

# Fault Diagnosis by Bayesian Network Classifiers with a Distance Rejection Criterion

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**Abstract:** In this paper, Bayesian network classifiers (BNCs) are used as a statistical tool to diagnosis faults with a distance rejection criterion. The proposed approach enhances significantly the structure of the use of Bayesian networks in the same context. Our framework is evaluated and compared to state of the art using data from the benchmark Tennessee Eastman Process (TEP).

## 1 INTRODUCTION

The existing monitoring techniques should always be subject to improvement to deal with uncertainties and complexities of modern systems. Therefore, developing novel fault diagnosis approaches has been a significant research topic during the past decades. We can find in the literature three main approaches, that are a) data-driven approach, that is concerned with the collected data from processes to develop a statistical model for monitoring, b) knowledge-based approach that is based on experts, and c) model-based approach that requires a prior physical and mathematical knowledge of the process.

The ultimate goal in fault diagnosis is to accurately identify various types of faults that may affect a process. Faults are commonly defined as changes either in the mean vector or in the covariance matrix, or both. This paper focuses on using Bayesian Networks (BNs) as a framework with decision rules to diagnosis and detect known and unknown faults.

BNs are powerful probabilistic tools. Previous studies have proposed various networks for fault diagnosis. BNs have shown great abilities to fault diagnosis (Wang et al., 2019), (Jin et al., 2017), (He et al., 2016), (Atoui et al., 2016), (Atoui et al., 2015b), (Zhao et al., 2013), (Yu and Rashid, 2013), (Yang and Lee, 2012). Though, one can observe from the literature that the proposed BNs i) rely only on the maximum posterior probability discrimination rule to make decisions; ii) don't consider the possibility of occurrence of new observations belonging to unknown faults/ operating conditions.

In this work, we will tackle the aforementioned issues when dealing with BN for Fault diagnosis. We shall propose a BN for fault diagnosis dealing with unknown class of faults. This paper is organized as follows: Section 2 briefly introduces the BN classifiers. In section 3, we present the proposed framework. Section 4 presents performances comparisons using the classical TEP benchmark. Finally, in Section 5, we give conclusions and outlooks of the further directions.

## 2 BAYESIAN NETWORK CLASSIFIERS

A Bayesian Network (BN) is a probabilistic graphical model (Nielsen and Jensen, 2009). It consists of the following:

- a directed acyclic graph  $G, G=(V, E)$ , where  $V$  and  $E$  are respectively its nodes' and arcs' sets,
- a finite probabilistic space  $(\Omega, \mathbb{Z}, p)$ , with  $\Omega$  a non-empty space,  $\mathbb{Z}$  a collection of the subspaces of  $\Omega$  and,  $p$  a probability measure (we use the same notation for both probability distributions and probability density functions. The meaning will be clear from the context) on  $\mathbb{Z}$  with  $p(\Omega) = 1$ ,
- a set of random variables  $\mathbf{X} = \mathbf{X}_1, \dots, \mathbf{X}_l$  assigned to  $V$  and defined on  $(\Omega, \mathbb{Z}, p)$ , such that:

$$p(\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_l) = \prod_{i=1}^l p(\mathbf{X}_i | pa(\mathbf{X}_i)) \quad (1)$$

where  $pa(\mathbf{X}_i)$  is the set of parent nodes of  $\mathbf{X}_i$  in  $G$ ,

- a conditional distribution associate to each node, given its parent nodes, describing probabilistic dependencies between variables,
- calculations named inference, used given the availability of a new evidence about one or several variables represented by the nodes of  $G$ , to update the network.

One particular form of Bayesian networks is the Conditional Gaussian Network (CGN). Each CGN's node represents a discrete or Gaussian random variable.

