A Semantic Approach for Handling Probabilistic Knowledge of Fuzzy Ontologies

Ishak Riali, Messaouda Fareh and Hafida Bouarfa
LRDSI Laboratory, Faculty of Science, University Blida 1, Soumaa BP 270, Blida, Algeria

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Abstract: Today, there is a critical need to develop new solutions that enable classical ontologies to deal with uncertain knowledge, which is inherently attached to the most of the real world's problems. For that need, several solutions have been proposed; one of them is based on fuzzy logic. Fuzzy ontologies were proposed as candidate solutions based on fuzzy logic. Indeed, they propose a formal representation and reason in presence of vague and imprecise knowledge in classical ontologies. Despite their indubitable success, they cannot handle the probabilistic knowledge, which is presented in most of the real world's applications. To address this problem, this paper proposes a new solution based on fuzzy Bayesian networks, which aims at enhancing the expressivity of the fuzzy ontologies to handle probabilistic knowledge and benefits from the highlights of the fuzzy Bayesian networks to provide a fuzzy probabilistic reasoning based on vague knowledge stored in fuzzy ontologies.

1 INTRODUCTION

Ontologies provide an important key factor for representing, reasoning, sharing, and reusing the knowledge of domains. They provide the key to machine-processbale data and permit to use it in more efficient way. They have been successfully used in order to represent and reasoning with the knowledge in several areas such semantic Web, artificial intelligence, etc. Despite the great success of the classical ontologies, they were deemed inappropriate and fail when handling uncertain knowledge that can appear inherently in most of the real world's problems. Indeed, the uncertainty is a ubiquitous aspect of most real world problems. It exists in almost every aspect of ontology engineering(Pan et al., 2005).

To cope with uncertainty in ontologies, great deals of efforts have been carried out. Fuzzy ontologies were proposed in this context as promising solution, they benefit from the power of fuzzy logic to cope with vagueness in classical ontologies (Zadeh, 1975). Fuzzy ontologies handle effectively the linguistic vagueness, which is attached inherently to the most of the natural language. Moreover, they permit to make lots of reasoning tasks based on vague concepts and assertions in fuzzy ontologies using some reasoners such as(Bobillo et al., 2012)(Bobillo and Straccia, 2016). Despite the fact that fuzzy ontologies were successfully applied in many challenging tasks, they suffer from their inability to handle the probabilistic knowledge.

Besides, Fuzzy Bayesian Networks (FBNs) have been proposed as an extension of the classical ones to cope with vagueness that may be attached to the random variables. They can deal with probabilistic and vague knowledge at the same time and execute probabilistic reasoning based on fuzzy evidences. However, they cannot represent the probabilistic knowledge in a semantic formal way that is treatable by machine.

In fact, uncertainty is present in most of the real world's problems, and it becomes a serious challenge to model and resonate with it in ontologies. Especially, when semantic Web agents are dealing with open data such in Internet, where information is combined from different sources and is often incomplete, vague, etc. Indeed, imperfection is a common property of the most real world applications. Therefore, it is important to develop hybrid models that allow handling these uncertainties simultaneously in ontologies. Thus, this paper proposes a novel solution that tackles this problem, it combines fuzzy ontologies with fuzzy bayesian networks in order to benefit from the advantages of the both, where fuzzy ontologies allow handling vagueness in ontologies and fuzzy Bayesian networks permit to cope with probabilistic knowledge and make probabilistic reasoning

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over vague knowledge.

The rest of this paper is organized as follows. In Section 2, we briefly introduce fuzzy ontologies then we give a summary about Fuzzy Bayesian Networks. In Section 3, we present our proposed approach. In section 4 we discuss the related work and section 5 concludes this paper and talks about future works.

2 BACKGROUND

In this section, some background knowledge is presented including fuzzy ontologies and fuzzy Bayesian networks.

2.1 Fuzzy Ontologies

Several definitions have been presented in the scientific literature for what is a fuzzy ontology, but there is no common definition. Indeed, many researchers have proposed to extend ontologies with fuzzy logic. The most shared characteristic of these propositions is that they extend the components of the classical ontologies in order to allow them representing vague and imprecise knowledge. Refereeing to (Bobillo and Straccia, 2011) , a fuzzy ontology is an ontology that uses fuzzy logic to represent the world in a natural representation of imprecise and vague knowledge. Therefore, eases reasoning over it.

