Dynamic Path Planning with Stable Growing Neural Gas

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Abstract: This paper considers the problem of path planning under dynamic aspects. We propose ”Neural Gas Dynamic Path Planning” (NGDPP), a novel algorithm that continuously provides a valid path between two points inside an environment that transforms in an unpredictable manner. These transformations can occur due to both, changes in the environment's shape and moving collision objects. The algorithm incorporates several techniques: Neural Gas, a dynamic discretization method; the A* Algorithm, a path planning algorithm for graphs; and the Potential Field method, which facilitates the avoidance of collisions. We empirically evaluate the proposed algorithm under various aspects providing performance information and guidance about situations and applications benefiting from the algorithm. The evaluation reveals that NGDPP is a solid algorithm for path planning in dynamic environments. Yet, the algorithm is based on heuristic information, i.e. a optimal result in term of the path length cannot be guaranteed.

1 INTRODUCTION

Autonomous path planning is an important topic in artificial intelligence research. From an evolutionary-biological point of view there is the assumption that the brain in living beings has arisen, among other things, from the need to move (Legendre et al., 1994). It is therefore logical that one goal in the field of Artificial Intelligence is to make purposeful and meaningful movements automatically, i.e. computer-controlled. A meaningful movement is only possible if it has been planned beforehand, which leads to the path planning problem.

We consider a problem domain consisting of an autonomous agent that is able to move freely in a two dimensional floor plan. The agent's goal is to navigate to given targets. Besides, an agent can either locate itself inside the floor plan or knows at least its starting position. We also allow that the walkable area can change dynamically. These changes may occur due to new areas that are made available or due to blockades that separate the previously walkable area. The agent shall adapt its plan to these changes. Finally, the agent should neither collide with static obstacles nor with dynamic obstacles like other agents.

In the following, the problem domain will be split into three subproblems: a) Discretization of the continuous domain into a graph structure to ease computation , b) Path planning on the previously generated graph structure, and c) Deployment of a suitable collision avoidance strategy.

We propose a complete solution (namely NGDPP) to the described problem domain that combines existing methods for the individual subtasks. We show the benefits arising by the combination of the chosen methods. Furthermore we propose improvements for the method chosen to discretize the continuous domain, namely Stable Growing Neural Gas (SGNG).

The paper is structured as follows. In Section II common related approaches are outlined. Our approach as well as the used methods are presented in Section III. This is followed by an evaluation in Section IV. The paper is concluded in Section VI, where also clues for future work are given.

2 RELATED WORK

There are multiple existing approaches that solely treat one of the named subproblems. We focus on two of the most common approaches for the discretization and path planning in a continuous domain.

The Probabilistic Roadmap (PRM) (Kavraki et al., 1996) method is a path planning algorithm for continuous spaces. The algorithm quantizes the space by randomly setting nodes and connecting all exist-

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ing nodes within a predefined distance with an edge, if the space allows it. Disjoint graphs, that possibly occur, are connected in a posterior step. The procedure might produce many redundant edges which densely cover certain areas of the space. Since the graph created by the Probabilistic Roadmap is only a snapshot of space in time, the question arises of how to deal with dynamically changing spaces. In this case, it is necessary to recalculate the roadmap unless one has specific information about the nature of the changes. Such a complete recalculation is very inefficient. However, there are procedures that make the replanning of Probabilisic Roadmaps more efficient (Belghith et al., 2006). These are essentially based on accelerating the path planning phase by using dynamic graph-based path planning algorithms that do not require a complete recalculation of the path (Likhachev et al., 2005).

Similar to the Probabilistic Roadmap, so-called Rapidly Exploring Random Trees (RRT) (LaValle, 1998) are also based on the random generation of waypoints. The intention behind RRT was to include physical phenomena that influence real path planning directly into the planned path. The path calculated by a Probabilistic Roadmap may have sudden turns at a certain angle: a path that a real agent (e.g. a car) with properties such as speed, acceleration or braking distance may not be able to drive. RRT is therefore particularly suitable for non-holonomic systems. As described, Probabilistic Roadmaps may return several disjoint graphs that can only be connected after a high number of iterations. The RRT procedure solves this problem by using a different data structure. A single tree is created from the starting position of the agent, to which each node that is generated must be connected. One can easily trace back from the target node to its topological parent node until the root of the tree is reached, i.e. the position of the agent. The inversion of this path then corresponds to the path the agent has to move. Advantageous is the possibility to define conditions for new nodes during generation that take properties such as speed, acceleration, etc. into account. A drawback, however, is the fact that a new tree must be generated for each agent when controlling several agents.

