Reliability Analysis of the Kalman Filter for INS/GPS Integrated Navigation System Applied to Train

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Keywords: INS/GPS, Train Application, Kalman Filter, Observability, RMSE.

Abstract: This paper aims to analyse the navigation performance that can be provided by the navigation system when applying the INS/GPS integrated navigation system to the train. The performance of the Kalman filter integrating INS and GPS can be summarized by the integrity of the measurement and the observability of the filter. Assuming the integrity of the GPS information used as a measurement is always satisfied, the performance of the filter can eventually be analysed by the observability. The observability of the filter depends on the dynamic trajectory of the train. Because the train has a non-holonomic constraint and one-dimensional motion, the filter design and the performance analysis are carried out considering this. We analyse the observability of the filter through simulation and explain the limit of the filter and the flaw of the observability. We also analyse the reliability of the navigation system and present additional research directions.

1 INTRODUCTION

Accurate navigation information of a train must be provided to control the train. Generally, trains are detected on the basis of a fixed infrastructure installed along the railway to track the position of trains. If the train detection fails, the location information of the train will not be provided. When new railways are built, therefore, new infrastructure must be installed at a high cost, infrastructure faults must be detected, and infrastructure must be maintained periodically. To overcome this realistic problem, in this paper, the application of the inertial navigation system (INS) /global positioning system (GPS) integrated navigation system to trains is considered. That is, a system that estimates the position of a train with only INS and GPS receiver mounted on a train without any additional infrastructure is dealt (Presti and Sabina, 2018).

The INS uses an inertial measurement unit (IMU) consisting of 3-axis accelerometers and gyros. If the sensor outputs are processed by the INS algorithm, the position, velocity, and attitude information is provided at a frequency faster than 100Hz. However, INS errors gradually increas over time due to the errors included in the sensor outputs, the initial attitude errors, the non-commutative errors occurring in the digital computer, and so on. This is a bigger problem if the low-level IMU is used (Titterton and Weston, 1997; Cho, 2014). This error can be estimated and compensated using the information provided by the GPS receiver. This system is called the INS/GPS integrated navigation system (Liu et al., 2010; Miller and Campbell, 2012; Cho, 2014). INS and GPS are integrated using a Kalman filter. Integration types can be divided into loosely coupling, tightly coupling, and ultra-tightly coupling depending on the type of measurement. In this paper, we discuss loosely coupling method considering simplicity of implementation.

The performance of INS/GPS integrated navigation system depends on the specifications of the IMU and GPS receiver and the performance of the integration filter. In this paper, it is assumed that 2.0

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Cho, S., Kang, C. and Shin, K.
Reliability Analysis of the Kalman Filter for INS/GPS Integrated Navigation System Applied to Train.
DOI: 10.5220/0007832602370242
In *Proceedings of the 16th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2019)*, pages 237-242
ISBN: 978-989-758-380-3

deg/hr grade MEMS type IMU and integrity guaranteed GPS receiver are used. Therefore, the performance of INS/ GPS integrated navigation can be predicted through the performance analysis of the integration filter.

The filter used in INS/GPS integration can be selected from nonlinear Kalman filters such as extended Kallman filter (EKF), unscented Kalman filter (UKF), cubature Kalman filter (CKF), etc. considering the nonlinearity of INS (Reif et al., 1990; Julier et al., 2000; Cho et al., 2017). In this paper, EKF is used in consideration of small initial attitude error. The performance of the filter can be explained by the observability. The observability of the INS/ GPS integration filter is determined by the trajectory of the trains. In this paper, considering the trajectory of trains, we analyze the observability of the filter through the estimation error covariance of the filter state variables (Cho et al., 2007). Since the train has a non-holonomic constraint and moves in one dimension, the filter is designed considering this. In this case, the observability is analyzed through covariance analysis. Also, the limitation of the filter and the loopholes in the analysis of the observability are discussed by analyzing the covariance of the filter and the root mean squared errors (RMSE) based on Monte-Carlo simulation results.

All this analysis is done through simulation. The information provided in this paper can be used to analyze the reliability of the location information provided by the INS/GPS integrated navigation system installed on trains without infrastructure.

2 INS/GPS INTEGRATION FILTER AND ANALYSIS

In this section, we design a loosely coupled INS/GPS integration filter based on EKF using the position and velocity information of GPS as a measurement, and analyse the navigation performance when it is applied to the train.