Gaussian nodes given their Gaussian parents follow a Gaussian linear regression models with parameters depending on the values of their discrete parents. Let's consider a Gaussian node  $\mathbf{Y}$  with discrete parents  $pa_D(\mathbf{Y}) = \{\mathbf{D}^1, \dots, \mathbf{D}^d\}$  and Gaussian parents  $pa_C(\mathbf{Y}) = \{\mathbf{Y}_1, \dots, \mathbf{Y}_c\}$ . Its conditional distribution could be written as below for each value  $k_{pa_D(\mathbf{Y})}$  of its discrete parents:

$$p(\mathbf{Y}|\mathbf{Y}_1, \dots, \mathbf{Y}_c, k = \mathcal{N}(\mu_k + R_k^{\mathbf{Y}_1} \mathbf{Y}_1 + \dots + R_k^{\mathbf{Y}_c} \mathbf{Y}_c; \Sigma_k), k \in I_{pa(\mathbf{Y})} \quad (2)$$

where  $\mu_k$  and  $\Sigma_k$  are respectively the mean and the covariance matrix of  $\mathbf{Y}$  given its discrete parents's value  $k$ .  $I_{pa(\mathbf{Y})}$  is a set of  $\mathbf{Y}$ 's discrete parents values.  $R_k^{\mathbf{Y}_1}, \dots, R_k^{\mathbf{Y}_c}$  are the regression coefficient associated respectively to  $\mathbf{Y}$ 's Gaussian parents  $\mathbf{Y}_1, \dots, \mathbf{Y}_c$ .

BNs and their ability to encode relationships between variables could be used naturally to solve classification problems; generally under the assumption that the data are normally distributed.

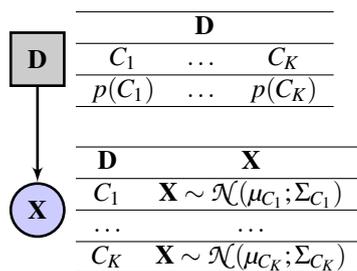


Figure 1: A basic CGN classifier.

Consider a new observation vector  $x$  of  $\mathbf{X} \in \mathbb{R}^m$  and  $K$  different classes  $C_k, i \in \{1, \dots, K\}$ . A basic conditional Gaussian network classifier equivalent to quadratic discriminant analysis, given in Figure 1, will assign  $x$  to the class  $C_k$  with the maximal a posterior probability  $p(C_k|x)$ . The Maximum A Posterior

(MAP) rule,  $\delta$ , can be written as follows:

$$\delta : x \in C_{k^*}, \text{ where } k^* = \underset{k=1, \dots, K}{\operatorname{argmax}} p(x|C_k) \quad (3)$$

Other discrimination rules, derived from (3), in respect of the BN's learned/ employed structure, can be derived by making assumptions on classes' covariance matrices and using equation (3).

In this paper, we shall propose a new set of rules to diagnosis known faults and detect unknown faults in respect of a distance rejection criterion.

### 3 FAULT DIAGNOSIS WITH DISTANCE REJECTION

Fault diagnosis consists of acknowledging the presence of a fault in a system, and then identifying which fault is it.

Fault diagnosis can be seen as a supervised classification problem. BNs can be used to define probabilities boundaries between the faults' classes (see an example in Figure 2). Therefore, a new observation is assigned to the fault with the higher a posteriori probability. Though, under the assumption that all the faulty operating conditions are known and well defined.

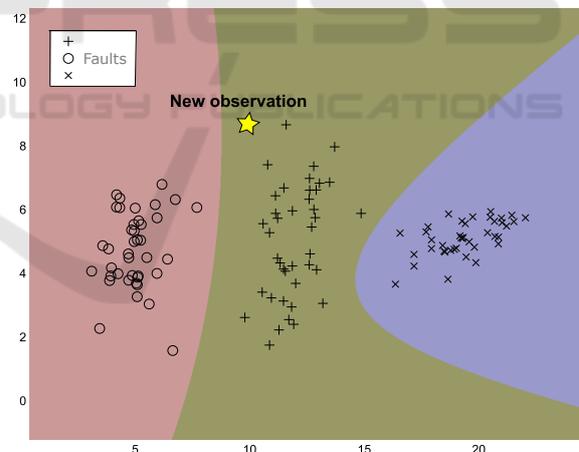


Figure 2: BNC - quadratic discriminant analysis - decisions discriminating between faults - standard approach.

