Stochastic Dynamic Pricing with Strategic Customers and Reference Price Effects

Rainer Schlosser
Hasso Plattner Institute, University of Potsdam, Potsdam, Germany

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Abstract: In many markets, customers strategically time their purchase by anticipating future prices and by checking offer prices multiple times. Typically, customers base their decisions on historical reference prices and their individual willingness-to-pay, which can change over time. For sellers it is challenging to derive successful pricing strategies as customers' reservation prices cannot be observed. In this paper, we present a stochastic dynamic finite horizon framework to determine optimized price adjustments for selling perishable goods in the presence of strategic customers, which (i) recur and (ii) anticipate future prices based on reference prices. We analyze the impact of different strategic behaviors on expected profits and the evolution of sales. Compared to myopic settings, we find that recurring customers lead to higher average prices, delayed sales, and increased profits. The presence of forward-looking customers has opposing but less intense effects.

1 INTRODUCTION

In electronic markets, it has become easy to adjust and to observe offer prices. Customers repeatedly check prices in order to strategically time their purchase decisions. Occasionally, they even try to anticipate future prices based on historical reference prices.

Applications can be found in a variety of contexts, particularly in the case of perishable products or when the sales horizon is limited. Prominent examples are, for instance, the sale of airline tickets, event tickets, accommodation services, fashion goods, and seasonal products.

To derive effective dynamic pricing strategies in the presence of strategic customers is an important problem in revenue management. Practical relevance is high, but the problem appears challenging. The challenge is (i) to account for returning and forward-looking customers, (ii) to include reference prices, and (iii) to derive pricing decisions with acceptable computation times.

In this paper, we study pricing strategies that take strategic customer behavior into account when updating offer prices. Existing dynamic pricing techniques cannot handle such scenarios efficiently and, hence, force practitioners to limit the scope of their strategies, e.g., by ignoring strategic behavior or by using simple heuristics. Naturally, this limits the potential quality of pricing strategies.

The inefficiency of existing techniques to account for strategic customers stems from several factors that reflect the challenges behind dynamic pricing: (i) the solution space of strategies is enormous, (ii) a customer's individual willingness-to-pay is not observable, and (iii) customers track the market to strategically *time* their buying decision.

1.1 Related Work

Selling products is a classical application of revenue management theory, cf., e.g., Talluri, van Ryzin (2004), Phillips (2005), and Yeoman, McMahon-Beattie (2011). An excellent overview about recent literature in the field of dynamic pricing is given by Chen, Chen (2015).

The analysis of the impact of strategic consumer behavior has been studied since Coase (1972). Surveys about strategic customer behavior in revenue management are proposed by Su (2007) and Goensch et al. (2013). Further, the recent survey article by Wei, Zhang (2018) provides a very detailed overview of publications studying strategic constumers in operations management.

Wei, Zhang (2018) distinguish three streams of counteracting strategic customer behavior: (i) pricing, (ii) inventory, and (iii) information. To account for strategic customers via pricing aims to minimize a customer's incentive to wait for future price drops.

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This includes strategies such as fixed price strategies, pre-announced increasing prices (price commitment strategy), or reimbursements for decreasing prices (price-matching strategy).

The second stream seeks to mitigate strategic waiting behavior by limiting the product availability in order to address a customer's concern of not being able to purchase the product in the future (cf. run-out). Similarly, sellers can also counteract strategic consumer behavior by strategically announcing (or hiding) partial inventory information to highlight the product's scarcity.

As typically observed in practice, in our model, we allow for free price adjustments. On average, however, this leads to comparatively stable price paths. These reference prices can be estimated by strategic customers, cf. Wu et al. (2015). Further models focusing on reference price effects are studied by Popescu, Wu (2007), Wu, Wu (2015), or Chenavaz, Paraschiv (2018).

Finally, the key question is how (i) reference prices, (ii) consumer's price anticipations, and (iii) a firm's pricing strategy affect each other. Further, the mutual dependencies will have to be determined by buyers and sellers based on their partially observable asymmetric (market) data. In addition, the complex interplay of their mutual beliefs is further complicated when multiple seller compete for the same market, cf., e.g., Levin et al. (2009), Liu, Zhang (2013).

1.2 Contribution

In the literature, for simplicity, mostly so-called myopic customers are considered. They simply arrive and decide; they do not return to check prices again and they do not anticipate future prices.

In our model, we consider the following two sources of strategic customer behavior. First, we allow that customers return with a certain probability in case they refuse to buy, i.e., if their willingness-to-pay (WTP) does not exceed the current offer price. To reflect planning uncertainty over time, we model a customer's future WTP as a random variable. Second, we allow a certain share of customers to anticipate future offer prices (based on the current offer price and predetermined reference prices) as well as their individual future WTP in order to check whether there is an incentive to delay their purchase decision - even if their current WTP exceeds the offer price. This allows customers to optimize their consumer surplus.