2.1 INS Algorithm

For INS, initial alignment is performed in the stopped state first. In the case of train application, the final navigation information can be stored in memory when the train operation is finished. This information can be used at the next operation. Therefore, INS can be performed without initial alignment.

INS calculates the navigation information in synchronization with the output period of the sensor data output from the IMU. First, the attitude is updated based on the following quaternion differential equation using the gyro output (Titterton and Weston, 1997).

$$\dot{q} = \frac{1}{2} q * \left(\omega_{ib}^{b} - C_{n}^{b} (\omega_{ie}^{n} + \omega_{en}^{n}) \right) \tag{1}$$

where q is the quaternion, $*$ is the quaternion product, ω_{ib}^{b} is the gyro output, ω_{ie}^{n} is the Earth rotation angular velocity, and ω_{en}^{n} is the rotation angular velocity of the navigation frame. And C_{n}^{b} is the direction cosine matrix (DCM) from the navigation frame to the body frame, and the quaternion, DCM, and Euler angles are mutually convertible.

The velocity is updated using the updated attitude information and accelerometer output, and the position is update by integrating the velocity.

$$\dot{V}^{n} = C_{b}^{n} f^{b} - (2\omega_{ie}^{n} + \omega_{en}^{n}) \times V^{n} + g^{n} \tag{2}$$

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} v_N / (R_m + h) \\ v_E / (R_t + h)\cos L \\ -v_D \end{bmatrix} \tag{3}$$

where $V^{n} = [v_N \quad v_E \quad v_D]^{T}$ is the velocity on the navigation frame, f^{b} is the accelerometer output, g^{n} is the gravitational acceleration vector, $P = [L \quad l \quad h]^{T}$ is the position (latitude, longitude, and altitude), and R_m and R_t are the Earth radii calculated in the latitude and longitude directions, respectively.

2.2 Filter Design

The integration filter is driven in synchronization with the GPS signal output. First, the error state variables are set as follows for EKF-based filter design.

$$\delta X = [\delta P \mid \delta V^{n} \mid \phi^{n} \mid \nabla^{b} \mid \varepsilon^{b}]^{T} \tag{4}$$

where δP is the position error, δV^{n} is the velocity error, ϕ^{n} is the attitude error shown in the navigation frame, and ∇^{b} and ε^{b} are the accelerometer bias and gyro bias, respectively.

The discrete-time system and measurement equations of the filter are expressed as follows:

$$\begin{aligned} \delta X_{k+1} &= \Phi_k \delta X_k + w_k, \ w \sim N(0,Q) \\ Z_k &= H_k \delta X_k + v_k, \ v \sim N(0,R) \end{aligned} \tag{5}$$

The systems matrix is derived by applying the linear perturbation method to (1) to (3) (Titterton and Weston, 1997). In case of using the position and velocity information of GPS, the measurement matrix can be denoted as follows:

$$H_k = [I_{6\times 6} \mid 0_{6\times 9}] \tag{6}$$

Table 1: Specification of the sensors used in simulation.

Sensor	Error	Spec.
Accelerometer	Bias Repeatability	3.0 *mg*
	Noise	0.05 $mg/\sqrt{hr}$
Gyro	Bias Repeatability	2.0 deg/hr
	Noise	0.07 $deg/\sqrt{hr}$
GPS	Position Noise	5.0 *m* (1σ)
	Velocity Noise	0.1 m/s (1σ)

2.3 Simulation Analysis

Simulation is performed to analyse the navigation performance considering the application of the INS/GPS integrated navigation system to the train. The specifications of the IMU and GPS used in the simulation are set as shown in Table 1. The model of IMU is Northrop Grumman's MEMS type μIMU-I, and the model of GPS receiver is u-blox.