However, in practice, it's not obvious to describe efficiently system's faulty operating conditions. Also, it is not always possible to identify the exact number of possible faults that could influence/ change the system from its normal operating conditions. Moreover, it is hard to obtain/ collect enough data of faulty operating conditions that are rare or too risky to simulate. Hence, it can be interesting to consider that some new observations could do not belong to any of the existing/ known fault classes.

A distance rejection criterion then can be used to handle and consider a new unknown class in a BNC. Thus, we propose a rule based on a new probabilistic limit (more details are given in the Appendix). The proposed distance rejection criterion, given a new observation  $x$ , compare the posterior probability of a class  $C_{k^*}$  with the higher posterior probability to its corresponding probabilistic limit, deduced from Appendix.(15) and given in (4), and decides statistically based on a considered significance level  $\alpha$ . It's obvious  $\alpha$  control the degree of exclusion, a higher value of  $\alpha$  would lead to shrinked ellipsoids and then more exclusions.

$$PL_{\Delta}^{C_k} = \frac{\tau}{1 + e^{-\frac{1}{2}(\varphi_1 - \beta_1)} + \dots + e^{-\frac{1}{2}(\varphi_{K-1} - \beta_{K-1})}} \quad (4)$$

with

$$\varphi_j = \Delta_{C_j} - \Delta_{C_k}, \quad (5)$$

$$\beta_j = 2 \ln(\omega_j \frac{|\Sigma_{C_k}|^{\frac{1}{2}}}{|\Sigma_{C_j}|^{\frac{1}{2}}}), \quad (6)$$

$$\omega_j = \frac{p(\mathbf{D} = C_j)}{p(\mathbf{D} = C_k)} \quad (7)$$

where  $j = 1, \dots, K$ .

If  $p(C_{k^*}|x) > PL_{\Delta}^{C_{k^*}}$  then we classify an observation  $x$  as  $C_k$ , else we attribute the observation to the class *UFC*. Hence, we divide the decision space into  $K + 1$  sub-spaces (an example is shown in Figure 3), where a new sub-space represents the class *UFC*, unknown/ not defined states class. It's clear that a BNC following our approach isolate statistically each class independently from the others classes. Basically, a new observation is compared to the boundary associated to each class if it does not belong statistically to any one of them then it belongs to the class *UFC*.

The following algorithm presents the steps we propose to diagnose faults while incorporating a distance rejection criterion under BNCs.

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Algorithm 1: Fault diagnosis with distance rejection criterion.

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**Input:** a new observation  $x$

**Outputs:** the fault class to which  $x$  belongs

Calculate  $p(\mathbf{D} = C_k|x)$ , for  $k \in 1, \dots, K$

**if**  $p(\mathbf{D} = C_{k^*}|x) \geq PL_{\Delta}^{C_{k^*}}$  **then**

$x \in C_{k^*}$  s.t.  $C_{k^*} = \text{argmax } p(C_{\hat{k}}|x)$

**else**

$x \in \text{UFC}$

▷ one can collect similar observations, define new class and add it to the classifier

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It worth noting to say that the proposed algorithm/ approach present a couple of advantages 1) it can be

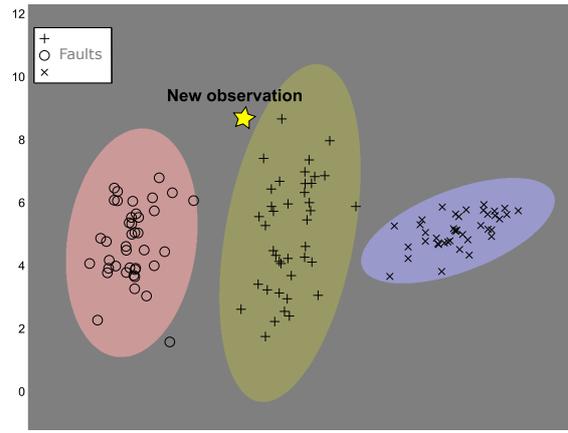


Figure 3: BNC - quadratic discriminant analysis - decisions discriminating between faults - our approach.

