New Alternative for Arithmetics Fuzzy Number

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Abstract: We will discuss about inverse of fuzzy matrix whose elements are fuzzy trapezoidal numbers. The discussion was prioritized from determining the new concept of positive and negative fuzzy numbers, namely by using the concept of broad positive areas. Based on the concept, there will be an alternative for multiplication concept will be discussed inverse fuzzy number matrix so that used directly to solve linear equation system of fully fuzzy trapezoidal.

1 INTRODUCTION

The famous introducer of the concept of fuzzy numbers in the world is (Zadeh, 1965) which explains the fuzzy set. Fuzzy numbers that are often discussed by researchers are fuzzy trapezoidal numbers which have some basic arithmetic between addition, subtraction, multiplication, inverse of numbers and divisions, and determinants and inverses of fuzzy matrices.

Some writers who have discussed about trapezoidal fuzzy numbers include (Kumar et al., 2010) applies a new method to the fuzzy trapezoidal number named the Mehar method, (Nasheri & Gholami, 2011) which resolves linear systems of fuzzy trapezoidal numbers, (Gemawati et al., 2018) gave a new algebra using the QR decomposition method on fuzzy trapezoidal numbers, then solved the linear equation system on trapezoidal fuzzy numbers with iterative solutions.

Some authors besides discussing the solution of fully fuzzy linear system, many of them also discuss the new arithmetic and new definitions in determining positive and negative fuzzy numbers offered in solving problems in fuzzy numbers including (Sari & Mashadi, 2019) and (Deswita & Mashadi, 2019) provide new definitions in determining positive and negative fuzzy numbers with broad concepts in triangular fuzzy numbers, (Kholida & Mashadi, 2019) and (Safitri & Mashadi, 2019) also provide new definitions with broad concepts in determining positive and negative fuzzy numbers but trapezoidal fuzzy numbers.

On the other hand some authors have discussed the inverse fuzzy numbers and the inverse fuzzy matrix inverse, namely (Sari & Mashadi, 2019) which provides a new definition in determining inverse fuzzy triangular numbers, other researchers also discuss methods for finding the rank and multiplication of inverse fuzzy trapezoid matrices, however, it does not provide a definition of the fuzzy matrix identity (Kaur, 2015), whereas (Mohana & Mani, 2018) provides a note for determining the adjoining fuzzy trapezoid matrix, using basic arithmetic which is identical to the same (Kaur & Kumar, 2017).

In this paper the author will provide and offer new arithmetic in determining the inverse fuzzy matrix with the same concept as the concept that has been given in the previous author's paper, namely (Safitri & Mashadi, 2019), (Kholida & Mashadi, 2019), (Abidin et al., 2019).

2 PRELIMINARIES

Fuzzy sets and fuzzy number are known in fuzzy, (Zadeh, 1965) and (Zimmermann, 1996) was given definition of fuzzy sets.

Definition 2.1. A fuzzy set $\widetilde{M} \subseteq X$ is a characterized by membership function $f_{\widetilde{M}}(x)$ which associates with each points in X real number in the interval $[0,1]$, with the value of $f_{\widetilde{M}}(x)$ at x representing the "grade of membership" of x in $\widetilde{M}$.

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Definition 2.2. Let X is a set of object collection that are denoted in general by x, the a fuzzy set $\widetilde{M}$ in X is a sequential set of pairs $\widetilde{M} = (X, \mu_{\tilde{A}}(x)|x \epsilon X)$ with $\mu_{\widetilde{M}}$ is a membership function of the fuzzy set $\widetilde{M}$ which a mapping of the universal set X in the interval $[0,1]$.

Some basic definition and theories related to fuzzy number has been discussed by (Gemawati et al., 2018) and (Cong-Xin & Ming, 1991).

Definition 2.3. Fuzzy number is a fuzzy set $\tilde{u}: R \rightarrow [0,1]$ with $\tilde{u} = (r, s, \varepsilon_1, \varepsilon_2)$ which satisfies

1. $\tilde{u}$ is upper semi continuous;
2. $\tilde{u}(x) = 0$ outside some interval $[r - \varepsilon_1, s + \varepsilon_2]$;
3. There are real number r, s in the interval $[r - \varepsilon_1, s + \varepsilon_2]$ such that
   (i) $\tilde{u}(x)$ monotonic non-decreasing in $[r - \varepsilon_1, r]$;
   (ii) $\tilde{u}(x)$ monotonic non-increasing in $[s, s + \varepsilon_2]$;
   (iii) $\tilde{u}(x) = 1$, for $r \leq x \leq s$.