The maximum speed of the train is 200 *km/hr* and the moving trajectory is set as shown in Fig. 1. The train accelerates first and then runs at constant

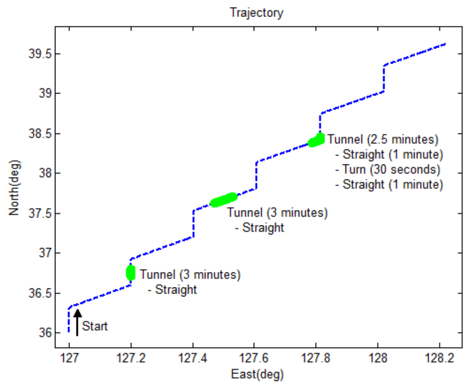

Figure 1: Moving trajectory.

speed and the total driving time is 2 hours. After 10 minutes of straight running, rotate 30.0 degrees at a rotational angular velocity of 1.0 *deg/s* for 30 seconds. And repeat this. There are three tunnels, which cannot receive GPS signals. The Monte-Carlo simulation was performed on this trajectory, and the number of simulations was 10 times. Fig. 2 shows the RMSE and standard deviation (SD) obtained through the covariance matrix of the filter. In the section where the GPS signal can be received, it is seen that the RMSE and SD are similar, and it is confirmed that the Monte-Carlo simulation is performed normally. It can be seen that accelerometer bias and gyro bias can be estimated according to time and train rotation. However, when the GPS signal cannot be received, the RMSE values of the position and velocity of the horizontal axis increase greatly. Therefore, it can be

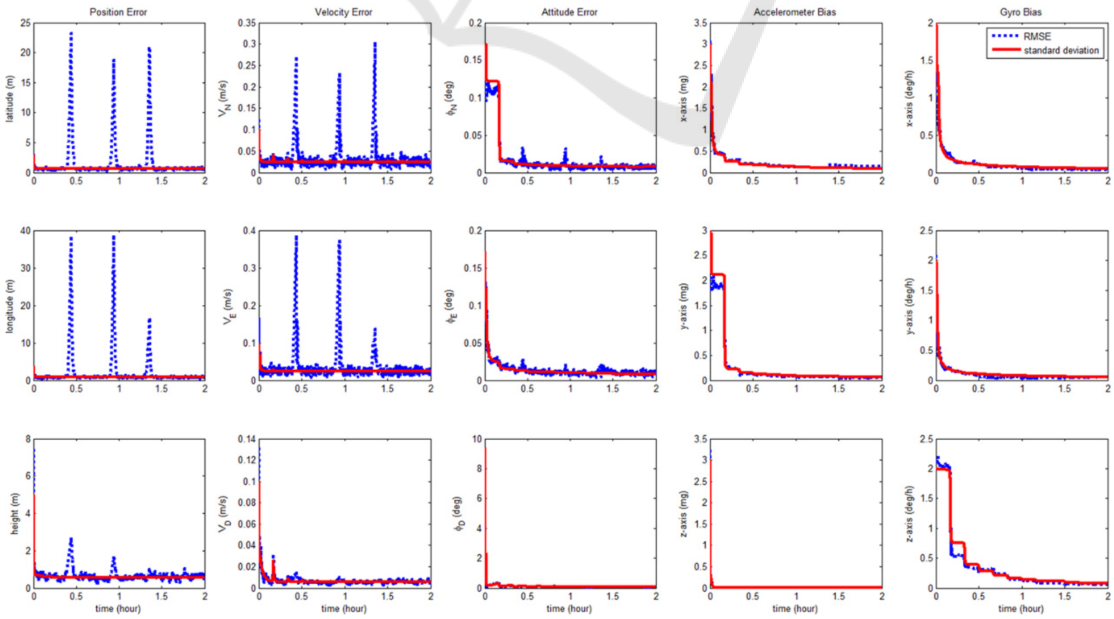

Figure 2: RMSE and standard deviation in INS/GPS.

confirmed that there is a limitation of INS/GPS integrated navigation in the section where the tunnel exists.

3 NON-HOLONOMIC CONSTRAINT FILTER AND ANALYSIS

3.1 Filter Design

Trains have six degree of freedom movement. However, it only moves on railways with non-holonomic constraints. In other words, there is velocity only in the longitudinal direction on the body frame, and the velocities are zero in the lateral and vertical directions. Using this information as an additional measurement, a new measurement matrix for the filter can be constructed. To do this, the velocity on the body frame is calculated.

$$V^b = C_n^b V^n \tag{7}$$

The linear perturbation method is applied to this equation.

$$V^b + \delta V^b = C_n^b \left(I + (\phi^n \times) \right)(V^n + \delta V^n) \tag{8}$$

Therefore, the velocity error model on the y and z axis of the body frame can be approximated as follows:

$$\begin{bmatrix} \delta v_y \\ \delta v_z \end{bmatrix} \cong C_n^b(y:z)\delta V^n - C_n^b(y:z)(V^n \times)\phi^n \tag{9}$$

where

$$C_n^b(y:z) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} C_n^b \tag{10}$$

Based on this equation, the following two measurement matrices can be derived.

$$H_k = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times6} \\ 0_{2\times3} & C_n^b(y:z) & -C_n^b(y:z)(V^n\times) & 0_{2\times6} \end{bmatrix} \tag{11}$$

$$H_k = \begin{bmatrix} 0_{2\times3} & C_n^b(y:z) & -C_n^b(y:z)(V^n\times) & 0_{2\times6} \end{bmatrix} \tag{12}$$

We define two filters as follows.