3 NEURAL GAS DYNAMIC PATH PLANNING

The algorithm proposed in this paper – Neural Gas Dynamic Path Planning (NGDPP) – tries to solve the path planning problem in a very general way by applying to dynamic spaces, both in the form of spatial changes and dynamic obstacles. Several agents should be able to approach any target inside a continuous space without colliding with walls, collision objects or each other.

NGDPP uses a matrix in form of a bitmap to represent the original space, whereby individual pixels can be either free or occupied. However, in order to maintain the continuous property of space, the bitmap can be as large or as fine as necessary. We call the set of all free pixels in space the signal space. Although the signal space is finite, it can be enlarged and thus retains its continuous character.

Basically, the algorithm is divided into three parallel running parts. These are discretization, path planning and collision avoidance.

3.1 Discretization through Stable Growing Neural Gas

The discretization creates a waypoint graph from the signal space. The graph consists of nodes and edges, whereas nodes represent positions in space and edges represent collision-free transitions between them. Since sporadic changes in space can occur, it is necessary to choose a method for discretization that also can dynamically co-develop a representation of this space. For this dynamic discretization we use the Stable Growing Neural Gas (SGNG) algorithm (Tencé et al., 2013), which is an iterative unsupervised learning technique that learns the underlying topology of an n-dimensional space in quantized form. It is an extension of the neural gas (Martinetz et al., 1991) and the growing neural gas algorithm (GNG) (Fritzke, 1995) for dynamic scenarios which is more stable in respect to the number of nodes that are created.

The term “gas” refers to the fact that the nodes of the formed topology evenly distribute in space, similarly to gas molecules. GNG has been used before for waypoint graph generation in a static fashion (Dellinger et al., 2017). However, the key feature of the SGNG, that we use, is that it continuously evolves with the space as it changes. Furthermore, the algorithm can be executed continuously as it converges to a stable state in regard to the number of nodes.

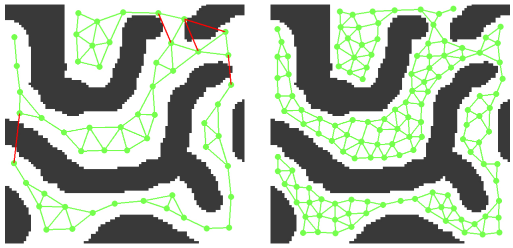
We understand sporadic changes as modifications in the matrix of space forming the signal space and assume that these changes have a relatively large surface area. In addition, the changes should not occur too dynamically as the neuronal gas would fail due to its latency to remedy erroneous edges or nodes (likewise Figure 1a). Therefore, small or fast dynamic changes have to be treated in a different manner with the help of collision avoidance, as explained later on.

The SGNG algorithm is continuously executed in order to build up a topological representation that can develop dynamically with the space. If a certain area of the space becomes unaccessible, no more signals are generated there. This in turn means that the nodes at this location are no longer selected by the algorithm and thus the "age" of the connected edges increases until they are finally removed. If all edges of a node have been removed, also the node is removed. If a new area is added to the space, signals are also generated there and nodes are pulled in their direction. Due to the usually longer distances between existing nodes and the signal, the "error" property of the node increases and thus new nodes are more likely to form which fill the newly created space. The high level pseudocode of NGDPP is shown in Algorithm 1. The agent uses the created waypoint graph in its step()-Method which is further explained in Section 3.3.

The discretization of the environment with SGNG might not be optimal. This error also strongly depends on a "maxError" variable which steers the creation of new nodes and with which the precision of the SGNG can be controlled. A comparison of two different settings of this "maxError" variable can be seen in Figure 1.

