Investigation on Viscoelastic Fluid Behavior by Modifying Deviatoric Stress Tensor

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Abstract: One of the most challenging issues is to simulate and visualize liquid behavior, especially viscoelastic fluid, which has both characteristics of viscosity and elasticity. Although Newtonian fluid, which is represented by water, is generally analyzed with the governing equations, which are Navier-Stokes equation and equation of continuity. However, viscoelastic behavior is so complex that there is no established governing equation such as Newton's equation of motion and Navier-Stokes equation. Some researchers employ Finite Element Method and others develop their own point based methods. In addition, there is a characteristic feature called "Spinnability" in viscoelastic fluid. That is, viscoelastic fluid is stretched so long and shows sudden shrink when the stretched fluid is broken. Then, we have been performing this spinnability simulation based on Cauchy's equation of motion by modifying the stress term in constitutive equation. In this paper, we report the simulation results on viscoelastic fluid behavior for four kinds of deviatoric stress tensors constructing Cauchy's equation of motion: only viscosity, only elasticity, linear combination of viscosity and elasticity, and complex modulus of elasticity.

1 INTRODUCTION

Nowadays, computer graphics can present almost everything from artificial objects to natural phenomena such as buildings, cars, robots, lighting, snowstorm, aurora and so on. Among these things, one of the most difficult and challenging tasks is to simulate and visualize liquid behavior since its deforms so dynamically yet the boundary of it is very clear, while solid body does not deform so largely, and the boundary of gas is not clear. In the liquid simulation, Newtonian fluid represented by water is comparatively simple to be simulated since the relation between shearing stress and velocity gradient is linear.

In the world, there are many non-Newtonian fluids, and one of them is called "viscoelastic fluid", which has both features of viscosity and elasticity, and the relation between shearing stress and velocity gradient is not linear. The behavior is so complicated that many researchers have tried to simulate and visualize the behavior of viscoelastic fluid. However, there is no established governing equation for viscoelastic fluid. Some researchers employ FEM (Finite Element Method) and SM (Spring Mass) model to visualize the deformation, and others are developing various kinds of methods, which are based on point method. Moreover, there are some works based on Navier-Stokes equation, which is the established governing equation of fluid. These researches are trying to simulate the viscoelastic fluid behavior by adding viscous and elastic stress terms as the external force.

In addition, viscoelastic fluid has the characteristic feature called "spinnability". Viscoelasic fluid can be stretched so long as if it is a string and shrinks suddenly when it is broken. This is "spinnability", and there are some works on spinnability; however, almost all of them do not quote the word of spinnability and just simulate that viscoelastic fluid is stretched so long like a string.

Then, we have been trying to simulate this characteristic feature of viscoelastic fluid on the basis of Cauchy's equation of motion, which is the basic equation of motion for continuum. In Cauchy's equation of motion, there is a term of "deviatoric stress", and we think that this stress term should have both characteristics of viscosity and elasticity, and have been trying to simulate the behavior and to measure the stretched length by replacing this term with a linear combination of viscous and elastic terms.

In this paper, we re-investigate the liner combination of the deviatoric stress term, and consider only viscosity term or only elasticity term for the simula-

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tion. In addition, we consider complex modulus of elasticity, which can handle both viscosity and elasticity in one term with complex number. This paper describes the method of the simulations with four different deviatoric stress terms, and shows the results of the simulations.

2 RELATED WORKS

Related to works on Newtonian fluid represented by water, (Mould and Yang, 1997) surveyed about water modeling and showed that there were two types of studies: hydrodynamic theory based research and experimental based works. (Iglesias, 2004) also investigated papers published during the 1980s and 1990s. In addition, (Darles et al., 2011) published a survey on computer graphics based ocean simulation and rendering. According to the survey, there are two types of researches. One is a physics based method using Navier-Stokes equation, and the other is an empirical law based oceanographic work.

(Hinsinger et al., 2002), (Cui et al., 2004) and (Dupuy and Bruneton, 2012) employed mesh modeling to represent ocean waves, irregular long crest waves, and vast ocean scene, respectively since ocean waves basically have continuous surfaces. However, as the drawbacks, re-meshing is required everytime the topology changes. On the other hand, (Müller et al., 2003) and (Kipfer and Westermann, 2006) utilized SPH (Smoothed Particle Hydrodynamics), which is one of particle methods, for solving Navier-Stokes equation with surface tension. They presented water pouring into a glass and river flowing from a rock, respectively.