- INS/GPS with NHC-1: The measurement matrix (16) is used where GPS signals can be received and the measurement matrix (12) is used where GPS signals cannot be received.
- INS/GPS with NHC-2: (11) is used in an open space where GPS signals can be received, and (12) is used in places where GPS signals cannot be received like a tunnel.

3.2 Simulation Analysis

Three types of INS/GPS integrated navigation were performed: INS/GPS, INS/GPS with NHC-1, and INS/GPS with NHC-2. Fig. 3 shows the RMSE for each type as a result of Monte-Carlo simulations.

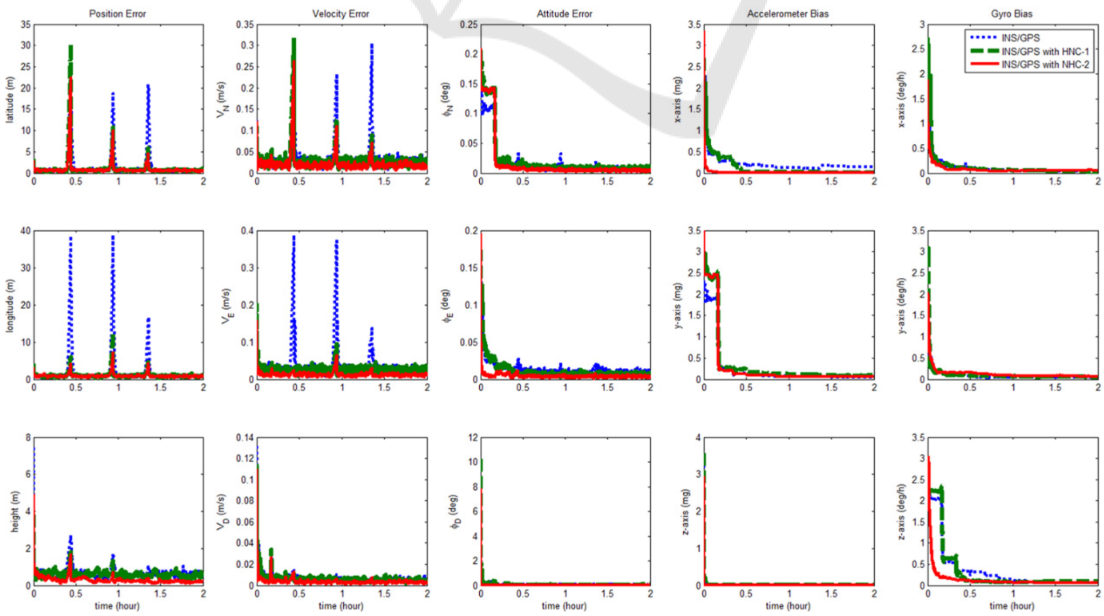

Figure 3: RMSEs according to the filters.

