# "Coherent Transitions" and Rabi-type Oscillations between Spatial Modes of Classical Light

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Abstract: In this paper we apply approaches and concepts from quantum mechanics to analyze the propagation of classical electromagnetic waves in the elements of integrated optical circuits. We consider here regions of transparent materials as potential wells between barriers of complex shape formed by opaque media. This allows us to build an analogy between coherent oscillations in a quantum system and the redistribution of the field strength of a classical wave in space in the framework of the slow-varying amplitude approximation for the wave equation. We also demonstrate the possibility of controlling the mode composition of a classical light in a spatially inhomogeneous waveguide structure. The proposed description is based on the analogy with Rabi-type oscillations in quantum mechanics.

## 1 INTRODUCTION

Photons (electromagnetic wave-packets) interact only weakly with an optically transparent medium but not with each other; they have several degrees of freedom for encoding of information (including quantum information) and provide fast propagation speed. So they are an attractive choice for creating information processing networks (Shvartsburg, 2007; Knill et al, 2001; Bogdanov et al, 2016; Kovalyuk et al, 2013; Khasminskaya et al, 2016; Crespi et al, 2013; Tillmann et al, 2013). Nowadays the implementation of several computational protocols with photons is possible in free space, but the requirement for a large number of optical components and their precise configuration push for new solutions. Integrated optical circuits seems to be the most promising due to their scalability, stability, no need for optical alignment as well as low power consumption and compatibility with traditional electronic circuits.

For the realization of such circuits silicon gallium arsenide and diamond platforms have been suggested. Each idea has its own advantages and disadvantages and is currently in different stages of development (Bogdanov et al, 2016). Nevertheless, all of these platforms rely on combining single photon sources (e.g. carbon nanotube), detectors (e.g. superconducting nanowire single-photon detectors) and linear optical elements (e.g. silicon-nitride waveguides). Some proof-of-principle concepts have already been implemented on a single chip (Khasminskaya et al, 2016). And in this article we propose a new approach to the analysis of nontrivial processes of light pulse propagation in structures along linear waveguides, interconnects and splitters.

We have already accomplished the analysis of tunneling processes for electromagnetic waves in opaque media regions on the basis of the well-known optical-mechanical analogy to solve the urgent communication blackout problem (Bogatskaya, et al, 2018a) as well as for increasing of the efficiency of bolometric photodetectors (Bogatskaya, et al, 2018b).



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## 2 NONSTATIONARY SCHROEDINGER EQUATION IN OPTICS

Let us consider spatially inhomogeneous nonmagnetic medium characterized by the susceptibility $\chi(\vec{r})$, or permittivity $\varepsilon(\vec{r}) = 1 + 4\pi\chi(\vec{r})$. Let us also suppose that these functions are slowly varying functions in space $|\nabla\varepsilon| << k\varepsilon$ (here $k = 2\pi/\lambda$ is the wave vector and $\lambda$ is the wave length) Then for the linearly polarized electromagnetic wave with slowly varying amplitude propagating in z - direction $E(\vec{r},t) = E_0(\vec{r})\exp(i(kz - \omega t))$, ($|\nabla E_0| << kE_0$, $\omega = 2\pi c/\lambda$ is the radiation frequency) we can use the well-known slow-varying amplitude approximation that results in the equation (Akhmanov and Nikitin, 1997)

$$ik\frac{\partial E_0}{\partial z} = -\frac{1}{2}\nabla_{\perp}^2 E_0 + \eta(\vec{r}_{\perp}, z)E \,. \tag{1}$$

Here $\vec{r}_{\perp} = \{x, y\}$ are coordinates perpendicular to the propagation direction, $\nabla_{\perp}^2$ is the Laplace operator over these coordinates and $\eta(\vec{r}_{\perp}, z) = -2\pi k^2\chi(\vec{r}_{\perp}, z)$. The equation (1) is mathematically similar to the nonstationary Schroedinger equation for the particle motion in two -dimensional space $\vec{r}_{\perp} = \{x, y\}$:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla_{\perp}^2\psi(\vec{r}_{\perp},t) + V(\vec{r}_{\perp},t)\psi(\vec{r}_{\perp},t)\,. \tag{2}$$