In relation to the simulation of fluid behavior, two kinds of methods are usually used. One is an Eulerian (grid based) method and the other is a Lagrangian (particle based) method. (Chentanez and Müller, 2011) and (Nishino et al., 2012) utilized Eulerian simulation methods to propose an optimized grid for GPU (Graphics Processing Unit) and to represent freezing ice with air bubbles, respectively. On the other hand, (Foster and Fedkiw, 2001) employed semi-Lagrangian method to represent viscous liquids interacting with 3D objects and (Busaryev et al., 2012) proposed a particle based algorithm to represent bubbles with Voronoi diagram.

Moreover, there is a hybrid method of Eulerian and Lagrangian methods. (Hong et al., 2008) and (Chentanez and Müller, 2010) used the hybrid method to represent bubbles in water and spray or splash, respectively. (Miller, 1989) also proposed a method to animate viscous fluid with collision between particles and obstacles, and (Sims, 1990) developed a parallel particle rendering system that allows to treat particles with different shapes, sizes, colors and transparencies. In addition, (Greenwood and House, 2004) and (Geiger et al., 2006) proposed particle level-set algorithms to visualize various kinds of bubble shapes and fine splash particles, respectively. In addition, (Kim et al., 2007) adopted a level set method to present bubbles in liquid and gas interaction, and (Losasso et al., 2008) also used a particle level set method for dense liquid volume and utilized a particle method for diffused regions.

As mentioned above, there are some basic types for the simulation of Newtonian fluid such as water. One is a Eulerian (grid based) method and the other is a Lagrangian (particle based) method, although mesh modeling is included in grid based modeling and particle level set method is part of particle based methods. Of course, some researches are based on experimental observations, and others obey Navier-stokes equation. However, experimental observations depend on the environment when the data are obtained. Then, Navier-Stokes equation should be used for the stable and precise fluid simulation as Newton's equation of motion is used for solid mechanics.

Now, for the previous works on the simulation of viscoelastic fluid, (Tamura et al., 2005) used spring-mass system to visualize an egg dropping on the floor. In addition, (Bargteil et al., 2007) and (Wojtan and Turk, 2008) employed Finite Element methods to represent large plastic deformation of solid materials and to simulate the complex elastic and plastic behavior of viscoelastic materials, respectively. These methods are in the group of Eulerian methods. On the other hand, (Clavet et al., 2005) employed a particle based method for viscoelastic fluid simulation; however, the method also added springs to accomplish elastic and non-linear plastic effects. (Ram et al., 2015) proposed a new method called “Material Point Method” to simulate foams and sponges, and employed Oldroyd-B model to preserve plastic volume. (Barreiro et al., 2017) developed a constrained dynamics solver by extending position based dynamics method to represent whipped cream and strawberry syrup. These methods are some kinds of particle methods or hybrid methods. They do not obey Navier-Stokes equation as the governing equation although some works employ conservation of mass and momentum.

(Goktekin et al., 2004) used a grid based method with level set to animate viscoelastic fluids such as mucus, liquid soap and so on. On the other hand, (Chang et al., 2009) utilized a particle based method called SPH (Smoothed Particle Hydrodynamics) to visualize melting and flowing viscoelastic fluid. Although these studies used different methods, they both employed Navier-Stokes equation as the governing equation of viscoelastic fluid because viscoleastic fluid also has characteristics of fluid and Navier-Stokes equation is the established governing equation to analyze fluid behavior. In addition, they both added viscosity and elasticity terms to Navier-Stokes equation as the external term.

Navier-stokes equation is the established governing equation of fluid, and viscoelastic fluid has both characteristic features of viscosity and elasticity. Viscosity is a feature of fluid, while elasticity is another feature of elastic body that is a kind of continuum. Then, the governing equation of viscoelastic fluid should be Cauchy's equation of motion, which is the governing equation of continuum as if Newton's equation of motion is the governing equation for solid mechanics. Cauchy's equation of motion has a term of "deviatoric stress", which should have both characteristic of viscosity and elasticity. Then, (Mukai et al., 2010) and (Mukai et al., 2018) have tried to simulate viscoelastic fluid behavior by introducing a linear combination of viscosity and elasticity terms for the deviatoric term of Cauchy's equation of motion, and to evaluate the length of the viscoelastic fluid when it is stretched.