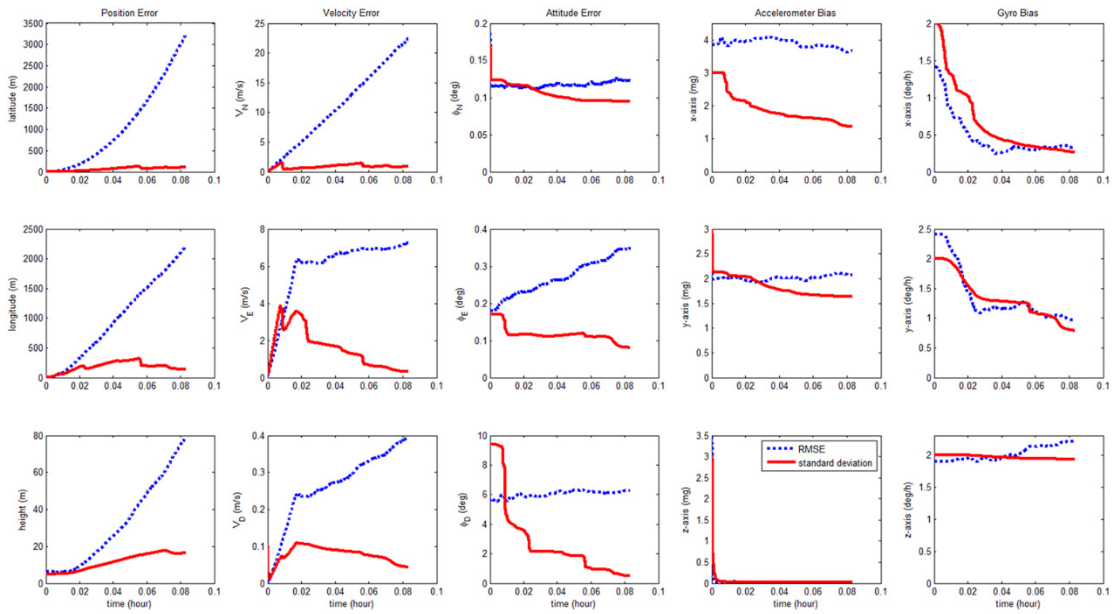

Figure 4: RMSE and standard deviation in INS with NHC.

First, in the case of NHC-2, the x-axis accelerometer bias is estimated faster than other methods. As a result, the E-axis attitude error is also estimated quickly. And the z-axis gyro bias estimation speed is increased. Based on this effect, the horizontal-axis velocity and position error estimation performance is improved. In particular, the increase in the error is also reduced in tunnels because the filter can be driven through the NHC.

In case of NHC-1, the sensor bias estimation performance is not significantly different from INS/GPS. However, overall performance is better than INS/GPS because it has the advantage of driving the filter even in tunnels.

What should be considered here is that until the train meets the tunnel after departure, the errors are reduced by driving the filter sufficiently through the GPS signal. Therefore, the errors in tunnels relatively small.

To further analyse this problem, the NHC filter was driven without a GPS signal for the first 5 minutes after departure and the results are shown in Fig. 4. Comparing RMSE and SD, we can see that the accuracy of the actual estimate of the gyro bias is within the covariance of the filter, which is the performance index of the filter. In the case of the horizontal-axis accelerometer bias, however, the two values seem to be somewhat different. The horizontal –axis attitude errors also has the same tendency. A problem lies in the D-axis attitude error. In the NHC filter, the covariance of the D-axis attitude error falls to a value close to zero. In other words, it is ensured that the observability is obtained. However, when starting with a relatively large initial attitude estimation error, the estimates do not converge to true values. This is a crucial factor, which results in velocity and position estimation errors. This is a hoe in the NHC filter.

3.3 Reliability Analysis

The performance of the filter can be expressed in terms of observability. In a time-varying system such as INS/GPS, the observability of the filter can be confirmed by covariance analysis rather than rank-based analysis. It is determined that the state variable having the covariance converging to a value close to 0 according to the filter update is estimable. However, the important point is that estimability does not always mean convergence to true values. As shown in Fig. 4, the state variables converge but may converge to the wrong values.

Therefore, it is difficult to judge the performance of the filter only by the observability. In this case, it is necessary to set various environments and judge the performance of the filter based on the RMSE analysis through Monte-Carlo simulations. It is important to carry out simulations using various trajectories for trains. In addition, reliability analysis of GPS signal and INS/GPS performance analysis according to IMU specification must be done.

4 CONCLUSION

In this paper, we designed the navigation filters and analysed its performance when applying the INS/GPS integrated navigation system to the train. NHC filters for NHC-based trains were designed and Monte-Carlo simulations were performed to performance analysis. Tunnels were set on a trajectory and simulations were performed. Here, the case of performing pure INS in the tunnels and the case of driving the NHC filter were considered, respectively. By analysing covariance and RMSE together, it was verified that the use of NHC filter regardless of GPS signal availability is good for filter performance. And the loophole in covariance analysis was point out. Based on this, it can be concluded that it is necessary to analyse covariance and RMSE together based on the Monte-Carlo simulations performed under various environment settings to analyse the reliability of the filter, therefore.

ACKNOWLEDGEMENTS

This research was supported by a grant from R&D Program (PK1904A1) of the Korea Railroad Research Institute, Republic of Korea.

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