Normalization of the Histogram of Forces

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Abstract: The histogram of forces is a quantitative representation of the relative position of two image objects. It is an image descriptor, like, e.g., shape descriptors. It is not invariant under similitudes, but can be made invariant under similitudes. These are two desirable properties that have been exploited in many applications. Making the histogram of forces invariant under similitudes is achieved through a procedure called normalization. In this paper, we formalize the concept of normalization, review the existing normalization procedures, introduce new ones, and compare all these procedures through experiments involving over 170,000 histogram computations or normalizations.

1 INTRODUCTION

A relative position descriptor, or RPD, carries quantitative information about the spatial arrangement of image objects—a feature people continuously rely on to understand and communicate about space. Several RPDs can be found in the literature (Naeem and Matsakis, 2015), but the histogram of forces might be the most popular. Its applications include human-robot interaction (Skubic et al., 2004), geospatial information retrieval (Shyu et al., 2007), scene matching (Sjahputera and Keller, 2007), technical document analysis (Debled-Rennesson and Wendling, 2010), satellite image analysis (Vaduva et al., 2013) and urban land use extraction (Li et al., 2016). Many other applications (e.g., the classification of skull orbits and sinuses, the translation of hand-sketched route maps into linguistic descriptions) are referenced in (Matsakis et al., 2010). Considerable attention has been paid in literature to the invariance of image descriptors under similitudes, especially rotations and scalings. The histogram of forces is not invariant under similitudes, but it can be made invariant under similitudes, and these are two desirable properties that have been exploited in many applications.

Consider the problem of locating a set of buildings in a map given an approximate description of their relative position in the form of a sketch. Assume relative positions are represented using some RPD. If the north direction is indicated on both the map and the sketch, rotating the buildings in the sketch amounts to changing the query. For example, finding a building to the east of the MoMA in New York City is not the same as finding a building to the south of it. Rotations should therefore affect the RPD. However, if the north direction is not indicated on the map or sketch, then rotations should not affect the RPD, i.e., it should be considered that the position of an object relative to another does not change if the same rotation is applied to both objects. Likewise, if the scale of the sketch is the same as the scale of the map, scalings should affect the RPD; and if the exact scale of the sketch is unknown, then scalings should not affect the RPD. This illustrates why it is desirable for RPDs not to be invariant under similitudes, and why it is also desirable for them to be normalizable, i.e., to have the ability to become invariant under similitudes.

Various normalization procedures for the histogram of forces can be found in the literature (Skubic et al., 2004) (Matsakis et al., 2004) (Buck et al., 2010) (Buck et al., 2013) (Vaduva et al., 2013) (Clement at al., 2016). However, they have not been assessed or compared; each procedure was introduced as part of a solution to a larger problem and was not the focus of the paper addressing that problem; invariance under direct similitudes only is actually achieved. Also note that the meaning of the term normalization varies from one author to another.

In Section 3, we formalize the concept of normalization, review the existing normalization procedures, and introduce new ones. Comparative experiments are conducted in Section 4. Conclusions

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and future work are in Section 5. First, in Section 2, we say a word about geometric transformations and give a brief review of the force histogram and its geometric properties.

2 BACKGROUND

2.1 Transformations

We are considering here the Euclidean affine plane and its associated vector plane. The *origin* is an arbitrary point of the affine plane. A *transformation* is a bijection from the affine plane to itself. An *affinity* is a transformation that preserves lines and proportions on lines. A *similitude* is an affinity that multiplies all distances by the same positive real number, which is called the *scale factor* of the similitude. An *isometry* is a similitude whose scale factor is 1.

A *scaling* is a similitude that fixes at least one point—the *center* of the scaling—and preserves the direction of all vectors. An isometry is a translation, a rotation, a reflection or a glide reflection. Any similitude is the composition of a scaling and an isometry, and any isometry is the composition of reflections.

A similitude is either direct or indirect. A *direct* similitude preserves orientation (e.g., scalings, translations, rotations), while an *indirect* similitude reverses orientation (e.g., reflections, glide reflections).

The five sets of all scalings, translations, rotations, reflections and glide reflections, whether considered alone or in combination, generate six groups under the operation of composition of functions: the translation group, the scaling-translation group, the direct isometry group, the isometry group, the direct similitude group, and the similitude group.

2.2 Force Histogram

An *object* is a nonempty bounded regular closed set of the affine plane. Consider two objects *A* and *B*. We may see them as physical plates with negligible thickness. Every particle *a* of *A* exerts on every particle *b* of *B* an infinitesimal force from *b* to *a* with magnitude $1/d^r$, where *d* is the distance between the two particles and *r* is a constant. For any real number θ, let $h_r^{AB}(\theta)$ be the (integral) sum of all the infinitesimal forces in direction θ (we say that a force is in direction θ if θ is a measure in radians of the angle from the positive *x*-axis to the force). The symbol h_r^{AB} denotes a periodic function from $\mathbb{R}$ to $\mathbb{R}$ with period 2π; we call it the *force histogram*—or *histogram*, for short—of the object pair *(A,B)*. It is a quantitative representation of the position of *A* relative to *B*. In practice, histograms are computed over a finite number *n* of evenly distributed directions: $\theta_i=2(i-1)\pi/n$, with $i\in 1..n$. See (Matsakis et al., 2010).