extended to handle multiple faults. We can do this by testing statistically the belonging of a new observation to every class of fault instead of considering the fault with highest posterior probability; 2) it can be associated to several BN classifiers (e.g. PCA as proposed in (Atoui et al., 2014)); 3) it can be easily integrated and very useful to complex Bayesian networks such as the ones proposed by (Roychoudhury et al., 2006), (Kawahara et al., 2005), (Schwall and Gerdes, 2002) and (He et al., 2016); 4) it outperforms, in the same context, the available approaches in terms of time complexity and classification error rate as in (Wang et al., 2017), (Verron et al., 2010). Furthermore, the number of the parameters and nodes is not proportional to the number of faults; and 6) it can be extended to detect and diagnosis known and unknown faults, which we present in this paper.

## 4 PERFORMANCE AND APPLICATION

In this section, we shall demonstrate and evaluate the performance of the proposed approach using a complex system: the Tennessee Eastman Process (TEP).

### 4.1 Presentation of the TEP

The Tennessee Eastman Process is a chemical process. It is not a real process but a simulation of a process that was created by the Eastman Chemical Company to provide a realistic industrial process in order to evaluate process control and monitoring methods (Downs and Vogel, 1993).

The TEP (flow sheet given on Figure 4) consists of five major operation units: a reactor, a condenser, a

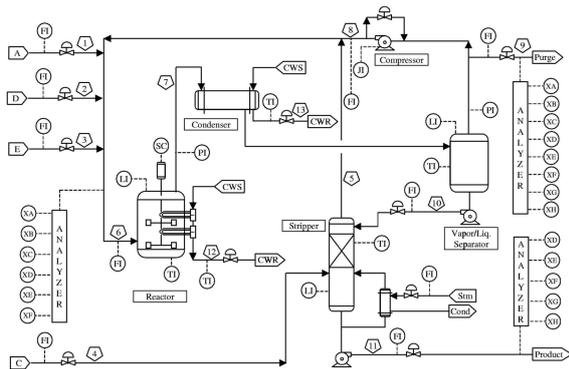


Figure 4: TEP flow sheet.

compressor, a stripper and a separator. Four gaseous reactant A, C, D, E and an inert B are fed to the reactor where the liquid products F, G and H are formed. This process has 12 input variables and 41 output variables. The TEP has 20 types of identified faults.

### 4.2 A BNC for Fault Diagnosis with a Distance Rejection Criterion

Table 1: Description of datasets.

| Class | Fault type   | Training data | Test data |
|-------|--|---------------|-----------|
| F4    | step change in the reactor cooling water inlet temperature       | 480           | 800       |
| F9    | random variation in D feed temperature                           | 480           | 800       |
| F11   | reactor variation in the reactor cooling water inlet temperature | 480           | 800       |

In the following, we shall compare the proposed approach to the one presented in (Verron et al., 2010) - to our knowledge it is so far the most popular and efficient method handling distance rejection in BNs' state of the art. The BN proposed by (Wang et al., 2017), (Verron et al., 2010) is given in Figure 5. One can notice its complex structure - five BNs (respectively representing a quadratic discriminant analysis, three control charts and a BN merging decisions). Indeed, two inference phases are needed. These phases involves the definition of several CPTs, transformation of probabilities and many redundant inputs. Basically, it depends on the number of faults which can lead to a very complex and time consuming BN. We propose to use a very simple BN structure presented in Figure 6, and representing, as an example, a quadratic discriminant analysis (similarly to (Wang et al., 2017) and (Verron et al., 2010)) associated to our proposed

algorithm.