In this paper, we re-investigate the linear combination between viscous and elastic terms of the deviatoric stress, and evaluate the behavior of viscoelastic fluid by considering three new terms: only viscosity term, only elasticity term and complex modulus of elasticity that can handle both features in one term.

3 METHOD

We employ MPS (Moving Particle Semi-implicit) method for the simulations. MPS is one of particle methods and was developed for incompressible fluid. In the research, the governing equations are equation of continuity and Cauchy's equation of motion described as follows.
Equation of continuity:

$$\frac{d\rho}{dt} = 0 \tag{1}$$

Cauchy's equation of motion with surface tension:

$$\rho\frac{d\boldsymbol{v}}{dt} = \nabla\cdot\boldsymbol{\sigma} + \boldsymbol{g} + \boldsymbol{f} = (-\nabla p\boldsymbol{I} + \nabla\cdot\boldsymbol{\tau}) + \boldsymbol{g} + \boldsymbol{f} \tag{2}$$

where, ρ is density, t is time, $\boldsymbol{v}$ is velocity, $\boldsymbol{\sigma}$ is stress tensor, $\boldsymbol{g}$ is the gravity, $\boldsymbol{f}$ is external force, p is pressure, $\boldsymbol{I}$ is unit matrix, and $\boldsymbol{\tau}$ is deviatoric stress.

The target is viscoelastic fluid that has both features of viscosity and elasticity. Then, $\boldsymbol{\tau}$ should have both features and can be written as the following.

$$\boldsymbol{\tau} = \alpha\boldsymbol{\tau}_v + (1-\alpha)\boldsymbol{\tau}_e \tag{3}$$

$$\boldsymbol{\tau}_v = 2\eta_0\boldsymbol{D} \tag{4}$$

$$\boldsymbol{D} = \frac{1}{2}(\boldsymbol{L}+\boldsymbol{L}^t), \qquad \boldsymbol{L} = \nabla\boldsymbol{V} \tag{5}$$

$$\boldsymbol{V} = (\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}) \tag{6}$$

$$\nabla\boldsymbol{V} = \begin{bmatrix} \frac{\partial\boldsymbol{u}}{\partial x} & \frac{\partial\boldsymbol{u}}{\partial y} & \frac{\partial\boldsymbol{u}}{\partial z} \\ \frac{\partial\boldsymbol{v}}{\partial x} & \frac{\partial\boldsymbol{v}}{\partial y} & \frac{\partial\boldsymbol{v}}{\partial z} \\ \frac{\partial\boldsymbol{w}}{\partial x} & \frac{\partial\boldsymbol{w}}{\partial y} & \frac{\partial\boldsymbol{w}}{\partial z} \end{bmatrix} \tag{7}$$

$$\boldsymbol{\tau}_e = 2\mu\boldsymbol{\varepsilon}, \qquad \mu = \frac{E}{2(1+\nu)} \tag{8}$$

where, $\boldsymbol{\tau}_v$ and $\boldsymbol{\tau}_e$ are viscosity and elasticity terms of deviatoric stress, respectively, and α is a linear combination coefficient. η_0 is zero shear viscosity, $\boldsymbol{V}$ is velocity, $\boldsymbol{\varepsilon}$ is distortion tensor, E is Young's modulus and ν is Poisson's ratio.

α is defined as follows by approximating the figure showing the relation between shearing velocity and viscosity researched by (Isogai, 2008) and by normalizing so that the maximum value becomes 1.0. In Eq.(4), there is an assumption that the volume of the viscoelactic fluid does not change.

$$\alpha = 4.99\times10^{-4}\,\dot{\gamma}^2 - 3.07\times10^{-2}\,\dot{\gamma} + 1 \tag{9}$$

where, $\dot{\gamma}$ is shearing velocity, which is calculated as follows.

$$\dot{\gamma} = \sqrt{2II_D} \tag{10}$$

$$II_D = \frac{1}{2}(D_{ii}D_{jj} - D_{ij}D_{ji}) \tag{11}$$

$$= \frac{1}{2}(D_{11}D_{22} + D_{22}D_{33} + D_{33}D_{11} - D_{12}D_{21} - D_{23}D_{32} - D_{31}D_{13}) \tag{12}$$

where, D_{ij} is the ith row jth column element of $\boldsymbol{D}$, and II_D is called the second invariant of deformation velocity tensor.