2.3 Geometric Properties of the Force Histogram

Let *tra* be a translation, *rot* an α-angle rotation, *ref* a reflection about a line in direction α, and *sca* a scaling with scale factor σ. We have (Matsakis et al., 2004):

$$h_r^{tra(A)tra(B)}(\theta) = h_r^{AB}(\theta) \quad (1)$$

$$h_r^{rot(A)rot(B)}(\theta) = h_r^{AB}(\theta-\alpha) \quad (2)$$

$$h_r^{ref(A)ref(B)}(\theta) = h_r^{AB}(2\alpha-\theta) \quad (3)$$

$$h_r^{sca(A)sca(B)}(\theta) = \sigma^{3-r}\, h_r^{AB}(\theta) \quad (4)$$

These equations show that the force histogram is not invariant under similitudes. They can be used, however, to normalize the histogram and make it invariant under similitudes. See Section 3. There is actually a more general equation, which describes how the histogram changes when an arbitrary affinity is applied to the objects (Ni and Matsakis, 2010). It is much more complex, however, and making the histogram invariant under affinities remains an unsolved problem.

3 NORMALIZATION

3.1 Normalization Procedure

Consider a group $\mathcal{G}$ of transformations. A *normalization procedure* w.r.t. (with respect to) $\mathcal{G}$ is a function that maps any force histogram *H* to a pair $(t_H, \bar{H})$, where t_H is an element of $\mathcal{G}$ called the *normalizing transformation* of *H*, and $\bar{H}$ is a histogram called the *normalized histogram*. This function satisfies two properties. If $H = h^{AB}$ then $\bar{H} = h^{t_H(A)t_H(B)}$, and the pair $(t_H(A), t_H(B))$ is the *normalized object pair*. Moreover, $\overline{h^{t(A)t(B)}} = \overline{h^{AB}}$ for any objects *A* and *B* and any element *t* of $\mathcal{G}$, i.e., the normalized histogram is invariant under $\mathcal{G}$. Note that a normalization procedure does not have to be a total function, i.e., some histograms may not be normalizable. If h^{AB} is normalizable, the object pair (A, B) is *well-behaved*; otherwise, it is *ill-behaved*.

3.2 Retrieving *T* from h^{AB} and $h^{t(A)t(B)}$

Consider a group $\mathcal{G}$ of transformations, and a normalization procedure w.r.t. $\mathcal{G}$. Let (A_0, B_0) and (A_1, B_1) be two well-behaved object pairs. Assume there exists an element t of $\mathcal{G}$ such that:

$$A_1 = t(A_0) \text{ and } B_1 = t(B_0) \tag{5}$$

It is possible to retrieve t from $h^{A_0B_0}$ and $h^{A_1B_1}$. Indeed, by definition of a normalization procedure (Section 3.1), the normalizing transformations t_0 and t_1 of $h^{A_0B_0}$ and $h^{A_1B_1}$ satisfy:

$$h^{t_0(A_0)t_0(B_0)} = h^{t_1(A_1)t_1(B_1)} \tag{6}$$

In practical situations, if two histograms are the same then the two object pairs they are associated with are most likely the same up to a translation (Matsakis et al., 2004). In other words, (6) usually implies

$$t_1(A_1) \equiv t_0(A_0) \text{ and } t_1(B_1) \equiv t_0(B_0), \tag{7}$$

where ≡ means equality up to a translation. Therefore,

$$A_1 \equiv t_1^{-1}(t_0(A_0)) = (t_1^{-1} \circ t_0)(A_0) \tag{8}$$

$$\text{and } B_1 \equiv t_1^{-1}(t_0(B_0)) = (t_1^{-1} \circ t_0)(B_0), \tag{9}$$

where ∘ denotes function composition. In the end:

$$t \equiv t_1^{-1} \circ t_0 \tag{10}$$

Now, assume (5) holds but the transformation t does not belong to $\mathcal{G}$. Then, (10) does not hold. However, the transformation $t_1^{-1} \circ t_0$ may be seen as the element of $\mathcal{G}$ that best approximates t, and the similarity between the normalized histograms $h^{t_0(A_0)t_0(B_0)}$ and $h^{t_1(A_1)t_1(B_1)}$ can be used to assess the quality of the approximation. This will be illustrated in Section 4.

3.3 Normalization w.r.t. the Translation Group

Let *id* be the identity transformation. The equations $t_H = id$ and $\bar{H} = H$ define a normalization procedure w.r.t. the translation group. All histograms are normalizable, and all object pairs are well-behaved. These results derive from (1). Note that any transformation of the form $tra \circ t_H$, where *tra* denotes a translation, could be chosen instead of t_H as the normalizing transformation of H. This is true with any histogram and any normalization procedure, whether it is w.r.t. the translation or another group.