Algorithm 1: NGDPP.

```
Req.: graph ← GRAPH(nodes ← [], edges ← [])
      area ← LOADINITIALAREA()
      agents ← LOADAGENTS()
      collisionObjs ← LOADCOLLISIONOBJS()
nodes ← [n1, n2]
i ← 0
while True do
    i ← i + 1
    # get a random free position in the area
    s ← GETSIGNAL(area)
    # find the nearest two nodes in the graph to s
    n1, n2 ← NEARESTNODES(s, graph)
    n1.error ← n1.error + DISTANCE(s, n1)^2
    # moves node n1 and its neighbors towards s
    ATTRACTNODES(s, n1)
    # new edge between n1 and n2 if not existing
    NEWEDGECONNECTION(n1, n2)
    # delete edges according to Section 3.1
    AGESELECTION(i)
    # heuristic creation of new nodes
    SPLITNODES()
    DECREASEALLERRORS()
    for each agent in agents do
        agent.STEP(area, collisionObjs, graph)
    end for
end while
```



(a) • (b) •

Figure 1: In the comparison of the SGNG with low (a) and high precision (b), it is noticeable that in the first case wrong (not collision-free) edges can be formed (colored in red).

In order to allow the agents to navigate to any destination in space, we propose to integrate the targets as nodes into the routing graph. Target nodes do not have to be defined at the beginning, but can be inserted at runtime. This is especially useful if not all targets are known at the beginning. Unlike a normal node, a target node has the following three properties:

1. Directly after a target node has been created, it must be connected with the node having the shortest distance to it. Else it would be immediately deleted.
2. If the target node has got only one edge and this edge exceeds the "maxAge", its "age" variable must be reduced by at least one in order to keep the edge.
3. Targets should be static. If the algorithm tries to move a target node, because it is the next node to a signal (or the topological neighbour of such a signal), this is not permitted and is therefore not executed.

From these properties it follows that it is not possible for the SGNG algorithm to remove a target node and they have to be removed otherwise for example, if an agent has reached its target.

Nearest Neighbor Problem. A large part of the SGNG's computational cost is caused by the need to find the nodes with the shortest distance to a certain position in every iteration. This search could be done simply by measuring the distance between each node in the graph and the position and then sorting the nodes accordingly. However, this is quite inefficient, especially with a high number of nodes. We therefore strive to optimize this search. Due to the highly dynamic character of SGNG, in which nodes move in space in every iteration, we do not consider advanced space-partitioning data structures like k-d Trees (Bentley, 1975) as suitable for our approach as these data structures had to be costly rebuilt or updated in every iteration.

We pursue a simpler approach and divide the graph space into a grid. Each node is stored in a two-dimensional array at the position where it is located within this grid. A node located at the position (x,y) can then be stored in the array at position $g_{i,j}$ with $(i,j) = (\lfloor(x/a)\rfloor, \lfloor(y/a)\rfloor)$, where a indicates the accuracy of the grid and $\lfloor\ldots\rfloor$ rounds the result off to integers. The accuracy must be defined in the beginning and must not be changed during the runtime of the algorithm. If we now save all nodes according to this scheme, we can use this to find the next node in the graph much more efficiently. Whenever a node is moved, its position within the array must also be checked and possibly updated. This is not a major problem, as the formula for $g_{i,j}$ is evaluated very fast.

Sorted List for Edges. Another factor that increases the calculation effort of the algorithm is the calculation of the "edge age". In principle, the SGNG algorithm requires that each edge receives an "age" property, which is increased by one at each iteration. In addition, it must be checked for each edge whether it has exceeded a "maxAge". We propose a more advanced strategy:

Each edge receives a property "birth" instead of an "age". This specifies the iteration step of the algorithm for which the edge was created. The edges are now stored in a list. The oldest edge, whose birth property is the lowest, comes first. As soon as a new edge is created, it has an age of zero ("birth" = current iteration step) and it is simply added at the tail of the list. The resulting list is naturally sorted as the iteration counter is strictly monotonically increasing. For the deletion of outdated edges the algorithm no longer has to check all edges in every iteration, but can simply inspect the last edge in the list. Subtracting its "birth" from the current iteration step gives its age. If this value exceeds the "maxAge", it is deleted from the list and from the graph. Then the following edge (from direction of the tail) also has to be checked as it might be the same age. If it has not exceeded the maximum age, it can be calculated when it exceeds it at the earliest time by subtracting the "birth" value from the current iteration count. By storing this value, we know for how many iterations it is not necessary to check whether an edge is too old. Thus, the algorithm does not only save iterating over all edges, but also does not have to check the edges at all for several iterations of the main loop.