The mathematical identity of equations (1) and (2) is the basis of so-called optical-mechanical analogy (Bohm, 1952) being widely used nowadays for analyzing of different problems both in quantum theory and wave optics. The time evolution of wave function of a one(two)-dimensional quantum system turns out to be analogous to the problem of calculating of electric field strength amplitude in a light beam propagating in an inhomogeneous medium. The coordinate *z* along which the light beam propagates is analogous to the time *t* in quantum theory, and the function $\eta(\vec{r}_{\perp}, z)$ determined by the susceptibility of the medium has the meaning of a potential $V(\vec{r}_{\perp}, t)$. In particular, if $\eta(\vec{r}_{\perp}, z) = 0$ (vacuum) then the problem (1) is equivalent to the study of free particle motion.

We will start our study from the situation when the media susceptibility depends on only perpendicular coordinate ($\eta = \eta(\vec{r}_{\perp})$). Such situation is similar to the stationary (time independent) potential in quantum theory ($V = V(\vec{r}_{\perp})$). Then general solution of eq.(2) can be found as a superposition of initially populated stationary states with amplitudes $C_n$:

$$\psi(\vec{r}_{\perp},t) = \sum_n C_n\varphi_n(\vec{r}_{\perp})\exp(-iE_n t/\hbar)\,. \tag{3}$$

Here wave functions and energies of stationary states should be found from the stationary Schroedinger equation

$$\left(-\frac{\hbar^2}{2m}\nabla_{\perp}^2 + V(\vec{r}_{\perp})\right)\varphi_n(\vec{r}_{\perp}) = E_n\varphi_n(\vec{r}_{\perp})\,. \tag{4}$$

By analogy with quantum mechanics we can find the solution of problem (1) in a form

$$E_0(\vec{r}_{\perp},t) = \sum_n C_n R_n(\vec{r}_{\perp})\exp\left(-i\frac{\lambda_n^2}{2k}z\right). \tag{5}$$

where $R_n(\vec{r}_{\perp})$ and $\lambda_n$ obey the eigenvalue problem similar to (4):

$$\left(\nabla_{\perp}^2 + 4\pi k^2\chi(\vec{r}_{\perp}) - \lambda_n^2\right)R_n(\vec{r}_{\perp}) = 0\,. \tag{6}$$

The solution of (6) gives rise to a number of transverse modes in the beam propagating in *z* - direction. We use the following normalization condition for eigenfunctions $R_n(\vec{r}_{\perp})$:

$$\int |R_n(\vec{r}_{\perp})|^2 d\vec{r}_{\perp} = \frac{\mathrm{E}_0^2}{8\pi}L_0^2\,. \tag{7}$$

Here $\mathrm{E}_0$ is the normalization field constant (this value can be chosen arbitrary), $L_0^2$ - normalization volume.

The solution of eigenvalue problem (6) gives rise to a number of transverse modes of light beam propagating in our structure.

Now we will study the possibility to control the "population" of transverse modes. Let us suppose that dielectric layer parameters (its thickness or permittivity) are slightly modulated along z - direction. In this case we can write the "potential" function $\eta(\vec{r}_\perp, z)$ in a form

$$\eta(\vec{r}_\perp, z) = \eta_0(\vec{r}_\perp) + \eta_1(\vec{r}_\perp, z) . \tag{8}$$

where the first term in the right part determines the parameters of the layer discussed above while the second one provides the spatial modulation of the layer. We will suppose that

$$\eta_1(\vec{r}_\perp, z) = \delta\eta(\vec{r}_\perp)\cos(Kz) . \tag{9}$$

with $\delta\eta(\vec{r}_\perp) << \eta_0(\vec{r}_\perp)$. Here $K$ is the wave vector of the longitudinal structure. Then the additional term in (8) can be taken into account as small perturbation to the "potential" $\eta_0(\vec{r}_\perp)$. The additional perturbation will cause the coherent transitions between transverse modes of the beam similar to the transitions between atomic states caused by the external perturbation, for example, laser field action. The transitions are governed by the set of equations