3.4 Normalization w.r.t. the Scaling-translation Group

The scaling-translation group can be generated by the set of all scalings. When applying a scaling to a pair of objects, the corresponding histogram H_r is shrunk or stretched vertically. See (4). To ensure invariance under scalings, this effect must be counterbalanced. The normalization can be achieved by dividing H_r by a particular value, which we are going to call the *characteristic force* of H_r and denote by $\varphi(H_r)$:

$$\overline{H_r} = \frac{1}{\varphi(H_r)} H_r \tag{11}$$

See Fig. 1. The normalizing transformation, t_{H_r}, is then the scaling with center the origin and with the following scale factor:

$$\frac{1}{\varphi(H_r)^{\frac{1}{3-r}}} \tag{12}$$

There are many ways to define the characteristic force $\varphi(H_r)$. For example, it may be set to the maximum value of the histogram. In practice, $\varphi(H_r)$ is then computed as follows:

$$\varphi(H_r) = \max_{i \in 1..n} H_r(\theta_i) \tag{13}$$

This is the approach used in (Clement at al., 2016). An alternative is to set $\varphi(H_r)$ to the mean value:

$$\varphi(H_r) = \frac{1}{n} \sum_{i \in 1..n} H_r(\theta_i) \tag{14}$$

This is the approach used in (Matsakis et al., 2004) — and it can be expected to be more robust. However, in many cases, the majority of the histogram values are zero, but the values of interest are the non-zero values. Equation (14) may therefore inappropriately pull the characteristic force towards 0. A better approach might be to set $\varphi(H_r)$ to the y-coordinate of the centroid of the region defined by the rectangular representation of the histogram on an arbitrary 2π-long interval (Fig. 2*a*). The x-coordinate of the centroid depends on the chosen interval, but the y-coordinate does not, and may be computed as follows:

$$\varphi(H_r) = \frac{\sum_{i \in 1..n} H_r(\theta_i)^2}{2 \sum_{i \in 1..n} H_r(\theta_i)} \tag{15}$$

All histograms are normalizable, and all object pairs are well-behaved, whether the characteristic force is defined by (13), (14) or (15). Moreover:

$$\varphi(\overline{H_r}) = 1 \tag{16}$$

3.5 Normalization w.r.t. the Direct Isometry Group

The direct isometry group can be generated by the set of all rotations. When applying a rotation to a pair of objects, the corresponding histogram H is shifted along the x-axis. See (2). To ensure invariance under rotations, this effect must be counterbalanced. The normalization can be achieved by shifting H to the "left" by a particular value, which we are going to call the *characteristic direction* of H and denote by $\delta(H)$:

$$\overline{H}(\theta) = H(\theta + \delta(H)) \tag{17}$$

See Fig. 1. The normalizing transformation, t_H, is then the rotation about the origin with angle $-\delta(H)$.

There are many ways to define $\delta(H)$. For example, it may be set to the direction θ in $[0, 2\pi)$ that maximizes $H(\theta)$:

$$\delta(H) = \theta_{\arg\max_i H(\theta_i)} \tag{18}$$

The approach is used in (Buck et al., 2010). However, H is not normalizable if multiple directions maximize $H(\theta)$. In practice, this means that the computed characteristic direction—and, therefore, the normalization procedure—is unreliable when multiple histogram values are very close to the maximum histogram value. The issue cannot be ignored, as many man-made object pairs exhibit symmetry and are ill-behaved.

Consider the centroid of the region defined by the rectangular representation of H on an arbitrary 2π-long interval. $\delta(H)$ cannot be set to the x-coordinate of that centroid, because it would depend on the chosen interval. However, $\delta(H)$ can be set to the angular coordinate of the centroid of the region defined by the polar representation of H (Fig. 2*b*):

$$\delta(H) = \operatorname{atan2}\left(\sum_i H(\theta_i)^3 \sin(\theta_i)\, , \sum_i H(\theta_i)^3 \cos(\theta_i)\right) \tag{19}$$

where atan2 is the two-argument variation of the arctangent function.

Equation (19) seems overly complicated. A similar but simpler approach is to see each pair $(\theta, H(\theta))$ as the polar coordinates of a vector and to define the characteristic direction $\delta(H)$ as the direction of the sum of all these vectors (Fisher, 1995):

$$\delta(H) = \operatorname{atan2}\left(\sum_i H(\theta_i)\sin(\theta_i)\, , \sum_i H(\theta_i)\cos(\theta_i)\right) \tag{20}$$

The histogram H is not normalizable if the arguments of the atan2 function in (20) are both zero. In practice, this means that the computed characteristic direction is unreliable when the two arguments are very close to zero. At any rate, an object pair is far less likely to be ill-behaved with (20) than with (18).