A fuzzy ontology consists of a set of components:

- Fuzzy Concepts: are the concepts that have no clear boundaries, they can be used to represent fuzzy sets of individuals and allow representing a gradual belonging of individuals to their classes;
- Fuzzy Roles: are divided into two classes, Fuzzy object properties that are fuzzy binary relations among concepts or individuals. They permit to assign some degree to the association among the instances of concepts (crisp or fuzzy). And fuzzy data properties that permit to assign a degree to the association among a data value and an instance of fuzzy concept;
- Fuzzy Data Types: fuzzy data types are used to fuzzify attributes values of concepts, such as the range of data properties. They can also be attached to a concept instance;
- Fuzzy Modifiers: are generally used in order to change the interpretation of fuzzy concepts and fuzzy datatypes.

2.2 Fuzzy Bayesian Networks

Fuzzy Bayesian networks proposed as hybrid models that enhance the classical ones in order to cope with vague and imprecise knowledge that may be attached to the random variables. They combine the capabilities of Bayesian networks and fuzzy logic to benefit from the advantages of the two models.

FBN is a Bayesian network extension that consists of two types of nodes; crisps nodes are the nodes whose meaning is precise, and the fuzzy nodes are the nodes whose meaning is vague. Many experiments have proved the benefits of FBNs in wide diversity domains and applications. Nonetheless, there is no unified model which defines fuzzy Bayesian networks. In fact, many solutions have been proposed in the scientific literature in order to incorporate the membership degrees with probabilities, which are:

- First, the weighted method (Mrad et al., 2012): the main idea of this method is to extend the different rules used in the Bayesian networks by associating a membership degree value to each rule as weight; then the fuzzy Bayesian rules can be defined to support the fuzzy Bayesian inference in FBN model.
- Second, fuzzy distribution method (Ryhajlo et al., 2013): in this method, the fuzzy membership degrees will be integrated directly in the probability distribution, where in the first step the fuzzy membership degree must be represented like a probability distribution, then this later will be integrated in the probability distribution in order to generate the Fuzzy Probability Distribution. The Fuzzy Probability Distribution is a hybrid representation of the fuzzy membership degree and the probability distribution.
- Finally, virtual evidence method (Li, 2009)(Peng et al., 2010): this method consists to add new nodes in the bayesian network called virtual evidence nodes. Then incorporate the fuzzy evidence in these latters, the fuzzy evidence will be represented as a probability distribution in the conditional probability table (CPT) of each virtual node. Then the fuzzy probabilistic inference can be done by setting the evidences on the virtual nodes and applying classical algorithms of inference.

3 THE PROPOSED APROACH

The study of the literature has showed that existing works, when dealing with uncertainty residing in ontologies are not enough expressive and suffer from the inability to deal with rich-uncertainty domains; where vague, imprecise and probabilistic knowledge appears simultaneously. Indeed, probabilistic knowledge and fuzzy knowledge have been treated separately in ontologies and unfortunately, there is no efforts have been devoted to cope with the two simultaneously in the ontologies. In this paper, we consider this problem; we propose a probabilistic extension of fuzzy ontologies, which provides a strong mean foundation to enable the fuzzy ontologies to cope with the probabilistic knowledge based on fuzzy Bayesian networks. It permits to handle both probabilistic knowledge and fuzzy knowledge at the same time in ontologies.

The main idea behind our proposal is to deal with the probabilistic knowledge in fuzzy ontologies by creating an FBN based on an existing fuzzy ontology in order to capture the probabilistic knowledge involved within. In addition, incorporate this probabilistic knowledge in the fuzzy ontology using a high-level ontology in order to represent it in a formal way. The proposed solution is described by a general process that eases creating probabilistic fuzzy ontologies. It refers to the phases illustrated in the Fig1.

The input of the proposed process is an fuzzy ontology (F1) that must confront probabilistic knowledge which is involved in its elements.