It also happens in SGNG that the age of edges is to be reset to zero. In this case we remove the edge from the list, no matter at which position it is located, set its "birth" variable to the current iteration count and add it at the tail of the list.

3.2 Path Planning with A*

Parallel to the waypoint graph, a path for the agent must be calculated. Due to the dynamic property of the graph, planned paths do not have to necessarily be valid over time. This problem can be solved by a continuous recalculation of the path. Of course, in case of multiple agents, each agent must calculate its own path in parallel using the globally available waypoint graph.

We use the A*-algorithm (Hart et al., 1968) for path planning, because the formed waypoint graph is located in Euclidean space and thus heuristic information can easily be extracted which provides a performance advantage compared to the Dijkstra algorithm (Dijkstra, 1959). For the A* algorithm, it is necessary to know the cost of the edges. In our case, these are described by the distance between nodes in space. The cost of an edge between two nodes can be calculated using the Euclidean distance. Since this calculation must be applied to all edges that are examined during path planning, it accounts for a large part of the calculation effort of path planning.

However, the SGNG algorithm spanning the graph has a crucial property that the NGDPP can take advantage of to save itself these calculations. The SGNG algorithm is designed in such a way that the nodes converge to a state in which they are evenly distributed within the space, similar to a gas on a molecular level, which implies that edges formed between nodes level off quite exactly to the same length. This property results from the fact, that the generated signals are uniformly distributed over the space and nodes are accordingly generated or displaced.

The fact that all edge lengths are almost the same makes the calculation of their exact length obsolete. It is sufficient to set the edge costs equal to one. The performance advantage resulting from this methodology and the fact that there is no deterioration of the path is one of our contributions and further evaluated in Section 4.

3.3 Collision Avoidance through the Potential Field Approach

A collision object, i.e. an object that moves within the space, does not directly alter the original space. Obviously, the agent cannot enter the area where a collision object is located, although this area is part of the space. In principle, the collision object could also be defined as a change in space. However, such an interpretation is not sensible due the following properties of a collision object.

A collision object does not change its size, is

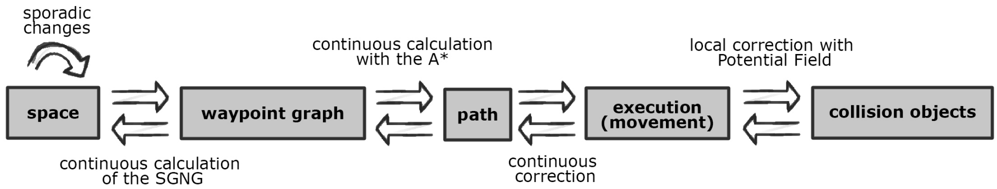


Figure 2: Structure of the NGDPP.

mostly in motion and it is usually much smaller than the sporadic changes that might occur to the space. As the neuronal gas takes some time to adapt to changes, it is not practical to apply it directly to small fast moving objects. The much more important reason is, that the graph should represent a globally valid topology of the space. If we assume an area in which two agents are to move independently, from an agent's point of view, the other agent is nothing more than a collision object that moves relatively unpredictable. If we applied the neural gas directly to avoid collision objects, it would be necessary to create a separate SGNG graph for each individual agent as the graph would no longer be globally valid. However, a globally valid graph yields the great advantage that it can be calculated by one central processing unit, while it can be used by multiple agents concurrently.

Therefore, we use the potential field approach (Koren and Borenstein, 1991) for collision objects. In order to move several objects in the same space without collisions, the phenomenon of electromagnetism is copied from nature. It describes mathematically the fact that two particles with the same charge repel each other. This concept is applied to collision objects. Objects within space are in principle treated like particles with the same charge, so that in the event of an impending collision they are repeled from each other and thus prevent a collision.

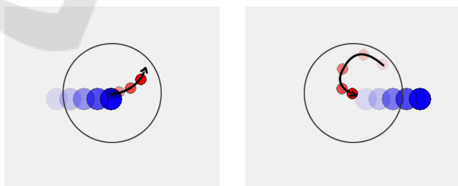
On its own, the potential field is not enough for planning the route. However, it is very effective at avoiding collisions between objects in local space. Therefore it is used in our NGDPP algorithm. In order to apply the "Potential Field" and allow an agent to leave its planned path locally, we extend the basic agent which was only defined by its position to a "force field agent" which is defined by the three properties *position of the force field*, *detection radius* and *position of the agent*.