To address the issue with (18), we can also replace H on the right-hand side with the *histogram of degrees of truth* $\widetilde{H}$:

$$\delta(H) = \theta_{\arg\max_i \widetilde{H}(\theta_i)} \tag{21}$$

Assume H represents the relative position of two objects A and B, i.e., $H=h^{AB}$. The value $\widetilde{H}(\theta)$ is the degree of truth of the proposition "A is in direction θ of B." It belongs to [0,1], with 0 for *false* and 1 for *true*. $\widetilde{H}$ is derived from H by categorizing forces into contradictory, compensatory and effective forces (Matsakis et al., 2001). Its particularity is that, in most cases, only one direction maps to the maximum degree of truth (Fig. 3). Note that (20) can be revised the same way:

$$\delta(H) = \operatorname{atan2}\left(\sum_i \widetilde{H}(\theta_i)\sin(\theta_i)\, , \sum_i \widetilde{H}(\theta_i)\cos(\theta_i)\right) \tag{22}$$

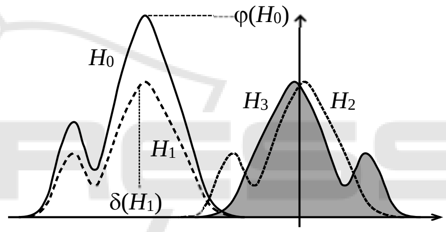


Figure 1: H_0: original histogram. H_1: after normalization w.r.t. the scaling-translation group. H_2: after normalization w.r.t. direct similitudes. H_3: after normalization w.r.t. similitudes. Note that: $\varphi(H_1)=\varphi(H_2)=\varphi(H_3)=1$, $\delta(H_0)=\delta(H_1)$, $\delta(H_2)=\delta(H_3)=0$, $\omega(H_0)=\omega(H_1)=\omega(H_2)=-1$, $\omega(H_3)=+1$.

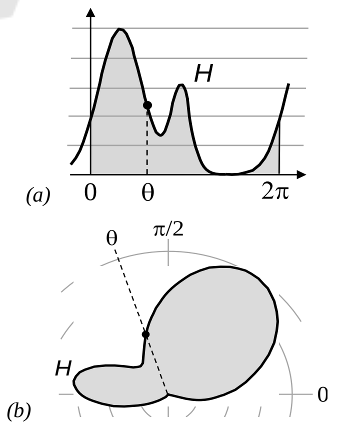


Figure 2: *(a)* Region (in grey) defined by the rectangular representation of some histogram H on the interval $[0,2\pi]$. *(b)* Region (in grey) defined by the polar representation of the same histogram.

Whatever the definition of the characteristic direction:

$$\delta(\overline{H}) = 0 \tag{23}$$

Note that (21) (22) are used in (Skubic et al., 2004); (Buck et al., 2013), respectively. In these papers, however, the authors rely on another histogram of degrees of truth, not derived from *H* alone; the described procedures are, therefore, not normalization procedures as defined in Section 3.1. The procedure described in (Vaduva et al., 2013) is not a proper normalization procedure either, since the characteristic direction is derived from the objects that produce the force histogram, not from the histogram itself.

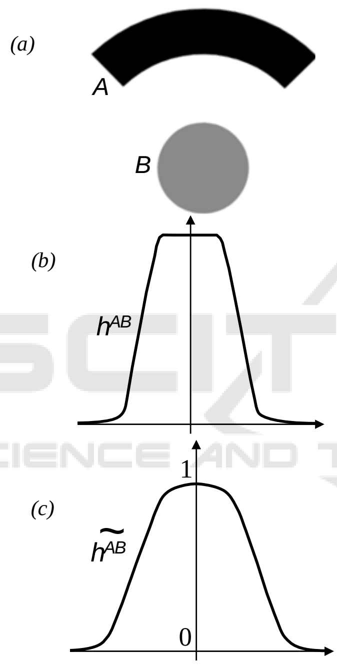


Figure 3: *(a)* Two objects *A* and *B*. *(b)* The corresponding histogram of forces. *(c)* The histogram of degrees of truth derived from the histogram of forces.

3.6 Normalization w.r.t. the Isometry Group

The isometry group can be generated by the set of all reflections, or by the set of all rotations (which generate the direct isometry group) and the reflection about the line in direction 0 that passes through the origin. When applying an isometry to a pair of objects, the corresponding histogram *H* is shifted along the *x*-axis—see (2)—and mirrored about the *y*-axis if the isometry is indirect—see (3). All this must be counterbalanced: first, normalize the histogram w.r.t the direct isometry group; then, consider mirroring the resulting histogram about the *y*-axis. In the end:

$$\overline{H}(\theta) = H(\omega(H)\theta + \delta(H)), \tag{24}$$

where the *characteristic orientation* $\omega(H)$ of *H* is either +1 (no mirroring) or −1 (mirroring). See Fig. 1. The normalizing transformation, t_H, is then the rotation about the origin with angle $-\delta(H)$, followed, if $\omega(H)$ is −1, by the reflection about the line in direction 0 that passes through the origin.