Definition 2.4. Fuzzy number $\tilde{u}$ in R are function pair $[\underline{u}(r), \overline{u}(r)]$, which satisfies the following:

1. $\underline{u}(r)$ is a bounded left continuous non decreasing function over $[0,1]$;
2. $\overline{u}(r)$ is a bounded left continuous non increasing function over $[0,1]$;
3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

(Kumar et al., 2010) provided a definition of membership functions of trapezoidal fuzzy numbers, given fuzzy numbers $\tilde{u} = (a, b, \alpha, \beta)$ where a and b are the center points, α distance from the center point jarak dari titik pusat a to the left, and β distance from the center point b to the right. Trapezoidal fuzzy numbers function have the following form :

$$\mu_{\tilde{u}}(x) = \begin{cases} 1 - \dfrac{r - x}{\varepsilon_1}, & r - \varepsilon_1 \leq x \leq r \\ 1, & r \leq x \leq s \\ 1 - \dfrac{x - s}{\varepsilon_2}, & s \leq x \leq s + \varepsilon_2 \\ 0, & \text{otherwise} \end{cases}$$

Trapezoidal fuzzy numbers have the following parametric form if $\tilde{u} = [\underline{u}(r), \overline{u}(r)]$ can be represented as:

$$\underline{u}(r) = r - (1 - r)\varepsilon_1$$

$$\overline{u}(r) = s + (1 - r)\varepsilon_2$$

Some authors have described arithmetic fuzzy trapezoidal numbers like (Malkawi et al., 2014). Two fuzzy number $\tilde{u} = (r, s, \varepsilon_1, \varepsilon_2)$ and $\tilde{v} = (t, u, \delta_1, \delta_2)$ we call same if only if $r = t$, $s = u$, $\varepsilon_1 = \delta_1$ and $\varepsilon_2 = \delta_2$.

Arithmetic on trapezoidal fuzzy numbers given by (Kumar et al., 2011) that is if there are two fuzzy numbers $\tilde{u} = (r, s, \varepsilon, \varepsilon)$ and $\tilde{v} = (t, u, \delta, \delta)$ then

1. Addition

$$\tilde{u} \oplus \tilde{v} = (r + t, s + u, \varepsilon + \delta, \varepsilon + \delta)$$

2. Substraction

$$\tilde{u} \ominus \tilde{v} = (r - u, s - t, \varepsilon + \delta, \varepsilon + \delta)$$

3. Multiplication

$$\tilde{u} \otimes \tilde{v} = \left(\left(\frac{r+s}{2}\right)\left(\frac{t+u}{2}\right) - w, \left(\frac{r+s}{2}\right)\left(\frac{t+u}{2}\right) + w, |s\delta + u\varepsilon|, |s\delta + s\varepsilon|\right)$$

where

$$w = \left(\frac{g - h}{2}\right)$$

and

$$h = min\{rt, ru, st, su\}$$
$$g = max\{rt, ru, st, su\}$$

4. Scalar Mutiplication

$$k \otimes \tilde{u} = \begin{cases} (kr, ks, k\varepsilon, k\varepsilon), & k \geq 0 \\ (ks, kr, -k\varepsilon, -k\varepsilon) & k \leq 0 \end{cases}$$

The weakness of the arithmetic above is that the definition given only applies to two fuzzy trapezoidal numbers which have a distance from the center to the left and to the right of the same value.