3.1 Phase 1 (Specification of Requirements)

This phase aims to specify explicitly the purpose that must be achieved by constructing a probabilistic fuzzy ontology. In this step, the ontology engineer must justify the needs and the requirements behind the creation of ProbFuzzOnto ontology for the studied domain. This step serves to identify the probabilistic components of the fuzzy ontology that are relevant for the considered problem and must be modeled in the next steps.Thus, the main objectives of this step are:

- To specify the objectives and the needs for creating probabilistic fuzzy ontology.
- To identify the probabilistic components of the fuzzy ontology that are relevant to the studied domain.

3.2 Phase 2 (Probabilistic Knowledge Design)

This phase is the core of our proposed solution, it aims to model the probabilistic knowledge involved in the fuzzy ontology using an Fuzzy Bayesian Network. This is done by constructing an FBN based on the requirements specified in the previous step, which captures the probabilistic knowledge of the studied domain. Therefore, the probabilistic knowledge design goes through three steps:

3.2.1 Constructing Structure of Bayesian Networks.

In this step ontology engineer has to represent the probabilistic components of the fuzzy ontology selected in the previous phase in an FBN. The main tasks of this step are:

- Node creation. Each selected concept will be represented by a node in the FBN.
- The ontology engineer must define the states of each created node.
- Arcs creation. Each probabilistic role will be represented by an arc in the FBN.

3.2.2 Estimate the Conditional Probability Tables (CPTs)

The aim of this step is to estimate the conditional probability tables of the Bayesian network (parameters). The most efficient solution is to use machine-learning algorithms in order to estimate the probability distributions from available data. However, in practical cases, the data are incomplete and usually contain missing values, where some random variables are observed only partially or never.

For this reason, we propose the use of the most applied method of estimating parameters with incomplete data. It is based on the iterative Expectation-Maximization (EM) algorithm proposed by Dempster in (Dempster et al., 1977). The EM algorithm alternates between executing an expectation (E) step, which creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters, and maximization (M) step, which computes parameters maximizing the expected log-likelihood found on the E step. These parameter-estimates are then used to determine the distribution of the latent variables in the next E step.

3.2.3 Fuzzify the States of each Fuzzy Node

The objective of this step is to attribute to each state of each fuzzy node a membership function, which is already defined in the fuzzy data types represented in the fuzzy ontology.

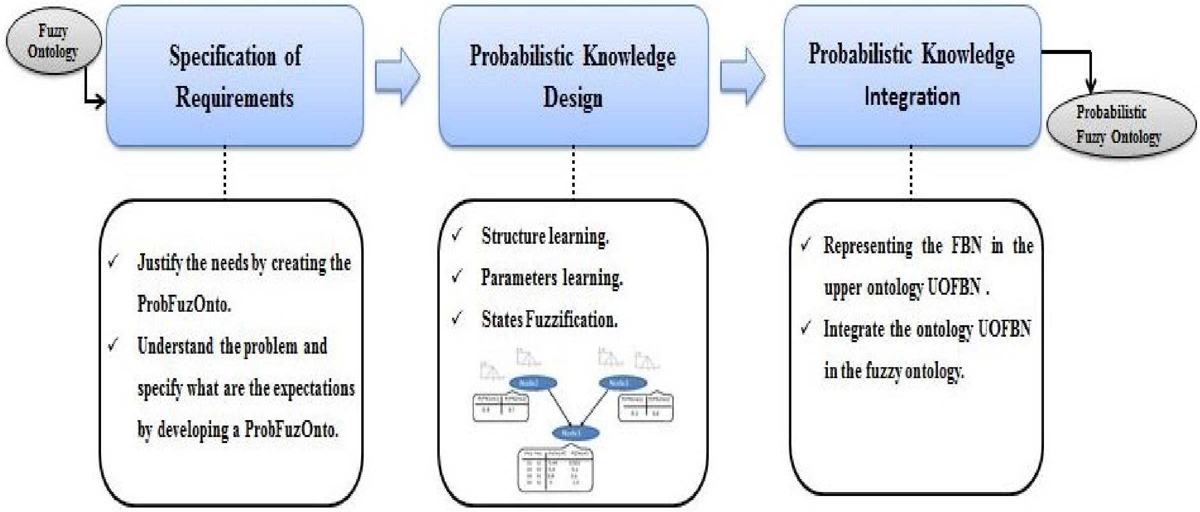

Figure 1: The proposed process for constructing Probabilistic Fuzzy Ontologies.