There are many ways to define $\omega(H)$. For example, it may be set to +1 if

$$\sum\nolimits_{\theta_i \in [0,\pi]} H(\delta(H) - \theta_i) < \sum\nolimits_{\theta_i \in [0,\pi]} H(\delta(H) + \theta_i) \tag{25}$$

and to −1 if the other strict inequality holds. Note that the left (resp. right) hand side of the inequality is the area of the half histogram to the left (resp. right) of the characteristic direction. *H* is not normalizable w.r.t. the isometry group if it is not normalizable w.r.t. the direct isometry group, or if the characteristic orientation is undefined (i.e., neither (25) nor the other strict inequality holds). In practice, the latter means that the computed characteristic orientation—and, therefore, the whole normalization procedure—is unreliable when the two halves of the histogram on each side of the characteristic direction have about the same area.

Another way to define $\omega(H)$ is to consider the characteristic directions of the half histograms instead of their areas. In other words, $\omega(H)$ may be set to +1 if

$$\delta(H_{LEFT}) < \delta(H_{RIGHT}) \tag{26}$$

and to −1 if the other strict inequality holds. H_{LEFT} is the histogram defined by $H_{LEFT}(\theta) = H(\delta(H)-\theta)$ if $\theta \in [0,\pi)$ and $H_{LEFT}(\theta) = 0$ if $\theta \in [\pi,2\pi)$. Likewise, H_{RIGHT} is defined by $H_{RIGHT}(\theta) = H(\delta(H)+\theta)$ if $\theta \in [0,\pi)$ and $H_{RIGHT}(\theta) = 0$ if $\theta \in [\pi,2\pi)$.

We may also want to consider the characteristic forces of the half histograms instead of their areas or characteristic directions. In other words, $\omega(H)$ may be set to +1 if

$$\varphi(H_{LEFT}) < \varphi(H_{RIGHT}) \tag{27}$$

and to −1 if the other strict inequality holds.

Whatever the definition of the characteristic orientation, we have:

$$\omega(\overline{H}) = +1 \tag{28}$$

3.7 Normalization w.r.t. the Direct Similitude Group

The direct similitude group can be generated by the set of all scalings (which generate the scaling-translation group) and rotations (which generate the direct isometry group). A normalization procedure w.r.t. that group can be obtained as follows: first, normalize the histogram w.r.t. the scaling-translation group, then normalize the resulting histogram w.r.t. the direct isometry group (or vice versa); compose the two normalizing transformations (in any order). We have:

$$\overline{H}(\theta)=\frac{1}{\varphi(H)}H(\theta+\delta(H)) \quad (29)$$

H is not normalizable w.r.t. the direct similitude group if it is not normalizable w.r.t. the direct isometry group.

3.8 Normalization w.r.t. the Similitude Group

Refer to Section 3.7: delete the word “direct” everywhere, replace “rotations” with “reflections” and (29) with (30):

$$\overline{H}(\theta)=\frac{1}{\varphi(H)}H(\omega(H)\theta+\delta(H)) \quad (30)$$

4 EXPERIMENTS

In this section, we conduct various experiments to evaluate the performance of and compare the normalization procedures discussed in Section 3. The objects and histograms considered in the experiments are presented in Sections 4.1 and 4.2. The experiments themselves are described in Section 4.3. The results are shown and analyzed in Section 4.4.

4.1 Objects

Two sets of objects were considered in our experiments. The first one, B1, is a map of 95 buildings (from downtown New York City) and the second one, B2, is a map of 86 buildings. Both were acquired through Google Maps. See Fig. 4. The two sets represent cases that are somewhat opposite: the distances between neighbour objects are about the same in B1, but vary significantly in B2; most objects have simple, regular shapes in B1, but have more complex and varied shapes in B2.

One hundred pairs of neighbour objects were chosen randomly from each set, with the constraint that each object of the set must be in at least one pair.

4.2 Histograms

Each histogram computed in our experiments was computed using n=360 evenly distributed directions. The two most common types of histogram were considered: the histogram of constant forces, i.e., r=0, and the histogram of gravitational forces, i.e., r=2. For the meaning of n and r, see Section 2.2.

As mentioned in Section 3.2, we need a way to assess the similarity between two histograms. In our experiments, we used the Tversky index (Pappis and Karacapilidis, 1993). See (31). It is a number between 0 (completely dissimilar) and 1 (completely similar). Several similarity measures for the comparison of force histograms were examined in (Matsakis et al., 2004), and the Tversky index appeared to be the most appropriate measure for the task.

$$sim(H,H')=\frac{\sum_i \min\{H(\theta_i),H'(\theta_i)\}}{\sum_i \max\{H(\theta_i),H'(\theta_i)\}} \quad (31)$$

4.3 Description of the Experiments

Many normalization procedures have been presented in Section 3. Finding the best ones comes down to finding the best ways to define the characteristic force, direction, and orientation of a histogram. Three experiments were therefore designed. The general idea is to find, within a map of buildings, two buildings in a given relative position. The position is specified by a query, which is like a very small map with only two buildings.