We study a finite horizon model with limited initial inventory (i.e., products cannot be reproduced or reordered). While in the literature demand is often assumed to be of a special highly stylized functional form, we allow for fairly general demand definitions. In our model, demand is characterized by randomized evolutions of individual WTP, which are not observable for the seller. To this end, demand is allowed to generally depend on time, the current offer price, and reference prices.

The main contributions of this paper are the following: We (i) present a demand model which is based on individual reservation prices, reference prices, and expected consumer surpluses, (ii) we compute optimized feedback pricing strategies, (iii) we study the impact of different strategic behaviors compared to myopic settings, and (iv) we propose a Hidden Markov version of our model.

This paper is organized as follows. In Section 2, we describe our model setup and define strategic customer behaviors. In Section 3, we present our solution approach and illustrate its results using different numerical examples. In Section 4, we study a relaxed version of our model with partially observable states. Conclusions are summarized in the final Section 5.

2 MODEL DESCRIPTION

We consider the situation in which a firm wants to sell a finite number of goods (e.g., airline tickets, event tickets, accommodation services) over a certain time frame. We assume a monopoly situation. Further, a certain ratio of customers acts strategically, i.e., they (i) repeatedly track prices and wait for acceptable offers and (ii) they are forward-looking, i.e., they compare their current consumer surplus with expected future consumer surpluses.

We assume that the time horizon T is finite. We assume that products cannot be reproduced or reordered. If a sale takes place, shipping costs c have to be paid, $c \geq 0$. A sale of one item at price a, $a \geq 0$, leads to a profit of $a - c$. Discounting is also included in the model. For the length of one period, we use the discount factor δ, $0 \leq \delta \leq 1$.

2.1 Individual Buying Behavior

We consider a discrete time model with T periods. We assume that consumers have individual (random) reservation prices denoted by R_t for periods $(t, t+1)$, $t = 0, 1, ..., T-1$. The reservation prices particularly account for a customer's planning uncertainty to be able to benefit from the product (e.g., an airline ticket or an event ticket in time T). In this context, the average planning uncertainty typically decreases over time as the time horizon T gets closer.

Further, customers may check current (ticket) prices at multiple points in time. While some customers start to check prices early (cf. economy class) other customers start to check prices shortly before the end of the sales period (cf. business class). Moreover, the evolutions of reservation prices of each individual customer are different. Figure 1 illustrates individual reservation prices R_t and average reservation prices (denoted by $\bar{R}_t$) over time. We observe that, on average, the willingness-to-pay is increasing over time. The individual paths resemble, e.g., random (sudden) changes of a customer's planning uncertainty. In other use-cases (e.g., the sale of fashion goods) reservation prices may also tend to decrease.

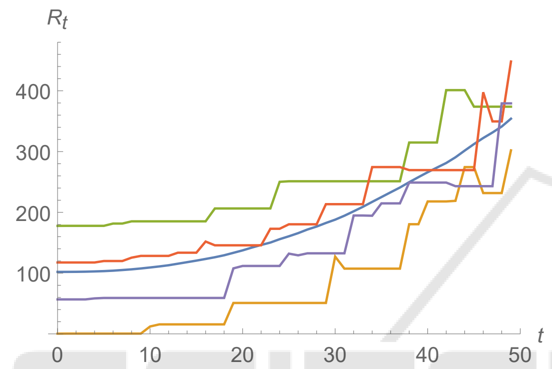

Figure 1: Examples of individual reservation prices R_t and average reservation prices $\bar{R}_t$ (smooth blue curve) over time.

We assume that forward-looking customers compare their current individual consumer surplus with their *expected* future surplus of the next period. Given a current offer price a_t and an individual reservation price R_t at time t, the current consumer surplus is $CS_t := R_t - a_t$. The expected future surplus CS_{t+1} of the next period depends on the (expected) offer price a_{t+1} and the (expected) individual reservation price R_{t+1} of the consumer at time $t+1$. Finally, a customer purchases a product at time t only if condition (i) is satisfied: The current consumer surplus is positive, i.e., $t = 0, 1, ..., T-1$,

$$CS_t \geq 0 \tag{1}$$

Further, for some consumer, also a *second* condition (ii) has to be satisfied: The expected future surplus $E(CS_{t+1})$ does not exceed CS_t plus a certain risk premium ε (e.g., mirroring a customer's risk aversion about a product's future availability), so that there is no incentive to wait for a higher consumer surplus), $t = 0, 1, ..., T-1$, $\varepsilon \geq 0$,

$$R_t - a_t \geq E(R_{t+1}|R_t) - E(a_{t+1}) - \varepsilon \tag{2}$$

Consumers are assumed to be able to estimate future prices $E(a_{t+1})$, e.g., based on average historical reference prices. In our model, we assume a known predetermined path of reference prices denoted by, $t = 0, 1, ..., T-1$,

$$a_t^{(ref)} \tag{3}$$

Based on a current offer price a_t and the reference price $a_{t+1}^{(ref)}$, cf. (3), consumers can estimate expected future prices, e.g., by, $t = 0, 1, ..., T-2$,