Here, as the external force $\boldsymbol{f}$ in Eq.(2), we consider the surface tension. In fact, there are two types of models for the surface tension. One is CSF (Continuum Surface Force) model, which is calculated with the shape of the free surface, and the other is a potential model, which is calculated by considering potential energy of each particle. CSF model is popular and is widely used; however, it is not stable in case there are not enough free surface particles. Then, we adopt a potential model proposed by (Koshizuka, 2014) to calculate the surface tension.

The following potential force $P(r_{ij})$ works between particles i and j, and the surface tension $\boldsymbol{f}$ is calculated with the potential coefficient C_p as follows.

$$P(r_{ij}) = \begin{cases} \frac{1}{3}C_p(r_{ij} - \frac{3}{2}l_0 + \frac{1}{2}r_e)(r_{ij} - r_e)^2 & (r_{ij} \leq r_e) \\ 0 & otherwise \end{cases} \tag{13}$$

$$\boldsymbol{f} = \Sigma_{j \neq i}\{C_p(r_{ij} - l_0)(r_{ij} - r_e)(\boldsymbol{r}_j - \boldsymbol{r}_i)/r_{ij}\} \tag{14}$$

where, $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ are the positions of particles i and j, r_{ij} is the distance between particles i and j, l_0 is initial distance between particles, and r_e is radius of influence. C_p is calculated from the wetting angle between the fluid and the solid contacted by the fluid.

In addition, we consider complex modulus of elasticity that handles both viscosity and elasticity in one term with complex number. Complex modulus of elasticity $E^*(\omega)$ is written as follows.

$$E^*(\omega) = \left(\frac{1}{E} + \frac{1}{i_u\omega\eta}\right)^{-1} \tag{15}$$

where, E is Young's modulus, i_u is imaginary unit, ω is angular velocity, η is viscosity coefficient.

Then, the deviatoric stress is calculated with the complex modulus of elasticity as the following.

$$\begin{aligned} \boldsymbol{\tau} &= E^*(\omega)\varepsilon & (16) \\ &= \left(\frac{E + i_u\omega\eta}{i_u E\omega\eta}\right)^{-1}\varepsilon & (17) \\ &= \left(\frac{i_u E\omega\eta}{E + i_u\omega\eta}\right)\varepsilon & (18) \\ &= \frac{E(\omega\eta)^2 + i_u E^2\omega\eta}{E^2 + (\omega\eta)^2}\varepsilon & (19) \\ |\boldsymbol{\tau}| &= \varepsilon\sqrt{\left\{\frac{E(\omega\eta)^2}{(\omega\eta)^2 + E^2}\right\}^2 + \left\{\frac{E^2(\omega\eta)}{(\omega\eta)^2 + E^2}\right\}^2} & (20) \end{aligned}$$

Here, η in the above equations is also calculated as follows by approximating the figure showing the relation between shearing velocity and viscosity (Isogai, 2008); however it is not normalized as in Eq.(9) because this value is not a coefficient.

$$\eta = 1.80 \times 10^{-4}\,\dot{\gamma}^2 - 1.11 \times 10^{-2}\,\dot{\gamma} + 3.62 \times 10^{-1} \tag{21}$$

4 SIMULATION

Table 1 and 2 show the specification of the PC and the parameters used for the simulation, respectively.

Fig.1 shows the initial position of the viscoelastic fluid, which sticks to two solid bodies on both the upper and the lower sides. The initial numbers of particles are about 30,000, 5,000 and 12,000 for the viscoelastic fluid, the upper and the lower solid bodies, respectively. In the simulation, the upper solid body is pulled up, while the lower one is fixed. Then, the viscoelastic fluid is stretched. The velocity to pull the upper solid body increases according to sinusoidal curve, and reaches the pulling velocity at 100 steps.

Table 1: PC specification.

OS	Windows 7 Professional 64 bit
CPU	Intel Core i5-2500K 3.3GHz
Main memory	4GB
GPU	GeForce GTX 570 with 4GB memory

Table 2: Parameters used for the simulation.

