of Simulation*, 10(1), pp.1–9. Available at: <http://www.scopus.com/inward/record.url?eid=2-s2.0-84961324333&partnerID=tZotx3y1> [Accessed November 5, 2014].
- Johnson, D.S., 1990. Local optimization and the Traveling Salesman Problem. In M. S. Paterson, ed. *Automata, Languages and Programming*. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 446–461.
- Juan, A.A. et al., 2015. A review of simheuristics: Extending metaheuristics to deal with stochastic combinatorial optimization problems. *Operations Research Perspectives*, 2(1), pp.62–72. Available at: <http://linkinghub.elsevier.com/retrieve/pii/S221471601500007X>.
- Juan, A.A. et al., 2014. A simheuristic algorithm for the Single-Period Stochastic Inventory-Routing Problem with stock-outs. *Simulation Modelling Practice and Theory*, 46, pp.40–52.
- Juan, A.A. et al., 2013. Using parallel and distributed computing for solving real-time Vehicle Routing Problems with Stochastic Demands. *Annals of Operations Research*, 2017(1), pp.43–65.
- Juan, A.A. et al., 2011. Using safety stocks and simulation to solve the vehicle routing problem with stochastic demands. *Transportation Research Part C: Emerging Technologies*, 19(5), pp.751–765. Available at: <http://linkinghub.elsevier.com/retrieve/pii/S0968090X10001439> [Accessed July 25, 2014].
- Lourenço, H.R., 1993. *A computational study of job shop and the flow shop scheduling problems*. Cornell UNiversity.
- Lourenço, H.R. et al., 2003. Iterated local search. In F. Glover & G. Kochenberger, eds. *Handbook of Metaheuristics*. Kluwer Academic, pp. 321–353.
- Lourenço, H.R., Martin, O. & Stützle, T., 2010. Iterated Local Search: Framework and Applications. In M. Gendreau & J. Y. Potvin, eds. *Handbook of Metaheuristics*. New York, New York, USA: Springer, pp. 363–397.
- Lourenço, H.R. & Zwijnenburg, M., 1996. Combining the large-step optimization with tabu-search : application to the job-shop scheduling problem. In I. H. Osman & J.

- P. Kelly, eds. *Meta-Heuristics: Theory and Applications*. Kluwer Academic Publishers, pp. 219–236.
- Martí, R., Panos, P. & Resende, M.G.C., 2018. *Handbook of Heuristics*, Springer, Cham.
- Martí, R., Resende, M.G.C.C. & Ribeiro, C.C., 2013. Multi-start methods for combinatorial optimization. *European Journal of Operational Research*, 226(1), pp.1–8. Available at: <http://linkinghub.elsevier.com/retrieve/pii/S0377221712007394> [Accessed March 14, 2014].
- Martin, O., S.W., O. & Felten, E.W., 1991. Large-Step Markov Chains for the Traveling Salesman Problem. *Complex Systems*, 5, pp.299–326.
- Martins, P., Ladrón, A. & Ramalinho, H., 2015. Maximum cut-clique problem: ILS heuristics and a data analysis application. *International Transactions in Operational Research*, 22(5), pp.775–809. Available at: <http://doi.wiley.com/10.1111/itor.12120> [Accessed September 23, 2014].
- Oberscheider, M. & Hirsch, P., 2016. Analysis of the impact of different service levels on the workload of an ambulance service provider. , 16(1).
- Pagès-Bernaus, A. et al., 2017. Designing e-commerce supply chains: A stochastic facility-location approach. *International Transactions in Operational Research*.
- Puchinger, J. & Raidl, G.R., 2005. Combining Metaheuristics and Exact Algorithms in Combinatorial Optimization: A Survey and Classification. *Lecture Notes in Computer Science*, 3562.
- Quintero-Araujo, C.L. et al., 2017. Using simheuristics to promote horizontal collaboration in stochastic city logistics. *Progress in Artificial Intelligence*, 6(4), pp.275–284. Available at: <http://link.springer.com/10.1007/s13748-017-0122-8>.
- Rais, A. & Viana, A., 2010. Operations Research in Healthcare: a survey. *International Transactions in Operational Research*, 18(1), pp.1–31. Available at: <http://doi.wiley.com/10.1111/j.1475-3995.2010.00767.x> [Accessed January 22, 2014].
- Ramalinho Lourenço, H., 1995. Job-Shop Scheduling: computational study of local search and large-step optimization methods. *European Journal of Operational Research*, 83, pp.347–364.
- Ramalinho Lourenço, H., Martin, O. & Stützle, T., 2019. Iterated Local Search: Framework and Applications. In M. Gendreau & J. Y. Potvin, eds. *Handbook of Metaheuristics*. Springer International Publishing.
- Stützle, T., 1998. *Applying iterated local search to the permutation flow shop problem*,
- Stützle, T. & Roos, H.H., 2002. Analyzing the run-time behaviour of iterated local search for TSP. In P. Hansen & C. Ribeiro, eds. *Essays and Surveys in Metaheuristics*. Kluwer Academic Publishers, pp. 589–612.
- Talbi, E.G., 2013. Combining metaheuristics with mathematical programming, constraint programming and machine learning. *4or*, 11(2), pp.101–150.

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