3.3 Phase 3 (Probabilistic Knowledge Integration)

Despite the probabilistic knowledge involved in the fuzzy ontology has been modeled and represented in the previous steps in an Fuzzy Bayesian network model. Nevertheless, this knowledge needs to be represented and formalized in a way that it can be read and processable by machine. In other words, it needs to be represented semantically. For this purpose, we propose a high level ontology of FBN that permits to represent semantically the fuzzy Bayesian networks and then incorporate it in the fuzzy ontology.

Besides, the fuzzy Bayesian network created in the previous step will be represented and described in this high level ontology, which facilitates the integration of the probabilistic knowledge in the fuzzy ontology.

Definition 1. Fuzzy Bayesian Networks. An Fuzzy Bayesian Network is defined by 3-tuple $F = \prec G, M, P \succ$
, where:

1. **G= (N, A)** is an acyclic graph.
   (a) N = $\{n_1, n_2, n_3, ..., n_m\}$ is the set of the nodes that constitute G. Where, N= $\Phi \cup \Psi$.
   (b) $\Phi = \{fn_1, fn_2, fn_3, ..., fn_k\}$ is the list of the fuzzy nodes of F with size of k, with $\Phi \subseteq$ N.
   (c) $\Psi = \{cn_1, cn_2, cn_3, ..., cn_j\}$ is the list of the crisp nodes of F with size of j, with $\Psi \subseteq$ N.
   (d) A = $\{(n_i, n_j)/n_i \in N and n_j \in N\}$ is a set of arcs, each $(n_i, n_j) \in$ A represents a dependency link between n_i and n_j (i.e, n_i influences directly on n_j).
2. M = $\{m_1, m_2, m_3, ..., m_l\}$ is a finite set of the membership functions used to fuzzify fuzzy nodes.
3. P is the probability distribution of F.

Furthermore, each node $n_i \in N$ has a set of finite states S=$\{s_1, s_2, s_3, ..., s_t\}$, when $n_i \in \Phi$. i.e., is fuzzy, per each $s_i \in S$, a membership function $m_i \in M$ will be associated to s_i in order to fuzzify this last.

Definition 2. Membership Function. Let Ω be the universe of the discourse of s.
Then, a membership function **m** is defined in Ω as follows:
m: $\Omega \rightarrow [0, 1]$.
Per each $x \in \Omega$, the value **m(x)** is called the degree of membership of x in s .

Definition 3. Mapping Function. A mapping function Γ is a function that maps to each state s, its membership function m.
Formally, Γ: S x M → False, True.

Property 1. Let s ∈ S and m ∈ M. Then Γ(s, m) = True if only if m is the membership function used to fuzzify s.

Each nodes $n_i \in N$ has a probabilistic distribution, when a node is a root node (without parents) its probabilistic distribution called Prior Probability represented in prior table, when a node has parents its probabilistic distribution is named Conditional Probabilistic Table represented by a set of conditional probabilities.