$$ik\frac{dC_f}{dz} = \sum_n C_n(z) M_{fi} \exp\left(\frac{i}{2k}(\lambda_f^2 - \lambda_n^2)z\right)\cos Kz . \tag{10}$$

which are similar to those obtained in quantum mechanical theory of light-atom interaction. Here

$$M_{fi} = \frac{8\pi}{E_0^2 L_0^2}\int R_f^*(\vec{r}_\perp)\delta\eta(\vec{r}_\perp)R_n(\vec{r}_\perp)d\vec{r}_\perp . \tag{11}$$

If we restrict ourselves by the first order of perturbation theory the expression for the probability of transition between different modes reads

$$\frac{dC_f(z)}{dz} = -\frac{i}{2k} M_{fi} \exp\left(\frac{i}{2k}(\lambda_f^2 - \lambda_i^2 \pm 2Kk)z\right). \tag{12}$$

where $R_i$ stands for the initially excited mode. We see that if $\lambda_f^2 - \lambda_i^2 \pm 2Kk = 0$ the coherent transitions between the transverse modes are possible.

## 3 COHERENT CONTROL OF CLASSICAL LIGHT BEAM

To be more specific let us consider a light beam propagating along the one-dimensional uniform dielectric layer of thickness $a$ and permittivity $\chi_0 > 1$ covered by dielectric substrate with permittivity $\chi(a)$ close to unity (see Fig.1). Such a structure is similar to the one-dimensional potential well that contains a number of discrete levels. Under the assumption that this "well" is deep the problem (5), (6) has approximate solution

$$R_n(x) \sim \begin{cases} \cos(n\pi x/a), n = 1,3,5,\ldots \\ \sin(n\pi x/a), n = 2,4,6,\ldots \end{cases} \tag{13}$$

where $x \in (-a/2, a/2)$ and

$$\lambda_n^2 \approx 4\pi k^2 \chi_0 - (n\pi/a)^2 . \tag{14}$$

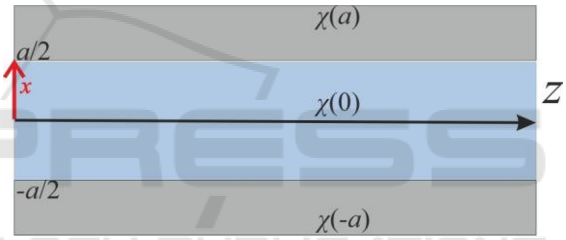


Figure 1: The three-layer planar structure. The central layer is characterized by the susceptibility greater than the covering layers. Such a structure represents the "potential well" for the propagation beam and can prevent it from the diffraction divergence.

These modes are stable against the diffraction divergence along the beam propagation. Any superposition of these modes is also stable against the divergence. Nevertheless, superposition of different transverse modes results in specific spatial beam oscillations and reconstruction during its propagation along the structure. For example, if only two lowest transverse modes are populated the spatial period of these oscillations will be given by the expression

$$L \approx 4ka^2/3\pi . \tag{15}$$

For example, if $\lambda = 2\pi/k = 10^{-4}$ cm and $a = 10^{-3}$ cm one obtains $L \approx 0.027$ cm. For the above mentioned parameters transverse mode oscillations are presented at. Fig.2.