$$E(a_{t+1}) \approx (a_{t+1}^{(ref)} + a_t)/2$$

We assume that a certain share of the consumers are forward-looking and consider condition (2). This share is denoted by, $t = 0, 1, ..., T-1$,

$$\gamma_t \in [0, 1] \tag{4}$$

Finally, given the distribution of the expected evolution of a random customer's reservation price R_t, from (1) - (3), we obtain the average purchase probability of a random customer arriving in period $(t, t+1)$, $t = 0, 1, ..., T$, $a \geq 0$, $\varepsilon \geq 0$,

$$p_t^{(buy)}(a) := (1 - \gamma_t) \cdot P(R_t > a)$$

$$+\gamma_t \cdot P \left(\begin{array}{l} R_t > a \quad and \\ R_t - a \geq E(R_{t+1}|R_t) - \frac{a_{t+1}^{(ref)} + a}{2} - \varepsilon \end{array} \right) \tag{5}$$

The purchase probabilities (5) are characterized by (i) the consumers' mixture of reservation prices, (ii) the share of forward-looking customers, cf. (4), and (iii) the reference prices, cf. (3).

Note, we do not assume that reservation prices are observable for the seller. As in practice, we only assume arriving customers and realized sales to be observable fro sellers. The probabilities (5) can be estimated by the conversion rate of interested and buying customers for different offer prices at different time t, cf., e.g., Schlosser, Boissier (2018).

2.2 Waiting Customers

Typically, customers tend to track the market and observe prices over time. In the literature, however, customers are often assumed to be myopic, i.e., they randomly occur, observe offers, and decide whether to purchase or not. In case of no purchase they do not further track the offer.

In reality, many customers are not myopic. In our model, we consider recurring customers. In case an interested customer does not purchase, we assume that he/she checks the next period's offer with a certain probability denoted by, $t = 0, 1, ..., T-1$,

$$\eta_t \in [0, 1] \tag{6}$$

Note, this nontrivially affects the arrival process of potential customers. In the following, we distinguish between initially arriving (new) customers and waiting/recurring (old) customers.

Arriving new customers are modelled as follows. We assume arbitrary given probabilities, $t = 0,1,...,T-1$, $j = 0,1,...$,

$$p_t^{(new)}(j) \tag{7}$$

that in period $(t,t+1)$ exactly j new customers arrive, where $\sum_{j\geq 0} p_t^{(new)}(j) = 1$ for all $t = 0,1,...,T-1$.

For the time being, we assume that the number of waiting customers can be effectively determined by the selling firm as new arriving customers and old recurring customers can be observed (cf. cookies, etc.). The random number of customers that did not purchase in period $(t-1,t)$ and recur in the next period $(t,t+1)$ are denoted by K_t, $t = 0,1,...,T-1$. A list of variables and parameters is given in the Appendix, cf. Table 5.

2.3 Problem Formulation

In our model, we use sales probabilities that depend on (i) the number of arriving new customers j, (ii) the number recurring waiting customers k, and (iii) the offer price a. The individual purchase decisions, cf. (5), are based on individual (expectations of future) reservation prices and predetermined reference prices.

The random inventory level of the seller at time t is denoted by X_t, $t = 0,1,...,T$. The end of sale is the random time τ, when all of the seller's items are sold, that is $\tau := \min_{t=0,...,T}\{t : X_t = 0\} \wedge T$. As long as the seller has items left to sell, for each period $(t,t+1)$, a price a_t has to be chosen. By A we denote the set of admissible prices. For all remaining $t \geq \tau$ the firm cannot sell further items and we let $a_t := 0$.

We call strategies $(a_t)_t$ admissible if they belong to the class of Markovian feedback policies; i.e., pricing decisions $a_t \geq 0$ will depend on (i) time t, (ii) the current inventory level X_t, and (iii) the current number of waiting customers K_t.

Depending on the chosen pricing strategy $(a_t)_t$, the random accumulated profit from time/period t on (discounted on time t) amounts to, $t = 0,1,...,T$,

$$G_t := \sum_{s=t}^{T-1} \delta^{s-t} \cdot (a_s(X_s,K_s) - c) \cdot (X_{s+1} - X_s) \tag{8}$$

The objective is to determine a (Markovian) feedback pricing policy that maximizes the expected total discounted profits, $t = 0,1,...,T$,

$$E(G_t|X_t = n, K_t = k) \tag{9}$$

conditioned on the current state at time t (cf. inventory level n and waiting consumers k). An optimized policy will balance expected short-term and long-term profits by accounting for the evolution of the inventory level and the number of waiting customers.

3 COMPUTATION OF OPTIMAL PRICING STRATEGIES WITH OBSERVABLE STATES

In this section, we want to derive optimal feedback pricing strategies that incorporate the strategic customer behavior described in Section 2.