Whereas (Kaur, 2015) provide the basic arithmetic definition of fuzzy trapezoid numbers, given fuzzy numbers $\tilde{u} = (m, n, p, q)$ and $\tilde{v} = (r, s, t, u)$ where $m \leq n \leq p \leq q$ and $r \leq s \leq t \leq u$ then:

1. Addition

$$\tilde{u} \oplus \tilde{v} = (m + r, n + s, p + t, q + u)$$

2. Substraction

$$\tilde{u} \ominus \tilde{v} = (m - u, n - t, p - s, q - r)$$

3. Multiplication

$$\tilde{u} \otimes \tilde{v} = (m\left(\frac{r+s+t+u}{4}\right), n\left(\frac{r+s+t+u}{4}\right), p\left(\frac{r+s+t+u}{4}\right), q\left(\frac{r+s+t+u}{4}\right))$$

4. Scalar Mutiplication

$$k \otimes \tilde{u} = \begin{cases}(km, kn, kp, kq), & k \geq 0 \\ (kq, kp, kn, km) & k \leq 0\end{cases}$$

5. Division

$$\frac{\tilde{u}}{\tilde{v}} = \left(\frac{4m}{r+s+t+u}, \frac{4n}{r+s+t+u}, \frac{4p}{r+s+t+u}, \frac{4q}{r+s+t+u}\right)$$

The weakness of this arithmetic is that the defined product does not give a case if $\tilde{u}$ and $\tilde{v}$ are positive or negtive fuzzy numbers, and in division operation $\frac{\tilde{u}}{\tilde{u}} \neq \tilde{\imath}$ or $\tilde{u} \otimes \frac{1}{\tilde{u}} \neq \tilde{\imath}$.

(Mohana & Mani, 2018) defines surgery on the trapezoidal fuzzy matrix. For example $\tilde{P} = (\tilde{p}_{ij})$ where $\tilde{p}_{ij} = (m_{ij}, n_{ij}, p_{ij}, q_{ij})$ and $\tilde{Q} = (\tilde{q}_{ij})$ where $\tilde{q}_{ij} = (r_{ij}, s_{ij}, t_{ij}, u_{ij})$

1. Addition

$$\tilde{P} \oplus \tilde{Q} = (\tilde{p}_{ij} \oplus \tilde{q}_{ij})$$

2. Substraction

$$\tilde{P} \ominus \tilde{Q} = (\tilde{p}_{ij} \ominus \tilde{q}_{ij})$$

3. Multiplication

$$\tilde{P} = (\tilde{p}_{ij})_{m\times n} \quad \text{and} \quad \tilde{Q} = (\tilde{q}_{ij})_{n\times k}$$

Then $\tilde{P} \otimes \tilde{Q} = (\tilde{r}_{ij})_{m\times n}$ where

$$\tilde{r}_{ij} = \sum_{m=1}^{n} \tilde{p}_{is} \otimes \tilde{q}_{sj}$$

4. Transpose

$$\tilde{P}^T = (\tilde{p}_{ji})$$

5. Scalar Multiplication

$$\mathrm{k}\tilde{P} = (k\tilde{p}_{ji})$$

The weakness of this arithmetic is that the multiplication that is defined does not give a case if $\tilde{u}$ and $\tilde{v}$ are positive or negative fuzzy numbers.

3 ALTERNATIVE ARITHMETIC FOR INVERS TRAPEZOIDAL FUZZY MATRIX

Now at this article, the basic arithmetic operations of fuzzy numbers used to determine the inverse matrix are arithmetic operations with broad concepts that have been given (Kholida & Mashadi, 2019), (Safitri & Mashadi, 2019) and (Abidin et al., 2019). Given two fuzzy numbers $\tilde{u} = (r, s, \varepsilon_1, \varepsilon_2)$ and $\tilde{v} = (t, u, \delta_1, \delta_2)$ are equal if only if $r = t$ and $s = u$. Two fuzzy numbers said to be the same pure if and only if $r = t$, $s = u$ and $\varepsilon_1 = \delta_1, \varepsilon_2 = \delta_2$.

1. Addition

$$\tilde{u} \oplus \tilde{v} = (r + t, s + u, \varepsilon_1 + \delta_1, \varepsilon_2 + \delta_2)$$

2. Substraction

$$\tilde{u} \ominus \tilde{v} = (r - u, s - t, \varepsilon_1 + \delta_2, \varepsilon_2 + \delta_1)$$

3. Scalar Multiplication

$$k \otimes \tilde{u} = k \otimes (r, s, \varepsilon_1, \varepsilon_2)$$
$$= \begin{cases}(kr, ks, k\varepsilon_1, k\varepsilon_2), & k \geq 0 \\ (ks, kr, -k\varepsilon_2, -k\varepsilon_1) & k \leq 0\end{cases}$$