The first experiment relies on the assumption that the North is indicated on both the map and the query, but the scale of the map, or of the query, is unknown. In other words, the normalization procedures considered in the experiment are w.r.t the scaling-translation group, and the aim is to determine the best way to define the characteristic force: is it through (13), (14), or (15)?

1. For each object pair repeat the following 10 times:

> 1.1. Scale the two objects, using a scale factor σ chosen randomly between 1 and 5.
>
> 1.2. For each normalization procedure and type of histogram:
>
> 1.2.1. Record the similarity between the normalized histograms of the object pair before and after transformation.
>
> 1.2.2. Let σ' be the retrieved scale factor as per (10). Record the scale ratio max{σ'/σ, σ/σ'} (it is greater than or equal to 1).

2. For each normalization procedure and type of histogram, derive some statistics from these records (e.g., min, max, mean, standard deviation, percentile curves).

The second experiment relies on the assumption that the scale is indicated on both the map and the query, but the North is not. In other words, the normalization procedures considered in the experiment are w.r.t the direct isometry group, and the aim is to determine the best way to define the characteristic direction: is it through (18), (19), (20), (21), or (22)? Steps 1.1 and 1.2.2 above are changed to:

> 1.1. Rotate the two objects, using a rotation angle α chosen randomly between 0° and 180°.
>
> 1.2.2. Let α' be the retrieved rotation angle (between −180° and 180°). Record the angle deviation 180−|180−|α'−α|| (it is greater than or equal to 0°).

The third experiment relies on the assumption that the scale is indicated on both the map and the query, but the North is not; moreover, the map may have been flipped about an arbitrary axis. In other words, the normalization procedures considered in the experiment are w.r.t the isometry group, and the aim is to determine the best way to define the characteristic orientation: is it through (25), (26), or (27)?

1. For each object pair repeat the following 5 times:

> 1.1. Rotate the two objects, using a rotation angle chosen randomly between 0° and 180°.
>
> 1.2. For each normalization procedure and type of histogram:
>
> 1.2.1. If the histograms of the object pair before and after transformation have both the same characteristic orientation, record a true negative (TN).

2. For each object pair, repeat the following 5 times:

> 2.1. Reflect the two objects; the reflection is about a line whose direction is chosen randomly.
>
> 2.2. For each normalization procedure and type of histogram:
>
> 2.2.1. If the histograms of the object pair before and after transformation have different characteristic orientations, record a true positive (TP).

3. For each normalization procedure and type of histogram, indicate TN and TP.

In practice, the querier does not know the exact shapes of the two buildings they are looking for; the focus is on the relative position of the buildings, not on their shapes. This is why each one of the three experiments was run three times with the 100 object pairs from B1, and three times with the 100 object pairs from B2. The second and third times, polygonal approximations of the objects—instead of the objects themselves—were scaled, rotated or reflected. The approximations were computed using Ramer-Douglas-Peucker algorithm (Ramer, 1972) (Douglas and Peucker, 1973), and were rougher the third times. See Fig. 5.

4.4 Results

Tables 1 and 2 summarize the results for the normalization procedures w.r.t the scaling-translation group. The bold values in the tables are the best results returned (highest average similarity and lowest average scale ratio), and the underlined values are the second best results. The results are best when the characteristic force is defined using the centroid-based approach; see (15). The similarities are almost always the highest, and the scale ratios the lowest (i.e., the retrieved scale factor is the most accurate). This is true for both maps of buildings and both types of histogram. There is no clear winner between the max-based approach, (13), and the mean-based approach, (14).

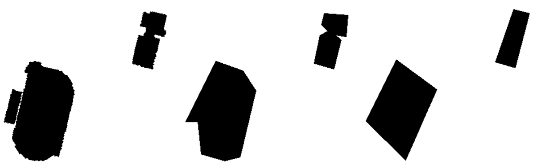
Tables 3 and 4 summarize the results for the normalization procedures w.r.t the direct isometry group. The results are best (highest similarities and lowest angle deviations) when the characteristic direction is chosen based on the vector sum of the histogram of degrees of truth; see (22). The approach based on the vector sum of the force histogram, (20), comes very close second; it is the approach we would recommend, as it is much simpler and faster. The worst way to choose the characteristic direction when normalizing a force histogram w.r.t the direct isometry group is the argmax-based approach, (18).