3.1 State Transition Probabilities

The state of the system to be controlled over time is described by time t, the current inventory level n, and the current number of waiting customers k. The transition dynamics can be described as follows. Given an inventory level n at time t and a demand for i items during the period $(t,t+1)$, we obtain the new inventory level $X_{t+1} := \max(n-i,0)$.

Given k waiting customers at time t and j new arriving customers, cf. (7), we have $k+j$ interested customers in period $(t,t+1)$. Assuming i buying customers, $i = 0,...,k+j$, cf. (1) - (5), we obtain $k+j-i$ customers that did not purchase an item. If m of them plan to recur in $(t+1,t+2)$ the new state, i.e., the number of waiting customers at time $t+1$ is m, $m = 0,...,k+j-i$.

Assuming k waiting customers and j new customers, the probability that exactly i items can be sold at period $(t,t+1)$ is binomial distributed, $i = 0,...,k+j$, $t = 0,1,...,T-1$, cf. (5),

$$p_t^{(demand)}(i|a,k,j) = \binom{k+j}{i} \cdot p_t^{(buy)}(a)^i \cdot \left(1 - p_t^{(buy)}(a)\right)^{k+j-i} \tag{10}$$

Assuming k waiting customers at time t, j new customers, and i customers that want to buy, the probability that m of the remaining $k+j-i$ customers return in period $(t+1,t+2)$ is also binomial distributed, $m = 0,...,k+j-i$, $t = 0,1,...,T-1$, cf. (6),

$$p_t^{(wait)}(m|k,j,i) = \binom{k+j-i}{m} \cdot \eta_t^m \cdot (1-\eta_t)^{k+j-i-m} \tag{11}$$

3.2 Solution Approach

The problem of finding the best pricing strategy can be solved using dynamic programming techniques. In this context, the so-called value function describes the best expected discounted future profits $E(G_t|n,k)$ for all possible states n and k at time t, cf. (9).

If either all items are sold or the time is up, no future profits can be made, i.e., the natural boundary conditions for the value functions V are given by, $t = 0,1,...,T-1$, $n = 0,1,...,N$, $k = 0,1,...,$

$$V_t(0,k) = 0 \quad and \quad V_T(n,k) = 0 \tag{12}$$

For the remaining states, the value function is determined by the Bellman equation, $n = 0,1,...,N$, $k = 0,1,...,M$, $t = 0,1,...,T-1$, cf. (7), (10), (11),

$$V_t(n,k) = \max_{a \in A} \Bigg\{ \sum_{\substack{j=0,1,...,J \\ i=0,1,...,j+k \\ m=0,1,...,j+k-i}} p_t^{(new)}(j) \cdot p_t^{(demand)}(i|a,j,k) \cdot p_t^{(wait)}(m|j,k,i) \cdot ((a-c) \cdot \min(i,n) + \delta \cdot V_{t+1}(\max(n-i,0),m)) \Bigg\} \tag{13}$$

Note, to obtain a bounded number of potential events, in (13) we use a maximum number of new customers J. To guarantee a limited state space, we use a maximum number M of waiting customers. Both bounds have to be chosen sufficiently large such that the optimal solution is not confined.

The nonlinear system of equations (13) can be solved recursively. The associated optimal pricing policy denoted by $a_t^*(n,k)$, $n = 0,1,...,N$, $k = 0,1,...,M$, $t = 0,1,...,T-1$, is determined by the arg max of (13), i.e.,

$$a_t^*(n,k) = \arg\max_{a \in A} \Bigg\{ \sum_{\substack{j=0,1,...,J \\ i=0,1,...,j+k \\ m=0,1,...,j+k-i}} p_t^{(new)}(j) \cdot p_t^{(demand)}(i|a,j,k) \cdot p_t^{(wait)}(m|j,k,i) \cdot ((a-c) \cdot \min(i,n) + \delta \cdot V_{t+1}(\max(n-i,0),m)) \Bigg\} \tag{14}$$

If $a_t^*(n,k)$ is not unique, we choose the smallest one.

3.3 Numerical Example

To illustrate our solution approach, we consider the following numerical example.

Example 3.1. We let $T = 50$, $N = 20$, $\delta = 1$, $c = 10$, $J = 5$, $M = 8$, $a \in A := \{10,20,...,500\}$, $\varepsilon := U(0,20)$, and $\gamma_t = \eta_t = 0.5$. Further, we use:

(i) Arrival of new customers, cf. (7), $j = 0,1,...,$

$$p_t^{(new)}(j) = \binom{J}{j} \cdot u_t^j \cdot (1-u_t)^{J-j}$$

where $u_t = 1 - e^{-|t/T-0.6|}$, $0 \le t < T$.