Parameter		Value	Unit
Density	ρ	1.16×10^3	kg/m^3
Young's modulus	E	1.05×10^3	Pa
Poisson's ratio	ν	0.5	
Zero shear viscosity	η_0	28	$Pa \cdot s$
Initial distance of particles (= Particle radius)	l_0	3.0×10^{-3}	m
Pulling velocity	$\boldsymbol{v}$	0.18	m/s
Time step	$\triangle t$	0.10×10^{-3}	s
Wetting angle	θ	30	degree
Angular velocity	ω	$\pi/4$	rad/s

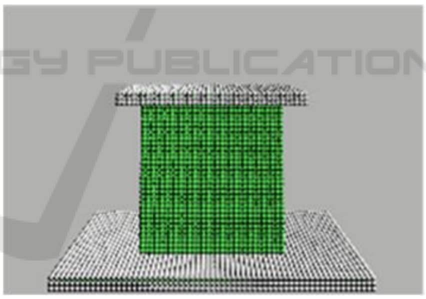

Figure 1: Initial position of the viscoelastic fluid.

There are four kinds of simulations for different deviatoric stresses $\boldsymbol{\tau}$ shown in Eq.(2).

Simulations for Different Deviatoric Stresses

Sim. 1 linear combination of viscosity and elasticity based on Eq.(3), where α changes according to Eq.(9).

Sim. 2 only viscosity term, where α is always 1.0 for Eq.(3).

Sim. 3 only elasticity term, where α is always 0.0 for Eq.(3).

Sim. 4 complex modulus of elasticity based on Eq.(20).

5 RESULTS

Figs.2 and 3 show the results of the simulations. Initial states are the same for all simulations; however, the viscoelastic fluid was stretched as time went for the simulations of 1 and 2, while particles were dispersed for the simulations of 3 and 4. In the simulations of 1 and 2, particles have the viscosity feature although the viscosity coefficient in the simulation 1 changes according to the shearing velocity, while the viscosity is constant in the simulation 2. Viscosity has the feature that puts particles together, and then it seems that this feature prevented for particles to be dispersed. The comparison between the simulations 1 and 2 shows that the particles in the simulation 1 are a little bit dispersed than that in the simulation 2. On the other hand, the simulation 3 has only elasticity feature and does not have any viscosity feature. Then, the particles in the simulation 3 were so dispersed. For the simulation 4, complex modulus of elasticity was used. This term can handle both viscosity and elasticity in one term; however, the value of E is larger than that of η so that the effect of viscosity becomes almost zero. This is the reason why the particles in the simulation 4 were also dispersed.

Figure 2: Simulation results on Sim. 1 and 2.

6 CONCLUSIONS

For the simulation of Newtonian fluid represented by water, there is the established governing equation called Navier-Stokes equation, and many studies have been performed; however, there is no established governing equation for the simulation of non-Newtonian fluid. Then, there are a lot of researches based on many ideas. On the other hand, Cauchy's equation of motion is the established governing equation for continuum. Then, we have been trying to simulate spinnability of viscoelastic fluid by modifying the deviatoric stress, and in this paper, we have tried four types of simulations: linear combination of viscosity and elasticity, viscosity only, elasticity only, and complex modulus of elasticity.

As the results of the simulations, the particles were stretched without being dispersed for the simulation with linear combination of viscosity and elasticity, and for the simulation with only viscosity. This is because viscosity has the feature to put particles together. Then, if viscosity term has some effect for the stretching, the fluid is stretched without being dispersed. On the other hand, if elasticity term has more effect for the stretching, the fluid is dispersed.

Then, we have to improve the viscosity term not to disperse the particles; however, if viscosity has large effect, the fluid is stretched without being broken. Even if it is broken, it does not shrink suddenly and does not show the characteristic feature of spinnability. In addition, we did not consider the volume change of the viscoelastic fluid in this paper; however, the volume might have changed because there are not so many particles in the middle of the fluid when it is stretched. In the future, we have to investigate how the spinnablity, which is a characteristic feature of viscoelastic fluid, can be visualized by con-