The agent is surrounded by a force field within which collision objects are detected. As a result, the agent no longer has an absolute position in space, but receives relative coordinates to the center of its force field. So instead of moving the agent directly along its path, now the agent's force field is moved along the path, with the agent normally located in the center of the force field $(x = 0, y = 0)$. If a collision with another agent is imminent, so if one enters the agent's force field radius, the agent can move within its force field (see Figure 3). Because an attracting force is constantly applied to the agent towards the center of the force field, the agent returns to its relative zero position when no collision object is within its detection radius. Some more insight in the movement of the agent is given in algorithm 2. The entire NGDPP process is summarized in Figure 2.

Algorithm 2: agent.STEP().

Precon.: *graph* $\leftarrow$ GRAPH(*nodes* $\leftarrow$ [], *edges* $\leftarrow$ [])
area $\leftarrow$ LOADINITIALAREA()
collisionObjs $\leftarrow$ LOADCOLLISIONOBJS()
target $\in$ *graph.nodes*

function STEP(*area, collisionObjs, graph*)()
  # move agent according to Section 3.3
  MOVEAGENTINFORCEFIELD(*collisionObjs*)
  # find nearest node in the graph to s
  n $\leftarrow$ NEARESTNODE(*agent.position*, *graph*)
  path $\leftarrow$ PATHPLANNING(*n*, *target*)
  # execute one step of the path
  MOVEFORCEFIELD(*path*)
end function



(a) • (b) •

Figure 3: An agent (red) evades a collision object (blue) within its force field. After the collision object leaves the detection radius, the agent returns to the center.

4 EVALUATION

In this section we will empirically evaluate the assumption about the evenly distributed edge lengths that we proposed and other properties of NGDPP, like occurring collisions. Also, NGDPP is compared to related work methods.

4.1 Simplification for the Usage of A*

In Section 3.2 the assumption was made, that the SGNG algorithm produces a graph with equal-sized edges. This simplifies the path planning as the real edge lengths do not have to be calculated, which yields a performance advantage. In Figure 4 both the mean values and the variances of the edge lengths over 100,000 iterations are shown. There is a strong rash in the variance at the beginning, but it converges quickly towards zero. Also the mean value converges against a certain value after a few thousand iterations of the algorithm. That means, that it is sensible to assume all edge lengths to be the same, i.e. to be one.

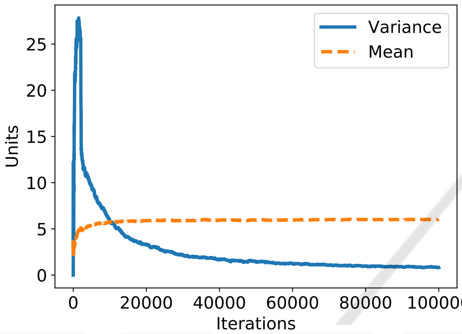


Figure 4: Mean and variance of all actual edge lengths over more than 100,000 iterations of NGDPP.

The above assumption was used to compare the performance of four different path planning algorithms. Figure 5 shows, that the "simple" procedures, which utilize the assumption, dominate the others in terms of performance. In addition, it was checked for more than 1 million iterations whether the methods calculate a different path length. This could theoretically happen due to inaccurate edge lengths, but also due to the heuristic mode of operation of the A*. However, not a single deviation in the length of the path was found. Not only does this improve performance, it is also highly likely that the calculated path

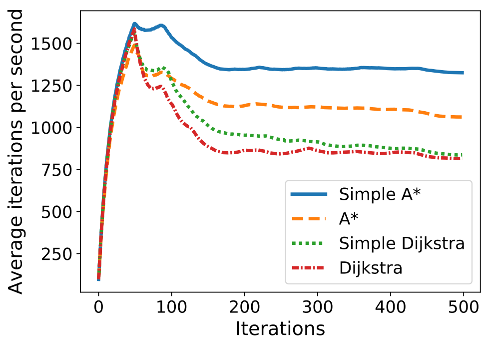


Figure 5: Performance comparison of path planning algorithms with and without the assumed simplification.

will retain its quality.