(ii) Individual reservation prices, cf. (5), $0 \le t < T$,

$$R_t = \begin{cases} L^{(\min)} & ,t=0 \\ if\ U(0,1) < 0.75\ then\ R_{t-1}\ else\ H_t & ,t>0 \end{cases}$$

where $L^{(\min)} := U(0,200)$ (initial reservation price), $L^{(\max)} := U(100,800)$ (upper reservation price), and $H_t := L^{(\min)} + D_t \cdot U(0.6,1.4)$ (random updates) with $D_t := E\left(L^{(\max)} - L^{(\min)}\right) \cdot (t/T)^2$ (average increase). By $U(\cdot,\cdot)$, we denote Uniform distributions.

(iii) Reference prices, cf. (3), $0 \le t < T$,

$$a_t^{(ref)} := 150 + 200 \cdot (t/T)^2$$

(iv) We use 10 000 random realization of R_t, cf. (ii), to determine the average reservation prices $\bar{R}_t := E(R_t)$ and the conditional expectations $E(R_{t+1}|R_t)$, which are the basis to derive $p_t^{(buy)}(a)$, $a \in A$, cf. (5).

Figure 2 depicts purchase probabilities $p_t^{(buy)}(a)$ in the setting of Example 3.1 for different periods t and prices a. Note, the seller cannot infer individual reservation prices from $p_t^{(buy)}$.

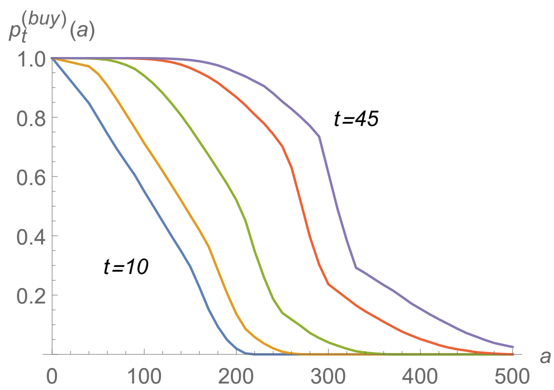

Figure 2: Purchase probabilities $p_t^{(buy)}(a)$, cf. (5), for different prices $a \le 500$ and periods $t = 10,20,30,40,45$; Example 3.1.

Table 1 illustrates the value function V_t for different inventory levels n and points in time t (for the case that the number of waiting customers is $k = 3$), cf. (13). We observe that expected future profits are (convex) decreasing in time and (concave) increasing in the number of items left to sell.

Table 1: Expected profits $V_t(n,3)$, Example 3.1.

$n \backslash t$	0	10	20	30	40	45
1	350	350	350	350	356	356
2	654	654	654	655	655	637
5	1409	1409	1410	1410	1351	1265
10	2413	2413	2414	2383	2037	1475
15	3246	3239	3219	3035	2114	1476
20	3924	3883	3763	3230	2115	1476

Table 2 shows the expected profits for different inventory levels n and different numbers of waiting customers k (for time $t = 45$). We observe that the expected future profits are increasing in the number of waiting consumers k. The impact of k is higher the larger the remaining inventory is.

Table 2: Expected profits $V_{45}(n,k)$, Example 3.1.

$n \backslash k$	0	1	2	3	4	5
1	288	318	339	356	369	380
2	492	552	599	637	667	693
5	744	953	1131	1265	1349	1408
10	762	1000	1238	1475	1711	1942
15	762	1000	1238	1476	1714	1952
20	762	1000	1238	1476	1714	1952

Table 3 illustrates optimal feedback prices a_t^*, cf. (14), for different inventory levels n and points in time t (for the case that the number of waiting customers is $k = 3$), cf. Table 1. We observe that offer prices are decreasing in n and increasing in time t. Further, prices are (slightly) increasing in k. The impact of k is higher if n is small and t is large.

Note, if n and t are small, optimal prices are chosen such that the probability to sell is basically 0, cf. Figure 2. This way, it is ensured that items are not sold too early. Due to increasing demand (cf. Figure 1) items can be sold later at higher prices.

Table 3: Optimal prices $a_t^*(n,3)$, Example 3.1.

$n \backslash t$	0	10	20	30	40	45
1	200	220	280	370	410	420
2	200	220	280	330	370	380
5	200	220	250	280	280	290
10	200	200	220	230	250	290
15	180	180	190	200	250	290
20	160	160	170	190	250	290

Remark 3.1. (Properties of expected profits)

(i) The expected profits are increasing in the inventory level.
(ii) If there is no discounting then the expected profits are increasing in time-to-go.
(iii) The expected profits are increasing in the number of waiting customers, especially if time-to-go is small and the inventory is large.

Remark 3.2. (Properties of feedback prices)

(i) The optimal prices are decreasing in the inventory level.
(ii) If demand is strongly increasing in time then the optimal prices are increasing in the time.
(iii) The optimal prices are increasing in the number of waiting customers, especially if time-to-go is small and the inventory is small.

Figure 3 illustrates evaluated average prices $E(a_t^*)$ (orange curve), which are, on average, increasing over time. Compared to $E(a_t^*)$ the average reservation prices $\bar{R}_t$ (blue curve) are smaller in the beginning and higher at the end of the sales period. Hence, sales are likely to occur late. The green curve depicts the predetermined reference prices $a_t^{(ref)}$, which are overall *consistent* with the evaluated offer prices $E(a_t^*)$.