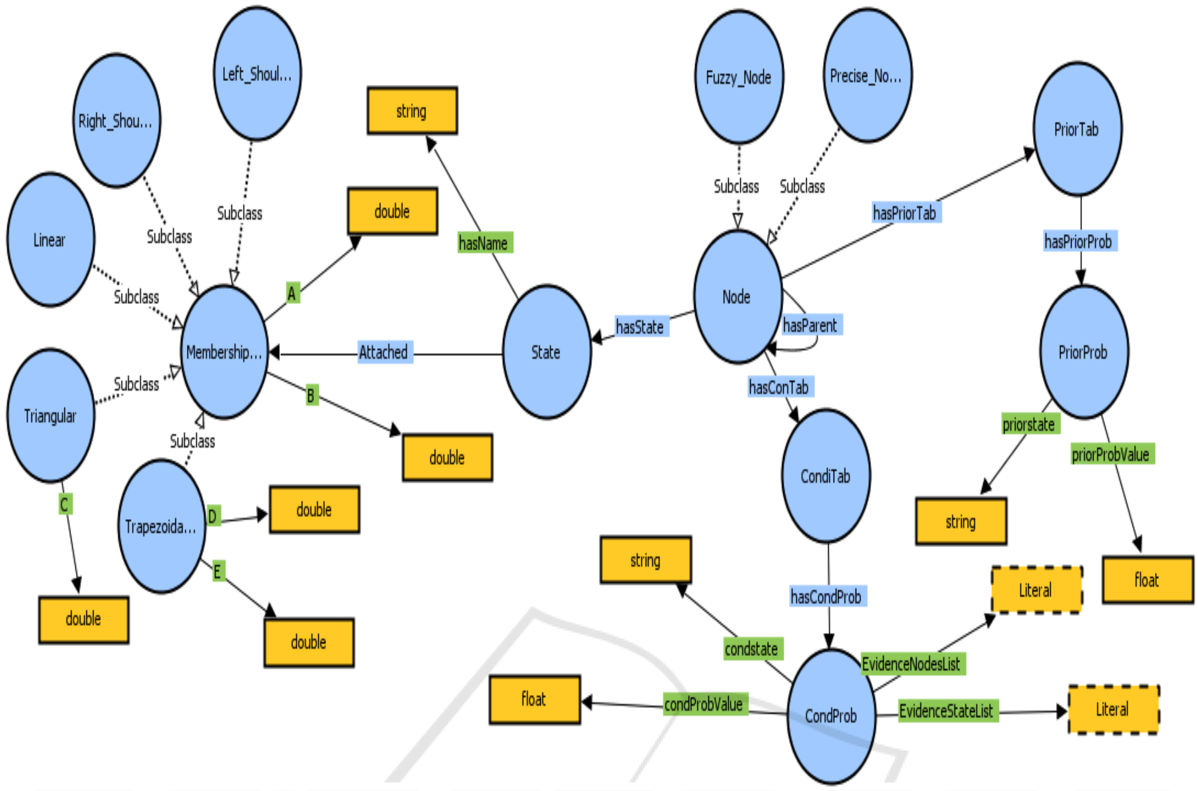

Figure 2: The upper ontology UOFBN.

Definition 4. A Prior Table. A prior table of a node $n \in N$ is a set of couples (s, P) that link to each state s_i with its prior probability $Prior_i$. Formally:

PriorTab (n) = $\{Prior_1, \ldots, Prior_a\}$, with a is the number of states of the node n.
$Prior_i = (s_i, P_i)$, where,
$s_i \in S$: represents a state of the node n.
$P_i \in [0, 1]$: represents the prior value.

Definition 5. A Conditional Probabilistic Table. A Conditional Probabilistic Table of a node n $\in$ N is a set of conditional probabilities, it is defined as:
CondTab (n) = $\{Cond_i, \ldots, Cond_b\}$, with b is the number of states of the node n.
$Cond_i = (s_i, ne_i, se_i, P_i)$, with :
$s_i \in S$: represents the state of the node n.
ne_i : represents the nodes of the evidence (observed nodes).
se_i : represents the states of the evidences nodes .
$P_i \in [0, 1]$: represents the conditional probability value.

The *UOFBN* allows representing the semantic of FBN and it is illustrated in the Fig. 2. It consists of a set of class and properties:

1. **Node Class.** The nodes of the FBN will be represented by individuals of the class Node. In fact, the nodes in an FBN can be crisp or fuzzy, for this purpose, we created two subclasses of the class Node in order to make a distinction between the two types. The Fuzzy Node class, which includes all the fuzzy nodes of the FBN, each fuzzy node $fn_i \in \Phi$, will be represented by an individual If_i of the class FuzzyNode. The Crisp Node class, which includes all the crisp nodes of the FBN, each crisp node $cn_j \in \Psi$ will be represented by an individual Ic_j of the class CrispNode. Moreover, the arcs between nodes can be represented by the object property "hasParent".

2. **State Class.** The elements of this class are the states related to each node n_i, thus, per each state $s_i \in S$, an individual I_s will be created to represent that state. Moreover, I_s will be linked with the node n_i via the object property "hasState". Moreover, when n_i is fuzzy, all its states will be fuzzified using membership functions, for this reason. An object property named "Attached" is defined, which links each state with its membership function m_i.

3. **Membership Function Class.** This class represents the membership functions, it contains five sub-classes (left-shoulder, right-shoulder, triangular, linear function, and trapezoidal), each membership function sub-class has a set of arguments represented by data properties (A, B, C, D, E). Thus, to each membership function m_i, an individual of the class ”Membership function” Im_i will be created to represent that function.