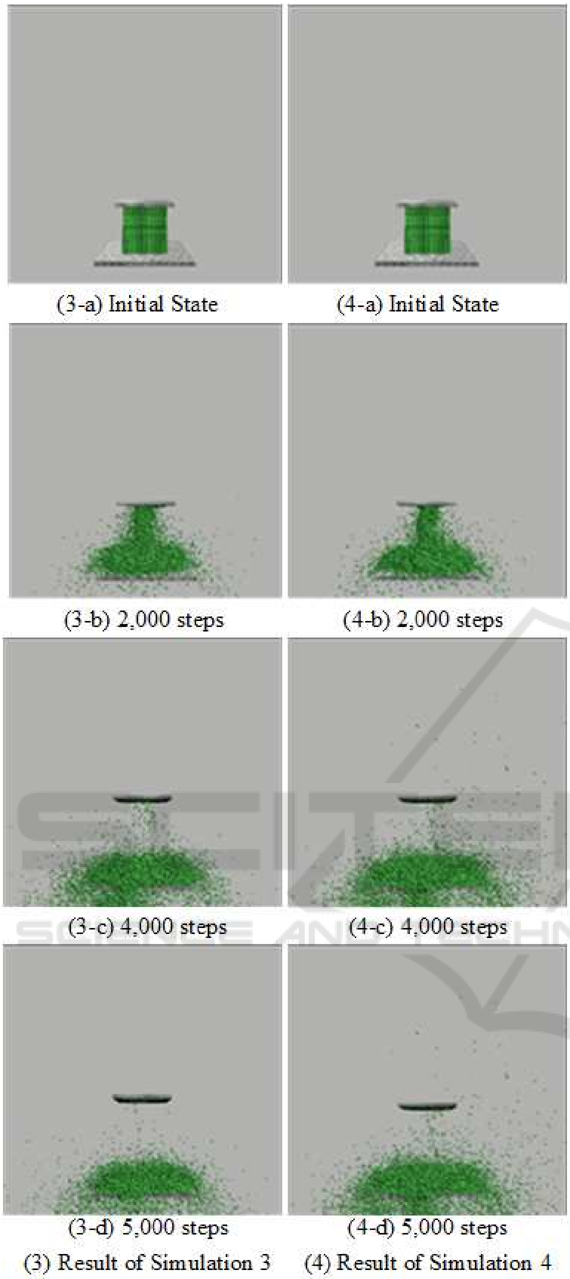

Figure 3: Simulation results on Sim. 3 and 4.

sidering the volume change of the fluid.

REFERENCES

Bargteil, A. W., Wojtan, C., Hodgins, J. K., and Turk, G. (2007). A finite element method for animating large viscoplastic flow. *ACM Transactions on Graphics*, 26(3):Article No.16.

Barreiro, H., García-Fermández, I., Alduán, I., and Otaduy, M. A. (2017). Conformation constraints for efficient viscoelastic fluid simulation. *ACM Transactions on Graphics*, 36(6):Article No.221.

Busaryev, O., Dy, T. K., Wang, H., and Ren, Z. (2012). Animating bubble interactions in a liquid foam. *ACM Transactions on Graphics*, 31(4):63:1–63:8.

Chang, Y., Bao, K., Liu, Y., Zhu, J., and Wu, E. (2009). A particle-based method for viscoelastic fluids animation. In *Proceedings of the 16th ACM Symposiumon Virtual Reality Software and Technology*, pages 463–468.

Chentanez, N. and Müller, M. (2010). Real-time simulation of large bodies of water with small scale details. *Proceedings of the 2010 ACM SIGGRAPH/Eurographics symposium on computer animation*, pages 197–206.

Chentanez, N. and Müller, M. (2011). Real-time eulerian water simulation using a restricted tall cell grid. *ACM Transactions on Graphics*, 30(4):82:1–82:10.

Clavet, S., Beaudoin, P., and Poulin, P. (2005). Particle-based viscoelastic fluid simulation. In *Proceedings of the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 219–228.

Cui, X., Yi-cheng, J., and Xiu-wen, L. (2004). Real-time ocean wave in multi-channel marine simulator. *Proceedings of the 2004 ACM SIGGRAPH international conference on virtual reality continuum and its application in industry*, pages 332–335.

Darles, E., Crespin, B., Ghazanfarpour, D., and Gonzato, J. (2011). A survey of ocean simulation and rendering techniques in computer graphics. *Computer Graphics Forum*, 30(1):43–60.