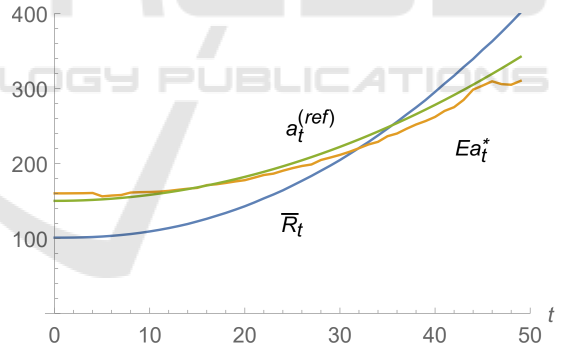

Figure 3: Average evaluated price paths $E(a_t^*)$ (orange curve), average reservation prices $\bar{R}_t$ (blue curve), and reference prices $a_t^{(ref)}$ (green curve); Example 3.1.

Remark 3.3. In order to make the model fully consistent, the resulting average prices obtained (cf. $E(a_t^*)$) can be used to define the expected reference price paths $a_t^{(ref)}$, cf. (3). This way, adaptive reconfigurations and iterated model solutions can be used to obtain converging equilibrium price paths where the evaluated optimized solution ($E(a_t^*)$) coincides with the underlying expectations of customers ($a_t^{(ref)}$).

3.4 Strategic vs. Myopic Customers

In this section, we study the impact of different setups of strategic customer behaviors, cf. (4) and (6).

Example 3.2. We assume the setting of Example 3.1. We consider different combinations of (time consistent) parameters $\eta_t = \eta$ (return probability), and $\gamma_t = \gamma$ (share of forward-looking customers) characterizing the customers' strategic behavior.

Table 4: Expected total profits $E(G_0)$ for different strategic customer behaviors; Example 3.2.

Setting	η	γ	$E(G_0)$
I	0	0	3 455
II	0	1	3 390
III	1	0	6 241
IV	1	1	4 940
V	0.5	0.5	3 924

Table 4 summarizes the expected profits for five different customer behaviors, cf. Example 3.2. As a reference, setting I represents the classical myopic customer behavior without any kind of strategic effects. In setting II, all consumers are forward-looking, cf. (4), but do not recur. In setting III, all consumers recur, cf. (6), but are not forward-looking. In setting IV, all consumers are forward-looking and steadily recur. Setting V corresponds to the mixed setup of Example 3.1. Comparing the expected profits in Table 4, we observe that the strategic behavior has a significant impact on a seller's sales results.

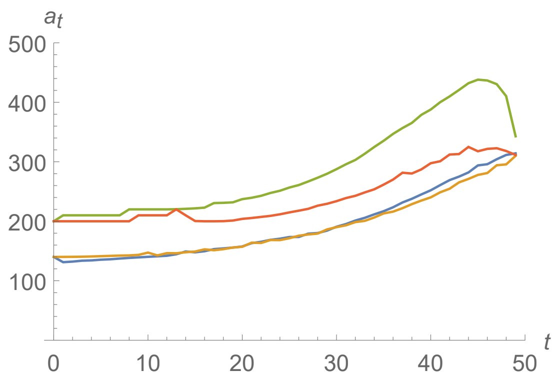

Figure 4: Expected evolution of optimal prices for different strategic behaviors (settings: I blue, II orange, III green, IV red); Example 3.2.

Figure 4 illustrates the expected price paths associated to the first four settings I - IV, cf. Table 4. While the increasing shape of the four paths is overall similar, the level of prices differs significantly. The highest (lowest) prices correspond to setting III (setting II). The moderate setting V, cf. Example 3.1 and Figure 2, corresponds to a mixture of setting I and IV, i.e. the blue and the red curve.

For setting I - IV, Figure 5 depicts the associated inventory levels over time. All four curves are of concave shape. Most of the sales are realized at the end of the time horizon, which is due to the increasing demand (i.e., reservation prices). The low return probability of setting I and II (blue and orange curve) leads to sales that occur comparatively early. Instead, in setting III and IV (green and red curve) with recurring customers sales can be realized later and the number of unsold items can be reduced.

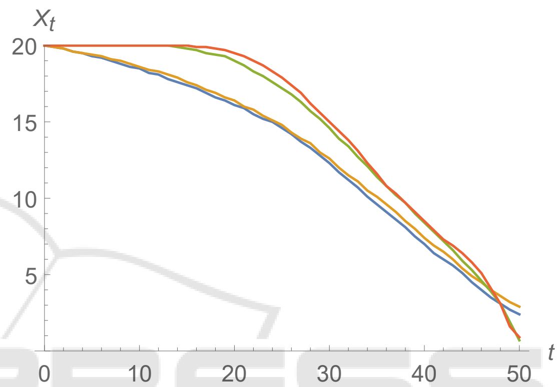

Figure 5: Expected evolution of the inventory level for different strategic behaviors (settings: I blue, II orange, III green, IV red); Example 3.2.