4. The probabilistic table of each node will be represented using the classes ”CondTab” and ”PriorTab”.
   (a) **CondTab.** For each root node, an instance ”pr” of this class will be created to represent its probabilistic table. Moreover, a set of prior probabilities P= $\{Prior_1,...,Prior_n\}$ will be created as instances of the class “PriorProb”. These instances will be linked with their table ”pr” via the object property ”hasPriorTab”.
   (b) **PriorTab.** For each node with parents, an instance ”cpt” of this class will be created to represent its probabilistic table. Moreover, a set of conditional probabilities C= $\{Cond_i,....,Cond_n\}$ will be created as instances of the class ”CondiTab”. These instances will be linked with their conditional table ”cpt” via the object property ”hasCondTab”.

At the end of the proposed process, the fuzzy ontology F1 will be augmented and enriched with probabilistic knowledge; it can represent both fuzzy knowledge and probabilistic one and make a probabilistic reasoning based on fuzzy evidences stored in the ontology.

4 RELATED WORK AND DISCUSSION

Many solutions have been provided in the scientific literature to cope with uncertainty in ontologies; in this section we investigate the most relevant works.

Authors in (Yang and Calmet, 2005), proposed an extension of the ontology Web language (OWL), which is named Bayes OWL; it allows a set of rules for translating OWL ontology to a Bayesian network. It also provides a method for incorporating available probability constraints when constructing the Bayesian network especially when constructing the conditional probability tables.

In (Ding et al., 2006), the authors proposed an extension called OntoBayes. It enhances knowledge representation in OWL and enables agents to act under uncertainty; the authors defined additional OWL classes that can be used to markup probabilities and dependencies in OWL. Both OntoBayes and Bayes OWL are unable to cope with the vague an imprecise knowledge.

In (Costa et al., 2008) (Carvalho et al., 2017), PROWL was proposed as an extension that enables ontologies represented in OWL to model and resonate under uncertain knowledge in complex real-world applications. It can encode probabilities distributions on the interpretation of an associated first-order theory as well as repeated structure based on the Multi-Entity Bayesian Network (MEBN) formalism (Laskey, 2008). Despite its robustness in terms of expressivity, its use remains very complicated for non-expert users of its modeling details. Moreover, it is based on MEBN. However, the MEBN community is not broad enough to consider it as a new standard for modeling uncertainty (Setiawan et al., 2015).

In (Ishak et al., 2011), an extension of ontologies based on Oriented Object Bayesian Networks (OOBNs) has been developed. In this approach, authors defined a set of mapping rules in order to construct an OOBN based on ontology. Despite the fact that this extension can cope with complex problems based on OOBNs, the vague and imprecise knowledge is not taken into account by this extension.

In (Emna et al., 2016a)(Emna et al., 2016b), a probabilistic extension of OWL 2 Meta-Model is presented which is named Probabilistic Ontology Definition Meta-Model (PODM). This is done by adding some new components such as Probabilistic Class, Probabilistic Individual, Probabilistic Data Property. . . etc. The main defect of this approach is that it cannot cope with the vague knowledge in the ontology.

Lately, in (Mohammed et al., 2016), an extension named hybrid probabilistic ontology based on the hybrid Bayesian networks is proposed. The merit of this extension is that it allows handling simultaneously distributions over discrete and continuous quantities in the ontology. Nevertheless, it cannot deal with vague and imprecise knowledge in the ontologies, neither reason with it.

Recently, a method to construct the probabilistic ontologies is developed in (Hlel et al., 2018), it is based on the classical Bayesian networks to cope with probabilistic knowledge in the classical ontologies, this method focuses on converting only the components of ontology that can support uncertainty in a BN graph(focus on instances and roles of the ontology). However, when constructing the CPTs, this method does not consider the missing data and it ig-