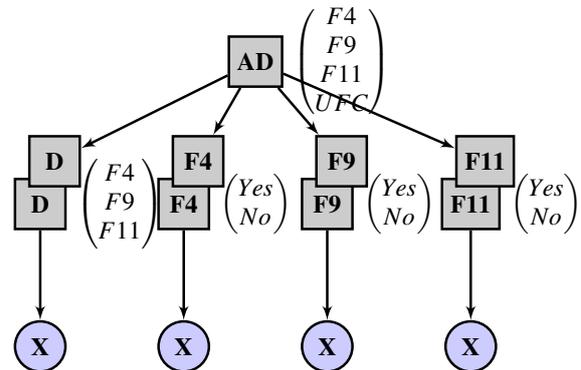


Figure 5: The structure of the BN proposed by (Wang et al., 2017) and (Verron et al., 2010).

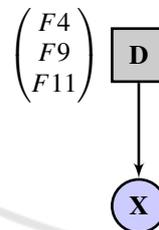


Figure 6: An example of a BNC's structure - other BNC could be considered - associated to our algorithm.

Consider now faults 4, 9 and 11 (see Table 6). These faults are widely used in literature to compare fault diagnosis methods. The three faults overlap, making the classification task difficult. Several classifiers have been used to discriminate between these faults. For instance, a learned BN classifier, equivalent to a QDA, provide 18.75% as a misclassification rate. More details are given in Table 2.

Table 2: Confusion matrix using the BN classifier without a distance rejection criterion.

| Class | F4  | F9  | F11  | Total |
|-------|-----|-----|------|-------|
| F4    | 659 | 0   | 141  | 800   |
| F9    | 0   | 582 | 218  | 800   |
| F11   | 28  | 66  | 706  | 800   |
| Total | 687 | 648 | 1065 | 2400  |

We tested both approaches on 7200 observations (800 observations from respectively fault 4, 9, 11 and 4800 observations representing the class UFC (collection of 800 observations from fault 7, 8, 10, 12, 13 and 14 (Chiang et al., 2012)). The obtained results are given in Tables 3 and 4.

From Table 4, the misclassification error rate obtained by our approach, in respect to the 3 faults, equals to 19.20 %, instead of 18.875% obtained by a QDA. However, our approach outperforms the one

Table 3: Confusion matrix using the BN integrating a distance rejection criterion proposed by (Verron et al., 2010).

| $C_k$ | F4  | F9  | F11  | UFC  | Total |
|-------|-----|-----|------|------|-------|
| F4    | 654 | 0   | 144  | 2    | 800   |
| F9    | 0   | 580 | 216  | 4    | 800   |
| F11   | 28  | 65  | 695  | 12   | 800   |
| UFC   | 132 | 122 | 194  | 4352 | 4800  |
| Total | 814 | 767 | 1249 | 4370 | 7200  |

proposed in (Verron et al., 2010) with a misclassification error rate equals 19.62%, see Table 3.

Also, we can see that 4454 from 4800 observations belonging to the class UFC have been recognized by our approach as unknown faults (misclassification error rate = 7.20%). Further, we have obtained better performance, in respect to UFC, compared with the the BN proposed in (Verron et al., 2010), 9.33%.

We have shown how our new approach outperforms and can be an alternative to the state of the art. Our approach is also able to simultaneously detect and diagnosis simultaneously known and unknown faults in a single Bayesian network.

Once again, the reader should notice that the results obtained by our approach depend considerably on the used BN classifier. Obviously, several BNCs can be associated to our proposal. The structure and parameters of a given BNC are generally learned from data. Different BNCs could be obtained in respect of variables relationships and considered assumptions (Friedman et al., 1997). BNCs are not only considered as powerful tools for classification but also as frameworks for different data-driven fault diagnosis schemes (Atoui et al., 2015a).