Tables 5 and 6 summarize the results for the normalization procedures w.r.t the isometry group. When normalizing w.r.t the isometry group, we need to first normalize w.r.t the direct isometry group (Section 3.6); the characteristic direction was chosen based on the vector sum of the force histogram, as recommended above (Tables 3 and 4). The question then is how to choose the characteristic orientation. The results are best (highest numbers of true positives and true negatives) with the characteristic force approach, (27); the characteristic force was computed using the centroid-based approach, as per the results above (Tables 1 and 2). The characteristic direction approach, (26), comes second. The worst way to choose the characteristic orientation is the approach based on the areas of the two halves of the force histogram, (25). These results stand for both maps of buildings and both types of histogram.

B1

B2

Figure 4: The two sets of objects used in the experiments. Note that B1 was recorded as a 1500×1285 binary image, and B2 as a 1306×1709 image.



P0 P1 P2

Figure 5: P0: no polygonal approximation of the objects, i.e., original object pair (first run of the experiments with B1 or B2). P1: polygonal approximation (second run). P2: rougher polygonal approximation (third run).

Table 1: Results of normalizing the force histogram of type *r* = 0 w.r.t the scaling-translation group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
average similarity						
(13)	0.9987	0.9676	0.9532	0.9960	0.9276	0.8924
(14)	0.9988	0.9668	0.9531	0.9964	**0.9317**	0.8947
(15)	**0.9988**	**0.9682**	**0.9545**	**0.9965**	0.9315	**0.8962**
average scale ratio						
(13)	1.0004	1.0210	1.0382	1.0012	**1.0238**	1.0494
(14)	1.0005	1.0269	1.0478	1.0012	1.0353	1.0571
(15)	**1.0004**	**1.0205**	**1.0377**	**1.0010**	1.0239	**1.0465**

Table 2: Results of normalizing the force histogram of type *r* = 2 w.r.t the scaling-translation group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
average similarity						
(13)	0.9981	0.9692	**0.9509**	0.9953	0.9346	0.8866
(14)	**0.9983**	0.9686	0.9487	0.9958	0.9349	0.8856
(15)	0.9983	**0.9701**	0.9501	**0.9958**	**0.9368**	**0.8891**
average scale ratio						
(13)	1.0022	**1.1026**	**1.1847**	1.0042	**1.1329**	1.2622
(14)	1.0022	1.1269	1.2347	1.0042	1.1811	1.3210
(15)	**1.0019**	1.1033	1.1940	**1.0036**	1.1419	**1.2561**

Table 3: Results of normalizing the force histogram of type *r* = 0 w.r.t the direct isometry group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
average similarity						
(18)	0.9845	0.9145	0.8562	0.9703	0.8735	0.7952
(19)	**0.9887**	0.9272	0.8724	0.9741	0.8860	0.8275
(20)	0.9886	0.9276	0.8738	0.9748	0.8904	0.8330
(21)	0.9877	0.9256	0.8725	0.9738	0.8859	0.8209
(22)	0.9884	**0.9278**	**0.8741**	**0.9755**	**0.8908**	**0.8334**
average angle deviation						
(18)	0.4504	1.3738	2.3920	0.3922	2.1481	4.3300
(19)	**0.3300**	0.7292	1.2361	0.3299	1.6419	2.2185
(20)	0.3366	**0.6781**	1.0684	0.3216	1.2542	1.7441
(21)	0.3562	0.8636	1.3523	0.3375	1.5833	2.7246
(22)	0.3414	0.6875	**1.0643**	**0.3099**	**1.2095**	**1.6593**

Table 4: Results of normalizing the force histogram of type *r* = 2 w.r.t the direct isometry group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
average similarity						
(18)	0.9771	0.8854	0.8261	0.9643	0.8559	0.7663
(19)	0.9888	0.8916	0.8346	0.9751	0.8789	0.7884
(20)	0.9892	**0.8922**	0.8365	0.9753	0.8812	0.7926
(21)	**0.9896**	0.8910	0.8360	**0.9762**	0.8776	0.7876
(22)	0.9893	0.8922	**0.8367**	0.9751	**0.8814**	**0.7929**
average angle deviation						
(18)	0.7986	1.4829	2.5389	0.5872	3.9653	5.6347
(19)	0.3547	0.8801	1.3538	0.3481	1.9990	4.0156
(20)	0.3417	**0.8378**	1.0093	0.3412	1.4147	3.2392
(21)	**0.3248**	0.9016	1.1300	**0.3283**	1.6755	3.9143
(22)	0.3378	0.8449	**0.9860**	0.3380	**1.3098**	**3.1818**

Table 5: Results of normalizing the force histogram of type *r* = 0 w.r.t the isometry group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
true negatives						
(25)	402	396	400	416	383	346
(26)	440	436	440	436	406	402
(27)	**483**	**487**	**470**	**487**	**456**	**421**
true positives						
(25)	420	423	402	424	415	383
(26)	454	440	448	450	415	375
(27)	**483**	**474**	**460**	**475**	**450**	**417**

Table 6: Results of normalizing the force histogram of type *r* = 2 w.r.t the isometry group.