Approach	Input	Output	Fuzzy Knowledge	Probabilistic Knowledge	Fuzzy Probabilistic Inference	Probabilistic Inference
OntoBayes(Yang et al., 2005).	Classical ontology	Probabilistic ontology	*	✓	*	✓
BayesOWL (Ding et al., 2006)	Classical ontology	Probabilistic ontology	*	✓	*	✓
PODM (Emna et al., 2016a, Emna et al., 2016b)	Classical ontology	Probabilistic ontology	*	✓	*	✓
(Fenz, 2012	Classical ontology	Probabilistic ontology	*	✓	*	✓
(Emna et al., 2018)	Classical ontology	Probabilistic ontology	*	✓	*	✓
(Ishak, et al., 2011)	Classical ontology	Probabilistic ontology	*	✓	*	✓
HyProb-Ontology (Mohammed et al., 2016)	Classical ontology	Probabilistic ontology	*	✓	*	✓
PROWL (Costa et al., 2005; Carvalho et al., 2017),	Classical ontology	Probabilistic ontology	*	✓	*	✓
Our Approach	**Fuzzy ontology**	**Probabilistic Fuzzy ontology**	✓	✓	✓	✓

✓ : Criterion treated by the solution.
* : Criterion not treated by the solution

Figure 3: The differences between our approach and the related work.

nores vague and imprecise knowledge.

Based on the study of the related work, we can see clearly that most of the related work are based on classical Bayesian networks and their extensions to deal with probabilistic knowledge in classical ontologies. Unfortunately classical BNs fail to handle the vague knowledge that may be attached to the nodes of these networks and their inference is based on certain evidence.

Indeed, the proposed solution in this paper aims to model probabilistic knowledge in fuzzy ontologies, it combines the advantages of fuzzy ontologies and fuzzy Bayesian networks to benefits from both of them. Indeed, Fuzzy Bayesian networks allow dealing with probabilistic events based on fuzzy evidence to perform a fuzzy probabilistic inference. Nevertheless, they cannot model this knowledge in a formal way that is treatable automatically by machine.

Fuzzy ontologies on the other side allow representing and make some reasoning tasks such as individual classification based on vague knowledge.
The table 1 compares between the different proposed solutions in the literature, according to their provided mechanisms in terms of modeling and reasoning.

In the table 1:

- **The Inputs Column:** represents the input taken by each solution, it can be classical ontology or fuzzy ontology.
- **Fuzzy Knowledge Column:** it indicates if the solution treats Fuzzy Knowledge and provides alternatives in order to represent vagueness in ontologies.
- **Probabilistic Knowledge Column:** it indicates if the solution considers Probabilistic Knowledge and provides alternatives in order to represent it semantically in ontologies.
- **Probabilistic Inference Column:** it indicates if the solution provides probabilistic reasoning or not.
- **Fuzzy Probabilistic Inference Column:** it indicates if the solution provides probabilistic reasoning (Fuzzy Probabilistic inference) based on fuzzy evidences or not.
- **The Outputs Column:** Represents the results given by the solutions. It can be probabilistic ontology or probabilistic fuzzy ontology.

Indeed, most of the proposed solutions in the literature focus on exploiting the classical BNs in order to deal with the probabilistic knowledge in classical ontologies. Unfortunately, classical BNs fail to handle the vague knowledge that may be attached to the nodes of these latters.

Moreover, all the probabilistic extensions are based on extending OWL language to deal only with the probabilistic knowledge. These extensions are limited to handle just the probabilistic knowledge and do not consider the vague and imprecise one.

In the contrary, our solution aims to combine the advantages of fuzzy ontologies and FBs. Thus, the underlying key of our proposed solution is that:

- It is a probabilistic extension of fuzzy ontologies, which is expressive enough to cover the needs of most of real world's problems. So far, it allows handling and representing formally vague, imprecise and probabilistic knowledge simultaneously in ontologies.
- It introduces a process for building ProbFuzOnto based on fuzzy ontologies.
- It provides several reasoning tasks, where all the tasks of reasoning that can be applied on fuzzy ontologies are still valid in our extension.

5 CONCLUSIONS

In this paper, we introduced a new solution that aims to improve the knowledge representation and reasoning with uncertain knowledge in fuzzy ontologies. The proposed solution is described by a general process, which takes as an input a fuzzy ontology and outputs a probabilistic fuzzy ontology. The merit of our proposal is that it can represent and reason with rich-uncertainty domains, where it models the vague, imprecise and probabilistic knowledge simultaneously and combines the ontological inference with the fuzzy probabilistic inference.
As future works, we are looking to present real cases studies with our proposed approach to show its potential applicability in terms of modeling and reasoning. Moreover, we are looking to implement an interactive protégé-plugin in order to help ontology developers to follow our proposed process.

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