Dupuy, J. and Bruneton, E. (2012). Real-time animation and rendering of ocean whitecaps. *Proceedings of the SIGGRAPH Asia 2012 Technical Briefs*, page Article No.15.

Foster, N. and Fedkiw, R. (2001). Practical animation of liquids. *Proceedings of the ACM SIGGRAPH 2001*, pages 23–30.

Geiger, W., Leo, M., Rasmussen, N., Losasso, F., and Fedkiw, R. (2006). So real it'll make you wet. *Proceedings of the 2006 ACM SIGGRAPH Sketches, Article No.20*.

Goktekin, T. G., Bargteil, A. W., and O'Brien, J. F. (2004). A method for animating viscoelastic fluids. *ACM Transactions on Graphics*, 23(3):463–468.

Greenwood, S. T. and House, D. H. (2004). Better with bubbles: Enhancing the visual realism of simulated fluid. *Proceedings of the 2004 ACM SIGGRAPH/Eurographics symposium on computer animation*, pages 287–296.

Hinsinger, D., Neyret, F., and Cani, M. P. (2002). Interactive animation of ocean waves. *Proceedings of the 2002 ACM SIGGRAPH/Eurographics symposium on computer animation*, pages 116–166.

Hong, J. M., Lee, H. Y., Yoon, J. C., and Kim, C. H. (2008). Bubbles alive. *ACM Transactions on Graphics*, 27(3):48:1–48:4.

Iglesias, A. (2004). Computer graphics for water modeling and rendering: A survey. *Future Generation Computer Systems*, 20(8):1355–1374.

Isogai, A. (2008). *Advanced Technologies of Cellulose Utilization*. Maruzen, Tokyo.

Kim, B., Liu, Y., Llamas, I., Jiao, X., and Rossignac, J. (2007). Simulation of bubbles in foam with the volume control method. *ACM Transactions on Graphics*, 26(3):98:1–98:10.

Kipfer, P. and Westermann, R. (2006). Realistic and interactive simulation of rivers. *Proceedings of Graphics Interface 2006*, pages 41–48.

Koshizuka, S. (2014). *Introduction to Particle Method*. Maruzen, Tokyo.

Losasso, F., Talton, J. O., Kwatra, N., and Fedkiw, R. (2008). Two-way coupled sph and particle level set fluid simulation. *IEEE Trans. on Visualization and Computer Graphics*, 14(4):797–804.

Miller, G. (1989). Globular dynamics: A connected particle system for animating viscous fluids. *Computers & Graphics*, 13(3):305–309.

Mould, D. and Yang, Y. (1997). Modeling water for computer graphics. *Computers & Graphics*, 21(6):801–814.

Müller, M., Charypsr, D., and Gross, M. (2003). Particle-based fluid simulation for interactive applications. *Proceedings of the 2003 ACM SIGGRAPH/Eurographics symposium on computer animation*, pages 154–159.

Mukai, N., Ito, K., Nakagawa, M., and Kosugi, M. (2010). Spinnability simulation of viscoelastic fluid. In *Proceedings of ACM SIGGRAPH 2010 Posters*, page Article No.18.

Mukai, N., Nishikawa, T., and Chang, Y. (2018). Evaluation of stretched thread lengths in spinnability. In *Proceedings of ACM SIGGRAPH 2018 Posters*, page Article No.62.

Nishino, T., Iwasaki, K., Dobashi, Y., and Nishita, T. (2012). Visual simulation of freezing ice with air bubbles. *Proceedings of the SIGGRAPH Asia 2012 Technical Briefs*, page Article No.1.

Ram, D., Gast, T., Jiang, C., Schroeder, C., Sromakhin, A., Teran, J., and Kavehpour, P. (2015). A material point method for viscoelastic fluids, foams and sponges. In *Proceedings of the 14th ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 157–163.

Sims, K. (1990). Particle animation and rendering using data parallel computation. *Proceedings of the ACM SIGGRAPH 90*, 24(4):405–413.

Tamura, N., Tsumura, N., Nakaguchi, T., and Miyak, Y. (2005). Spring-bead animation of viscoelastic materials. In *Proceedings of the 2005 ACM SIGGRAPH Sketches*, page Article No.64.

Wojtan, C. and Turk, G. (2008). Fast viscoelastic behavior with thin features. *ACM Transactions on Graphics*, 27(3):Article No.47.