4. Multiplication
   a. Case 1, if $\tilde{u}$ positive and $\tilde{v}$ positive, then:

$$\tilde{u}\otimes\tilde{v} = \big(rt, su, (r\delta_1 + t\varepsilon_1), (s\delta_2 + u\varepsilon_2)\big)$$

   b. Case 2, if $\tilde{u}$ positive and $\tilde{v}$ negative, then:

$$\tilde{u}\otimes\tilde{v} = \big(st, ru, (s\delta_1 - t\varepsilon_2), (r\delta_2 - u\varepsilon_1)\big)$$

   c. Case 3, if $\tilde{u}$ negative and $\tilde{v}$ positive, then:

$$\tilde{u}\otimes\tilde{v} = \big(ru, st, (u\varepsilon_1 - r\delta_2), (t\varepsilon_2 - s\delta_1)\big)$$

   d. Case 4, if $\tilde{u}$ negative and $\tilde{v}$ negative, then:

$$\tilde{u}\otimes\tilde{v} = \big(su, rt, -(u\varepsilon_2 + s\delta_2), -(r\delta_1 + t\varepsilon_1)\big)$$

5. Identity of Fuzzy Number

The identity for fuzzy numbers is divided into two, namely pure identity where $\tilde{\imath}_m = (1,1,0,0)$ and identity where $\tilde{\imath} = (1,1,\varepsilon_1,\varepsilon_2)$.

6. Inverse of Fuzzy Number

$$\tilde{u}\otimes\tilde{v} = (r, s, \varepsilon_1, \varepsilon_2) \otimes (t, u, \delta_1, \delta_2)$$
$$= \big(rt, su, (r\delta_1 + t\varepsilon_1), (s\delta_2 + u\varepsilon_2)\big)$$

So that inverse of trapezoidal fuzzy number with $r, s \neq 0$ are obtain :

$$\tilde{v} = \left(\frac{1}{r}, \frac{1}{s}, \frac{-\varepsilon_1}{r^2}, \frac{-\varepsilon_2}{s^2}\right)$$

7. Inverse of Fuzzy Matrix

Similar to fuzzy numbers, fuzzy matrices also have two identities namely pure identity is defined as follows:

$$\tilde{I}_m = [a_{ij}]$$

Where

$$a_{ij} = \begin{cases}(0,0,0,0), & for\ \ i \neq j \\ (1,1,0,0), & for\ \ i = j\end{cases}$$

To get the result of $\tilde{P} \otimes \tilde{P}^{-1} = \tilde{I}_m$ is very difficult so that another alternative to the matrix identity is defined as follows :

$$\tilde{I} = [a_{ij}]$$

where

$$a_{ij} = \begin{cases} (0,0,\varepsilon_{ij},\varepsilon_{ij}), & for\ i \neq j \\ (1,1,\varepsilon_{ij},\varepsilon_{ij}), & for\ i = j \end{cases}$$

Let

$$\tilde{P} = [\tilde{a}_{ij}]$$

where

$$\tilde{a}_{ii} = (a_{ii}, b_{ii}, \alpha_{ii}, \beta_{ii})$$
$$\tilde{a}_{ij} = (a_{ij}, b_{ij}, \alpha_{ij}, \beta_{ij})$$

for $i \neq j$.

Then it can be partitioned into:

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$N = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$P = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}$$

$$Q = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{bmatrix}$$

Then $\tilde{P}$ can be written $\tilde{P} = (P, Q, E_1, E_2)$ and $\tilde{Q} = (R, S, D_1, D_2)$, then the fuzzy matrix $\tilde{P} = \tilde{Q}$ if $P = R,\ Q = S,\ E_1 = D_1,\ E_2 = D_2$. Matrix fuzzy $\tilde{Q}$ is said to be an inverse of the fuzzy matrix $\tilde{P}$ if

$$\tilde{P} \otimes \tilde{Q} = \tilde{I}$$

Assuming each element the fuzzy matrix $\tilde{P}$ and $\tilde{Q}$ are positive fuzzy numbers, then

$$(P, Q, E_1, E_2) \otimes (R, S, D_1, D_2) = (I, I, \varepsilon_1, \varepsilon_2)$$

$$(PR, QS, NG, PD_1 + RE_1, QD_2 + SE_2) = (I, I, \varepsilon_1, \varepsilon_2)$$

was obtain

$$\begin{cases} PR = I \\ QS = I \\ PD_1 + RE_1 = \varepsilon_1 \\ QD_2 + SE_2 = \varepsilon_2 \end{cases}$$