	B1			B2		
	P0	P1	P2	P0	P1	P2
true negatives						
(25)	393	385	388	392	388	334
(26)	418	427	426	426	393	**387**
(27)	**463**	**450**	**448**	**481**	**445**	380
true positives						
(25)	382	394	374	385	370	296
(26)	429	413	426	433	387	**370**
(27)	**465**	**451**	**458**	**490**	**421**	353

5 CONCLUSION

Making the histogram of forces invariant under similitudes is achieved through a procedure called normalization. Various normalization procedures can be found in the literature, but they had not been assessed or compared, and invariance under direct similitudes only was actually achieved.

We have shown that the histogram of forces can be made invariant under the similitude group or under a subgroup of that group, and that any normalization procedure to achieve such goal relies on one or more of three values derived from the histogram: the characteristic force, the characteristic direction, and the characteristic orientation.

We have reviewed the existing procedures, we have introduced new ones, and we have shown through comparative experiments involving over 170,000 histogram computations or normalizations that many of these new procedures outperform the existing ones.

Making the histogram of forces invariant under the affine group remains an unsolved problem, and we will tackle it in future work. We will also examine normalization procedures for other relative position descriptors.

REFERENCES

A. R. Buck, J.M. Keller, M. Skubic, "A modified genetic algorithm for matching building sets with the histograms of forces," *IEEE Congress on Evolutionary Computation (CEC)*, Proceedings, 1-7, 2010.

A. R. Buck, J.M. Keller, M. Skubic, "A memetic algorithm for matching spatial configurations with the histogram of forces," *IEEE Trans. on Evolutionary Computation*, 17(4): 588-604, 2013.

M. Clement, C. Kurtz, L. Wendling, "Bags of spatial relations and shapes features for structural object description," *23rd Int. Conf. on Pattern Recognition (ICPR)*, Proceedings, 1994-9, 2016.

I. Debled-Rennesson, L. Wendling. "Combining force histogram and discrete lines to extract dashed lines," *20th Int. Conf. on Pattern Recognition (ICPR)*, Proceedings, 1574-7, 2010.

D. Douglas, T. Peucker, "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature," *The Canadian Cartographer*, 10(2):112–22, 1973.

N. I. Fisher, *Statistical Analysis of Circular Data*, Cambridge University Press, 1995.

M. Li, A. Stein, W. Bijker, Q. Zhan, "Urban land use extraction from very high resolution remote sensing imagery using a Bayesian network," *ISPRS J. of Photogrammetry and Remote Sensing*, 122:192-205, 2016.

P. Matsakis, J.M. Keller, O. Sjahputera, J. Marjamaa, "The use of force histograms for affine-invariant relative position description," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 26(1):1-18, 2004.

P. Matsakis, J.M. Keller, L. Wendling, J. Marjamaa, O. Sjahputera, "Linguistic description of relative positions in images," *IEEE Trans. on Systems, Man, and Cybernetics, Part B*, 31(4):573-88, 2001.

P. Matsakis, L. Wendling, J. Ni, "A General Approach to the Fuzzy Modeling of Spatial Relationships," In: R. Jeansoulin, O. Papini, H. Prade, S. Schockaert (Eds.): *Methods for Handling Imperfect Spatial Information*, Springer-Verlag, 49-74, 2010.

M. Naeem, P. Matsakis, "Relative position descriptors: A review," *4th Int. Conf. on Pattern Recognition Applications and Methods (ICPRAM)*, Proceedings, 286-95, 2015.

J. Ni, P. Matsakis, "An equivalent definition of the histogram of forces: Theoretical and algorithmic implications," *Pattern Recognition*, 43(4):1607-17, 2010.

C. Pappis, N. Karacapilidis, "A Comparative Assessment of Measures of Similarity of Fuzzy Values," *Fuzzy Sets and Systems*, 56(2):171-4, 1993.

U. Ramer, "An iterative procedure for the polygonal approximation of plane curves," *Computer Graphics and Image Processing*, 1(3):244–56, 1972.

C.-R. Shyu, M. Klaric, G.J. Scott, A.S. Barb, C.H. Davis, K. Palaniappan, "Geoiris: Geospatial information retrieval and indexing system—content mining, semantics modeling, and complex queries," *IEEE Trans. on Geoscience and Remote Sensing*, 45(4):839-52, 2007.

O. Sjahputera, J.M. Keller, “Scene matching using f-histogram-based features with possibilistic c-means optimization,” *Fuzzy Sets and Systems*, 158(3):253-69, 2007.

M. Skubic, D. Perzanowski, S. Blisard, A. Schultz, W. Adams, M. Bugajska, D. Brock, “Spatial language for human-robot dialogs,” *IEEE Trans. on Systems, Man, and Cybernetics, Part C*, 34(2):154-67, 2004.

C. Vaduva, I. Gavat, M. Datcu, “Latent Dirichlet allocation for spatial analysis of satellite images,” *IEEE Trans. on Geoscience and Remote Sensing*, 51(5): 2770-86, 2013.