4.2 Collisions

NGDPP is a heuristic algorithm and Collisions can occur between agents as well as between agents and their environment.

As mentioned in Section 3.1 the discretization of the environment with SGNG is not perfect as the precision is bound to parameters of the algorithm and also due to the fact that it iteratively adapts to changes in the environment. That is why we measured the number of wrong edges, that means, edges that intersect with the environment (see Figure 1), after a change of the map over time. The experiment shows that the proportion for wrong edges (or nodes) decreases significantly over time after a map change (Figure 6).

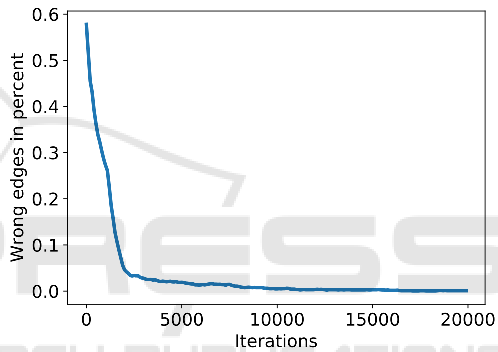


Figure 6: Percentage of wrong edges over time.

Figure 7 shows the evaluation of occurring collisions between agents. A number of 1-20 agents collected random targets over 100,000 iterations in a free space. Since collisions are probabilistic, the experiment was repeated eight times for each number of agents. The curve shows the average number of collision per agent amount. It can be seen that a larger number of agents leads more frequently to collisions.

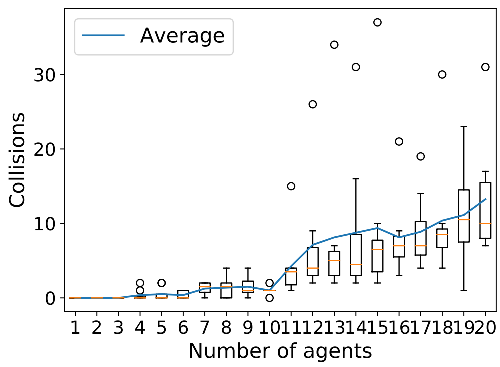


Figure 7: Evaluation of the collision of multiple agents.

4.3 Comparison with Other Methods

In Figure 8 the NGDPP algorithm is compared with another, very simple pathfinding procedure to show its efficiency. The method used for comparison utilizes the A* algorithm for path finding, however, the underlying graph results from the fact, that every walkable pixel of the spatial plan is transformed into a node, with edges between adjacent nodes. With larger spaces, the runtime of this method increases quadratically while the NGDPP algorithm increases linear.

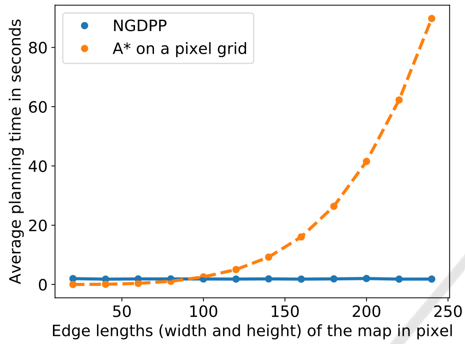


Figure 8: Performance comparison of NGDPP to A* routing on a graph that results from interpreting every pixel of the input floor plan as node.

Comparison with PRM

In Section 2 the PRM approach was mentioned with the point of criticism that the Probabilistic Roadmap has to be completely rebuild after every change in the space. During this reconstruction of the waypoint graph no routing can be executed. In comparison, our NGDPP approach dynamically adapts to changes and continuously allows routing queries (with the restriction that edges might collide with obstacles during the adaption of changes and with inappropriate parameter settings as mentioned in Section 3.1).

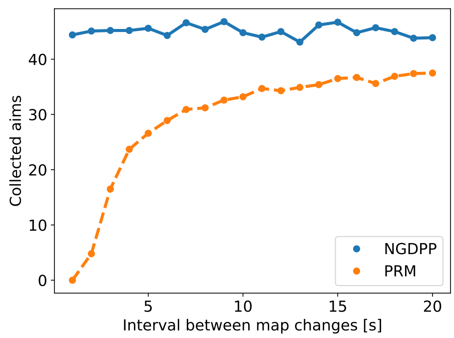


Figure 9: Performance comparison of NGDPP to a simple PRM implementation (adapted from (Sakai et al., 2018)).