So

$$R = P^{-1}$$
$$S = Q^{-1}$$
$$D_1 = P^{-1}(\varepsilon_1 - RE_1)$$
$$D_2 = Q^{-1}(\varepsilon_2 - SE_2)$$

Based on the algebra above P^{-1} and Q^{-1} can be searched directly, so the authors provide the definition of the matrix $\tilde{Q}$ or $\tilde{P}^{-1}$ as follow :

$$\tilde{P}^{-1} = (P^{-1}, Q^{-1}, {E_1}^{-1}, {E_2}^{-1})$$

Furthermore, it will be proven that

$$\tilde{P} \otimes \tilde{P}^{-1} = \tilde{I}$$

then

$$(P, Q, E_1, E_2) \otimes (P^{-1}, Q^{-1}, {E_1}^{-1}, {E_2}^{-1}) = \tilde{I}$$

Assuming each element of the fuzzy matrix is a positive fuzzy number, arithmetic multiplication of positive fuzzy number and positive fuzzy number is obtained:

$$\tilde{P} \otimes \tilde{P}^{-1} = (PP^{-1}, QQ^{-1}, P^{-1}E_1 + P{E_1}^{-1}, Q^{-1}E_2 + Q{E_2}^{-1})$$

$$= (I, I, P^{-1}E_1 + P{E_1}^{-1}, Q^{-1}E_2 + Q{E_2}^{-1})$$

Assume if $P^{-1}E_1 + P{E_1}^{-1} = \varepsilon_1$ and $Q^{-1}E_2 + Q{E_2}^{-1} = \varepsilon_2$.
then

$$\tilde{P} \otimes \tilde{P}^{-1} = (I, I, \varepsilon_1, \varepsilon_2)$$

Thus it is evident that $\tilde{P} \otimes \tilde{P}^{-1}$ have identity results.

Given two trapezoidal fuzzy matrices $\tilde{A} = (A, B, C, D)$ and $\tilde{B} = (E, F, G, H)$ with
$\tilde{A}^{-1} = (A^{-1}, B^{-1}, C^{-1}, D^{-1})$
and
$\tilde{B}^{-1} = (E^{-1}, F^{-1}, G^{-1}, H^{-1})$
will show that :

$$(\tilde{A} \otimes \tilde{B})^{-1} = \tilde{B}^{-1} \otimes \tilde{A}^{-1}$$

$$\tilde{A} \otimes \tilde{B} = (A, B, C, D) \otimes (E, F, G, H)$$
$$= MK, NL, MR + KP, NS + LQ$$

$$(\tilde{A} \otimes \tilde{B})^{-1} = ((MK)^{-1}, (NL)^{-1}, (MR + KP)^{-1}, (NS + LQ)^{-1})$$
$$= K^{-1}M^{-1}, L^{-1}N^{-1}, (MR + KP)^{-1}, (NS + LQ)^{-1}$$

and

$$\tilde{B}^{-1} \otimes \tilde{A}^{-1} = (K^{-1}, L^{-1}, R^{-1}, S^{-1}) \otimes (M^{-1}, N^{-1}, P^{-1}, Q^{-1})$$

$= (K^{-1}M^{-1}, L^{-1}N^{-1}, K^{-1}P^{-1}$
$+M^{-1}R^{-1}, L^{-1}Q^{-1} + N^{-1}S^{-1})$

From the algebra above it is obtained that:
$(MR + KP)^{-1} \neq M^{-1}R^{-1} + K^{-1}P^{-1}$
and
$(NS + LQ)^{-1} \neq N^{-1}S^{-1} + L^{-1}Q^{-1}$

The next step will be indicated
$(\tilde{A} \otimes \tilde{B}) \otimes (\tilde{A} \otimes \tilde{B})^{-1} = \tilde{I}$
$= (MK, NL, MR + KP, NS + LQ)$
$\otimes ((MK)^{-1}, (NL)^{-1}, (MR + KP)^{-1}, (NS + LQ)^{-1})$
$= (MK(MK)^{-1}, NL(NL)^{-1}, MK(MR + KP)^{-1} +$
$(MK)^{-1}(MR + KP), NL(NS + LQ)^{-1} +$
$(NL)^{-1} + (NS + LQ))$
$= (I, I, \varepsilon_1, \varepsilon_2) = \tilde{I}$
So it can be conclude that $\tilde{P} \otimes \tilde{P}^{-1}$ produces identity, not pure identity.