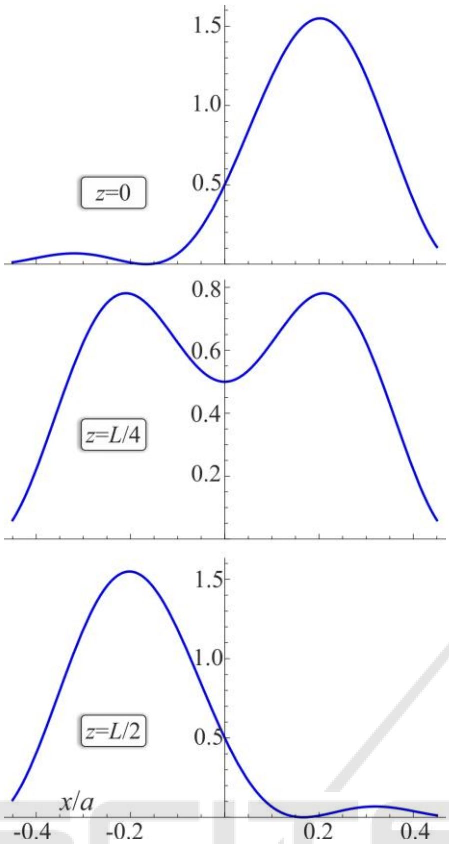


Figure 2: Illustration to the spatial beam oscillations and reconstruction during its propagation along the fiber-like structure for the case of initially excited two lowest transverse modes.

More interesting situation is the possibility of Rabi - type oscillations. Here we mean the energy transfer from one mode to another and back in a spatially inhomogeneous waveguide. For example, for two lowest modes ($i=1,\ f=2$) such Rabi - type oscillations are possible if $K \approx (\lambda_2^2 - \lambda_1^2)/2k$. For our simple one-dimensional structure discussed above this relation is satisfied for the period $\Lambda = 2\pi/K$ of the longitudinal structure given by (15). The length of the structure $L_R$ (Rabi length) for the total conversion of the energy from the one transverse mode to another and back depends on the matrix element (11) and is given by the expression: $L_R = 4\pi k/M_{21}$. If we suppose that $\delta\eta(x) = 2\pi k^2 \cdot \delta\chi(x)$ with $\delta\chi(x)$ given by the step-like structure $\delta\chi(x) = \delta\chi_0 \begin{cases} -1, x \in (-a/2, 0), \\ 1, x \in (0, a/2), \end{cases}$ it is easy to obtain $M_{21} \approx k^2 \delta\chi_0$. For $\delta\chi_0 \approx 0.001$ one

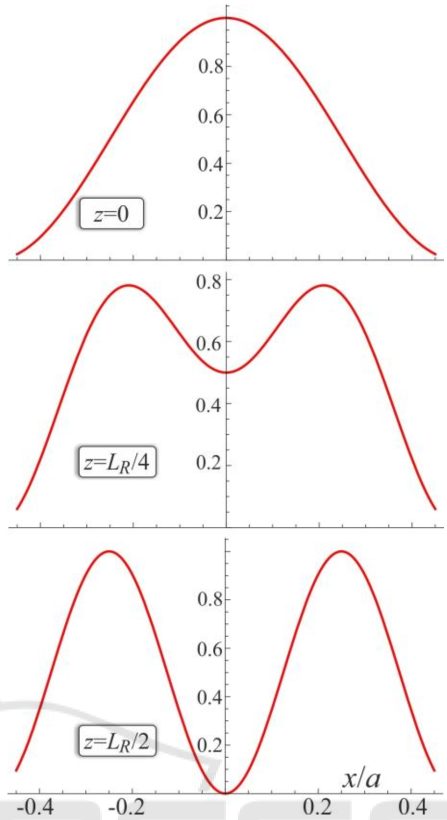


Figure 3: Rabi - type oscillations between two lowest transverse modes in a spatially inhomogeneous waveguide.

obtains $L_R \sim \lambda/\delta\chi_0 \sim 0.01$ cm. Typical distribution of electromagnetic energy in the regime of Rabi-type oscillation between two lowest spatial modes is presented at fig.2.

## 4 CONCLUSIONS

Thus, the discussed coherent oscillations of the spatial structure of the light beam as well as the Rabi-type oscillation can be observed in experiments with photonic circuits designed for bosonic sampling simulations (Spring, et al, 2013; Lund, et al, 2014; Motes, et al, 2015). Even in the simplest Y-splitter in order to demonstrate the idea we can launch light with different weights of only two modes to the single input. This test will lead to the radiation intensities at two outputs, governed by the geometric dimensions of the structure according to the formulas given above. In the future, the use of superposition of various classical light modes in such networks together with the proposed spatial methods of control, can be useful in solving of a number of practical problems of analog modeling.

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