Further numerical experiments led to similar results. Finally, we summarize our observations in the following remark.

Remark 3.4. (Impact of strategic behavior)

(i) Compared to myopic settings a higher share (η) of patient recurring customers leads to higher profits. The (predictable) higher number of interested customers results in higher prices and delayed sales.

(ii) A higher share (γ) of forward-looking customers leads to lower profits. Customers strategical time their purchase in order to increase their consumer surplus. Further, due to suspended decision-making, sales – that would have been realized in myopic settings – may even get lost. Hence, a higher number of anticipating customers forces the seller to lower prices which, in turn, leads to earlier sales.

(iii) Finally, the two effects, i.e., the quantities η and γ, are counteractive. When comparing both effects, we observe that the impact of anticipating customers is overcompensated by those of recurring customers.

4 PRICING STRATEGIES WHEN WAITING CUSTOMERS ARE NOT OBSERVABLE

In real-life applications, the number of returning customers is typically not exactly known. Further, it might not always be possible to distinguish between new and recurring customers. In this section, we show how to adjust the model presented in Section 3 by using probability distributions of waiting customers. This makes it possible to still compute effective strategies although less information is available.

4.1 Return Probabilities

While the number of recurring customers is often not observable, the average return probability can be easily estimated. Based on average return probabilities it is possible to derive a probability distribution for the number of recurring customers.

The key idea is to exploit the model with full information, cf. Section 3, and to use probabilities for waiting customers (cf. Hidden Markov Model).

The probability that k customers return in period $(t,t+1)$ is denoted by $\pi_t^{(k)} := P_t(K_t = k)$. We can estimate $\pi_t^{(k)}$ based on the observable number of interested customers v, i.e., the sum of old and new costomers, cf. $v := k + j$, see Section 3.1. Assume in a period $(t-1,t)$, we observed v interested customers and i buyers. Hence, we have, cf. (11), $k = 0,1,...,M$, $t = 0,1,...,T$, $v = 0,1,...,J+M$, $i = 0,1,...,v$,

$$\pi_t^{(k)} := P(K_t = k|v,i)$$
$$= \binom{v-i}{k} \cdot \eta_t^k \cdot (1-\eta_t)^{v-i-k} \qquad (15)$$

For all $k > v-i$, we obtain $\pi_t^{(k)} := 0$. Note, as (15) is based on the observable number of interested (v) and buying customers (i), we do not need to be able to distinguish between new and old customers.

4.2 Computation of Prices

Next, we compute optimized strategies. We use given (state) probabilities $\pi_t^{(k)}$, cf. (15), and the value function $V_t(n,k)$, cf. (13), of the scenario with full information, see Schlosser, Richly (2018) for a similar HMM approach. We define the following heuristic pricing strategy denoted by $\tilde{a}_t(n)$ for the scenario with unobservable recurring customers, $n = 0,1,...,N$, $k = 0,1,...,M$, $t = 0,1,...,T$,

$$\tilde{a}_t(n) = \arg\max_{a \in A} \Bigg\{ \sum_{k=0,...,M} \pi_t^{(k)} \cdot \sum_{\substack{j=0,1,...,J \\ i=0,1,...,j+k \\ m=0,1,...,j+k-i}} p_t^{(new)}(j)$$
$$\cdot p_t^{(demand)}(i|a,j,k) \cdot p_t^{(wait)}(m|j,k,i)$$
$$\cdot \left((a-c)\cdot \min(i,n) + \delta \cdot V_{t+1}(\max(n-i,0),m)\right)\Bigg\} \qquad (16)$$

Algorithm 4.1. Use (13), (15), and (16) in the following order to compute $\tilde{a}_t(X_t)$, $t = 0,1,...,T-1$:

(i) Compute the values $V_t(n,k)$ for all $n = 0,1,...,N$, $k = 0,1,...,M$, and $t = 0,...,T$ via (13).

(ii) In $t = 0$ let $\pi_0^{(k)} := 1_{\{k=0\}}$. Compute the price $\tilde{a}_0(N)$ using (16) for the initial inventory $X_0 := N$.

(iii) For all $t = 1,...,T-1$ observe the number v of interested customers in period $(t-1,t)$ and the number i of buying customers. Given v and i compute $\pi_t^{(k)}$, $k = 0,1,...,M$, via (15). Let $X_t = X_{t-1} - i$. Use $\pi_t^{(k)}$ to compute the price $\tilde{a}_t(X_t)$ for the current inventory level X_t, cf. (16).

4.3 Numerical Examples

In the following example, we demonstrate the applicability and the quality of our Hidden Markov approach, cf. Algorithm 4.1.

Example 4.1. We assume the setting of Example 3.1. We assume that recurring customers cannot be observed. We consider $\eta_t = 0.5$ and $\gamma_t = 0.5$.