Numerical Example

Given

$$\tilde{A}\tilde{x} = \tilde{b}$$

where

$$\tilde{A} = \begin{bmatrix} (5,6,2,3) & (3,5,4,2) \\ (9,11,2,1) & (3,7,1,2) \end{bmatrix}$$

$$\tilde{b} = \begin{bmatrix} (34,56,45,37) \\ (54,94,52,32) \end{bmatrix}$$

and will be find $\tilde{x}$.
From $\tilde{A}$ we get

$$M = \begin{bmatrix} 5 & 3 \\ 9 & 3 \end{bmatrix}, N = \begin{bmatrix} 6 & 5 \\ 11 & 7 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 4 \\ 2 & 1 \end{bmatrix}, Q = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

and from $\tilde{b}$ we get

$$b = \begin{bmatrix} 34 \\ 54 \end{bmatrix}, g = \begin{bmatrix} 56 \\ 94 \end{bmatrix}, h = \begin{bmatrix} 45 \\ 52 \end{bmatrix}, t = \begin{bmatrix} 37 \\ 32 \end{bmatrix}$$

We get

$$\tilde{A}^{-1} = \begin{bmatrix} \left(\frac{-1}{4}, \frac{-7}{13}, \frac{-1}{6}, \frac{1}{2}\right) & \left(\frac{1}{4}, \frac{5}{13}, \frac{2}{3}, \frac{-1}{2}\right) \\ \left(\frac{3}{4}, \frac{11}{13}, \frac{1}{3}, \frac{-1}{4}\right) & \left(\frac{-5}{12}, \frac{-6}{13}, \frac{-1}{3}, \frac{3}{4}\right) \end{bmatrix}$$

then

$$\tilde{x} = \tilde{A}^{-1}\tilde{b}$$

So we get the results of $\tilde{x}$ is

$$\tilde{x} = \begin{bmatrix} (5,6,4,1) \\ (3,4,1,1) \end{bmatrix}$$

Then to check the truth of the results from $\tilde{A}^{-1}$ will be shown $\tilde{A} \otimes \tilde{A}^{-1} = \tilde{I}$ and $\tilde{A}^{-1} \otimes \tilde{A} = \tilde{I}$.

$$\tilde{A} \otimes \tilde{A}^{-1} = \begin{bmatrix} \left(1,1,2\frac{2}{3}, 1\frac{43}{52}\right) & \left(0,0,1\frac{1}{6}, \frac{51}{52}\right) \\ \left(0,0,7\frac{1}{4}, \frac{47}{52}\right) & \left(1,1, \frac{11}{12}, \frac{-41}{52}\right) \end{bmatrix}$$

$$= (I, I, \varepsilon_1, \varepsilon_2) = \tilde{I}$$

and

$$\tilde{A}^{-1} \otimes \tilde{A} = \begin{bmatrix} \left(1,1,2\frac{2}{3}, 1\frac{7}{26}\right) & \left(0,0,1\frac{1}{6}, -1\frac{37}{78}\right) \\ \left(0,0,-\frac{1}{4}, 2\frac{17}{52}\right) & \left(1,1,5\frac{1}{12}, \frac{133}{156}\right) \end{bmatrix}$$

4 CONCLUSIONS

In this article, arithmetic alternatives to fuzzy numbers and alternative elements of identity are pure identity and identity in fuzzy trapezoidal numbers and fuzzy trapezoidal matrices. After the arithmetic alternative is obtained, the inverse of a matrix can be determined. The matrix inverse obtained can be used to solve system of fully fuzzy linear equations $\tilde{A} \otimes \tilde{x} = \tilde{b}$ directly, with $\tilde{x} = \tilde{A}^{-1} \otimes \tilde{b}$.

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