In the experiment both approaches were used for the navigation of a single agent which had to collect targets, that were spawned at random positions on the map over the course of 5 minutes. In those 5 minutes changes of the map occurred at different rates. The changes had the character, that parts of the map became accessible and were blocked again. The experiment was repeated 10 times for each algorithm and map change rate. The mean of the results is plotted in Figure 9.

The results show that the agent using NGDPP collects an almost constant amount of targets over the course of 5 minutes as the adaption of the waypoint graph to map changes and the path planning of the agent are interleaved. On the other hand the agent using the PRM approach has to wait for the completion of the waypoint graph before navigation can start. If the interval between map changes is low, for example 2 second, the generation of the graph takes up most of the time, leaving the agent no time to move before the next change occurs and the graph as to be recreated.

Comparison with RRT

As mentioned in Section 2, one problem with RRT, a method often used for path planning, is that a separate graph (tree) has to be created for each agent. The consideration here was that with an increasing number of agents our method (NGDPP) should be superior to the RRT method. This is based on the fact that in our case all agents can use the same waypoint graph. In order to verify this thesis empirically, we have carried out an experiment.

We have used the two algorithms to let a number of 1 - 10 agents collect randomly placed targets (but the same for both) for 3 minutes in 10 different environments. The results of this experiment can be seen in Figure 10.

The experiment shows that the NGDPP is superior to the RRT approach. Agents using NGDPP collect more targets in the same time which is attributable to

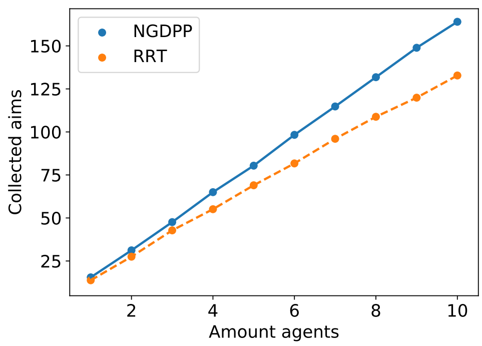


Figure 10: Performance comparison of NGDPP to a simple RRT implementation (adapted from (Sakai et al., 2018)).

the higher planning time of RRT while the NGDPP waypoint graph can continuously be used. This difference increases, as already assumed, with a higher number of agents.

5 CONCLUSION AND FUTURE WORK

In this paper, a new holistic method for path planning in dynamic environments was presented: the NGDPP algorithm. The sub problems of the problem domain mentioned in the introduction, discretization, path planning and collision avoidance, were dealt with separately. The dynamic discretization of the space was solved using the SGNG algorithm. The resulting waypoint graph could then be used by the A* algorithm to plan a valid path. During execution, this path is locally adjusted using the "Potential Field" method, so that collisions with dynamic obstacles are avoided. Section 3 explains how these methods work together to form the new NGDPP algorithm and which synergies arise by the combination of them. In the evaluation, the proposed changes that can be made to methods for the subproblems in order to obtain computational advantages, particularly the observation that the used SGNG algorithm produces edges of equal length, which accelerates the path planning, yield the anticipated performance improvements. Overall, the algorithm shows good performance compared to related work.

Up until now, our algorithm has only been applied in simulations of holonome systems. However, most real agents cannot take sudden turns through a 90° angle, for example. Such agents would probably not be able to execute most paths planned by the NGDPP algorithm. A possible solution for this could be smoothing the path using Splines (Catmull and Rom, 1974) respectively through the De-Casteljau algorithm (Alt et al., 1997). To what extent this can be applied to a path calculated by the NGDPP algorithm has to be examined.

We assume, that our algorithm can be straightforwardly adapted for three-dimensional spaces, since all used methods, i.e. both neural gas and the "Potential Field" as well as our proposed improvements, would be possible in a three-dimensional application. The algorithm could then be used e.g. for the movement planning of flying drones. However, the applicability of our algorithm and its performance in three-dimensional spaces has not been tested and leaves room for further investigation.

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