Table 4: Confusion matrix of an example of a BN classifier integrating our distance rejection criterion.

| $C_k$ | F4  | F9  | F11  | UFC  | Total |
|-------|-----|-----|------|------|-------|
| F4    | 655 | 0   | 141  | 4    | 800   |
| F9    | 0   | 582 | 217  | 1    | 800   |
| F11   | 28  | 65  | 702  | 5    | 800   |
| UFC   | 0   | 132 | 214  | 4454 | 4800  |
| Total | 683 | 779 | 1274 | 4464 | 7200  |

## 5 CONCLUSIONS

In this paper, a new approach able to diagnosis faults with a distance rejection criterion is proposed. Its performances on the TEP gives excellent results comparatively to the literature. Obvious outlook of this work is to expand our approach to simultaneously detect and diagnose known and unknown faults. Also, develop it for others data-driven BNCs to fault detec-

tion and diagnosis. Furthermore, it can be interesting to enhance the decisions made by our approach by considering different types of information.

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## APPENDIX

Assume each class  $C_k$ ,  $k = \{1, \dots, K\}$ , follow a normal distribution

$$x|C_k : \frac{1}{2\pi^{\frac{m}{2}} |S_{C_k}|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu_{C_k})^T S_{C_k}^{-1}(x-\mu_{C_k})} \quad (8)$$

where  $m_{C_k}$  and  $S_{C_k}$  are respectively the mean and covariance of  $C_k$ .

Let's call  $\Delta_k$  the quadratic form associated to the class  $C_k$

$$\Delta_{C_k} = (x - m_{C_k})^T S_{C_k}^{-1} (x - m_{C_k}) \quad (9)$$

The form  $\Delta_k$  based on its statistical distribution (usually the chi-squared distribution is considered), given significance level  $\alpha$ , help to decide whether or not a new observation belongs to the class  $C_k$ . This is done by comparing  $\Delta$  to its deduced limit  $CL_\Delta$  (Control limit) as below

$$x \in C_k, \Delta_{C_k} \leq CL_\Delta^{C_k} \quad (10)$$

By developing the inequality equation presented above we obtain

$x \in C_k$ , if

$$\begin{aligned} \Delta_{C_k} &\leq CL_\Delta^{C_k} \\ -\frac{1}{2}\Delta_{C_k} &\geq -\frac{1}{2}CL_\Delta^{C_k} \\ e^{-\frac{1}{2}\Delta_{C_k}} &\geq e^{-\frac{1}{2}CL_\Delta^{C_k}} \\ p(x|\mathbf{D} = C_k) &\geq p(x^*|\mathbf{D} = C_k) \end{aligned} \quad (11)$$

where  $x^*$  is an observation of  $\mathbf{X}$  with  $x^* \in C_k$  such as  $\Delta_{C_k} = CL_\Delta^{C_k}$ .

Let's multiply each side of (11) by  $p(x)$  as below

$$\begin{aligned} \frac{p(x)}{p(x^*)} p(x|\mathbf{D} = C_k) p(\mathbf{D} = C_k) \\ \geq p(x^*|\mathbf{D} = C_k) p(\mathbf{D} = C_k) \end{aligned} \quad (12)$$

where

$$\begin{aligned} p(x) &= p(x|\mathbf{D} = C_1) p(\mathbf{D} = C_1) + \dots \\ &\quad + p(x|\mathbf{D} = C_K) p(\mathbf{D} = C_K) \end{aligned} \quad (13)$$

Thus, we deduce the following rule

$$x \in C_k, \text{ if } p(\mathbf{D} = C_k|x) \geq PL_\Delta^{C_k} \quad (14)$$

with

$$PL_\Delta^{C_k} = \frac{p(x^*|\mathbf{D} = C_k) p(\mathbf{D} = C_k)}{p(x)} \quad (15)$$

It's worth to mention that  $p(\mathbf{D} = C_k|x)$  corresponds to the posterior probability of an observation  $x$  given the value  $C_k$  of the node  $\mathbf{D}$ . The observation  $x$  could concern one or several nodes.