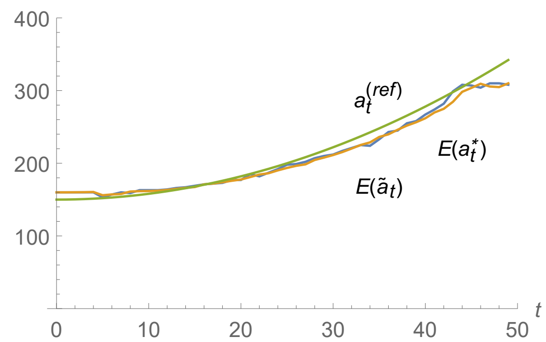

Figure 6: Average evaluated price paths $E(\tilde{a}_t)$ (blue curve), cf. Algorithm 4.1, compared to optimal price paths $E(a_t^*)$ (orange curve) of the model with full information, cf. Fig. 2, and reference prices $a_t^{(ref)}$ (green curve); Example 4.1.

Figure 6 illustrates average evaluated price curves of our heuristic strategy $\tilde{a}_t$ compared to the optimal strategy a_t^*, which, in contrast to the heuristic, takes advantage of being able to observe waiting customers. We observe that both curves are almost identical which indicates that realized prices of both strategies are similar.

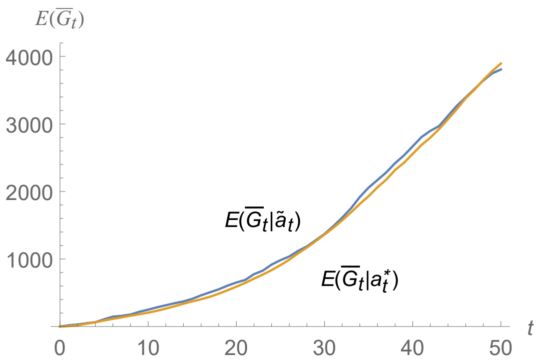

Figure 7: Average accumulated profits $E(\bar{G}_t|\tilde{a}_t)$ (blue curve) of strategy $\tilde{a}_t$, cf. Algorithm 4.1, compared to average accumulated profits $E(\bar{G}_t|a_t^*)$ (orange curve) of the optimal policy a_t^* of the model with full information, cf. (14); Example 4.1.

Figure 7 shows the corresponding evolutions of accumulated profits up to time t (denoted by $\bar{G}_t$). The curves verify that the performance of the heuristic strategy is close to optimal. For other settings of the customer behavior we obtain similar results.

5 CONCLUSION

In this paper, we proposed a stochastic dynamic finite horizon framework for sellers to determine price adjustments in the presence of strategic customers. Compared to classical myopic setups, we consider customers that (i) recur and (ii) compare the current consumer surplus against an expected future surplus, which is based on anticipated future prices.

Given an initial inventory level our pricing strategy maximizes expected profits by accounting for time-dependent demand and the number of returning consumer, which constitute a (predictable) additional demand potential.

The strategic behavior of the customers has a significant impact on prices and expected profits. We find that recurring customers lead to higher average prices, delayed sales, and most importantly higher profits. The presence of forward-looking customers has opposing effects. However, it turns out that the latter impact is overcompensated by the effect of recurring customers.

Using a Hidden Markov model (HMM), we also consider the case in which returning customers cannot be observed by the seller. By comparing solutions of the extended model and the previous model which exploits full information, we verified that the performance of the HMM model is close to optimal.

Our framework is characterized by the average evolution of customers' reservation prices and the predetermined reference prices. In future work, we will study the case in which the reference prices are updated by the evaluated average offer prices of the model. To this end, the current model can be adaptively resolved until reference prices and evaluated average prices are fully consistent. Alternatively, it might also be possible to endogenize the impact of a seller's price adjustments on current reference prices.

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APPENDIX

Table 5: List of variables and parameters.

T	time horizon / number of periods
t	time / period
N	initial inventory level
X_t	random number of items to sell in t
K_t	random number of waiting customers
G_t	random future profits from t on
c	shipping costs
δ	discount factor
R_t	individual reservation price in t
$\bar{R}_t$	average reservation price in t
$a_t^{(ref)}$	reference price in t
A	set of admissible prices
n	current inventory level
j	number of new customers
i	number of buying customers
k	number of waiting customers
$V_t(n,k)$	value function
a	offer price
CS_t	individual consumer surplus in t
$p_t^{(buy)}(a)$	purchase probability for price a
$p_t^{(new)}(j)$	probability for j new customers
$p_t^{(wait)}(k)$	probability for k waiting customers
$p_t^{(demand)}(i)$	probability for i sales in period t
$a_t^*(n,k)$	optimal prices (full information)
$\pi_t^{(k)}$	beliefs for k waiting customers
$\tilde{a}_t(n)$	heuristic prices (HMM model)
η_t	share of waiting customers
γ_t	share of anticipating customers
$\bar{G}_t$	random accumulated profits up to t