Optimum Stocks Portfolio Selection using Fuzzy Decision Theory

Liem Chin, Erwinna Chendra and Agus Sukmana
Department of Mathematics, Parahyangan Catholic University, Ciumbuleuit 94, Bandung, Indonesia

Keywords: Portfolio Selection, Fuzzy Decision Theory, Mixed-integer Linear Programming.

Abstract: An investor wants the value of his or her money does not have a decline in value against inflation. For this reason, investors need to invest in financial instruments, one of which is stocks. Thus, investors need to create an optimum stock portfolio. Generally, the factors considered by investors in creating an optimum portfolio are expectations of return and portfolio risk. However, besides these two factors, the liquidity is also an important factor to be considered. These three factors will be discussed in this paper to form an optimum portfolio. In addition, because stock transactions use lot units, the optimization problem here is a mixed-integer linear programming optimization problem that will be solved using the Branch and Bound algorithm that is available in toolbox Matlab 2016. This optimization problem will be applied to the formation of a portfolio consisting of stocks in LQ45 index. The LQ45 Stock Index was chosen because shares in this index have high liquidity levels according to the Indonesia Stock Exchange. The computation results show that the portfolio rebalancing model can form a portfolio based on the level of satisfaction of investor.

1 INTRODUCTION

In 1952, Markowitz selected the optimum portfolio by minimizing portfolio risk expressed by the covariance matrix (Markowitz, 1952). However, this is not efficient for large-scale portfolios because the model proposed by Markowitz is quadratic programming. Moreover, in this Markowitz model only two factors are considered, namely return expectations and portfolio risk.

Then, Fang et. al. developed this Markowitz model by adding the liquidity factor to assets (Fang et al., 2005). Liquidity is an important role in investment. Liquidity is the level of possibility in converting investments in cash without losing significant value. Thus, this liquidity measures how easily investors can buy and sell their assets. An investor's portfolio is not good if it only has a high return and low risk while the assets in the portfolio are not liquid. If the assets in the portfolio are illiquid, it means that investors will have difficulty changing the assets in cash when investors need money. Moreover, Fang et. al. do not measure portfolio risk using the covariance matrix but rather using semi-absolute deviation. By using this semi-absolute deviation, the optimization problem becomes a linear programming problem that can be solved by the simplex method. Because risk is measured by semi-absolute deviations, the problem becomes simpler and more efficient for creating large-scale portfolios. However, this model cannot be directly used if the formed portfolio consists only of shares because the purchase of shares must be in lots. This lot unit is a nonnegative integer.

In our previous studies, we discussed portfolio selection by considering the number of lots of shares an investor needs to buy (Chin et al., 2018; Sukmana et al., 2019). The model discussed is quite complex because the objective function of the optimization problem is non-linear function. For this reason, in this study we will not measure portfolio risk using covariance matrices but instead using semi-absolute deviations as suggested by Fang et. al. In addition, we will also use fuzzy decision theory to solve portfolio optimization problems by considering the lot units in stock purchases. Using this theory, the optimization problem is a mixed-integer linear programming problem because the objective function is linear and there are constraints in the form of non-negative integers. Then, with this fuzzy decision theory, we will also make portfolio rebalancing to maintain the target expected by investors.

Chin, L., Chendra, E. and Sukmana, A.
Optimum Stocks Portfolio Selection using Fuzzy Decision Theory.
DOI: 10.5220/0010137400002775
In *Proceedings of the 1st International MIPAnet Conference on Science and Mathematics (IMC-SciMath 2019)*, pages 125-131
ISBN: 978-989-758-556-2

The developed model will be applied to create an optimum portfolio where this portfolio consists only of stocks listed in the LQ45 index. The LQ45 stock index was first launched in February 1997. This index is one of the benchmark indexes in the Indonesian capital market. The Indonesia Stock Exchange prescribe the principle of LQ45 index and it consist of 45 shares. The criteria include liquidity and market capitalization. In the regular market, the liquidity is mainly measured by the transaction value. However, since January 2005 the authority of capital market in Indonesia added the transaction frequency and the number of trading days as a measure of liquidity (Indonesia Stock Exchange (ISE), 2010). Stocks that are included in the LQ45 index calculation will be evaluated every three months and the replacement of stocks in the LQ45 index is completed every six months, namely in early February and August.

To solve the optimization problem in this study, we will use the Branch and Bound method (Chen et al., 2010). Then, this method will be implemented with software Matlab. We will use five years the shares price data, that is 1 July 2014 to 30 June 2019 (Yahoo Finance, 2019).

2 DISCUSSION OF THE MODEL

Assume that there are n assets in a portfolio and r_{ij} represents the percentage return rate from the asset-i in the j-th period with $i = 1,2,\dots,n$ and $j = 1,2,\dots,T$ and suppose that $m > n$. Moreover, let y_i $(i = 1,2,\dots,n)$ represents the proportion of the amount of investment for the asset-i. The semi-absolute deviation of return on the portfolio under the expected return over the past period j, $j = 1,\cdots,T$ given as

$$v_j(\boldsymbol{y}) = \left|\min\left\{0, \sum_{i=1}^{n}(r_{ij} - r_i)y_i\right\}\right| = \frac{\left|\sum_{i=1}^{n}(r_{ij} - r_i)y_i\right| + \sum_{i=1}^{n}(r_i - r_{ij})y_i}{2} \quad (1)$$

with $y = (y_1, y_2, \cdots, y_T)^T$. If r_i denote expected rate of return of asset i, the portfolio risk (V) can be determined as (Fang et al., 2005)

$$V(\boldsymbol{y}) = \frac{1}{T}\sum_{j=1}^{T} v_j(y) = = \sum_{j=1}^{T} \frac{\left|\sum_{i=1}^{n}(r_{ij} - r_i)y_i\right| + \sum_{i=1}^{n}(r_i - r_{ij})y_i}{2T} \quad (2)$$

A fuzzy number F is called trapezoidal with tolerance interval $[a, b]$, left width α and right width β if its membership function takes the following form

$$F(v) = \begin{cases} 1 - \dfrac{a - v}{\alpha}, & \text{if } a - \alpha \le v \le \text{a} \\ 1, & \text{if } a \le v \le b \\ 1 - \dfrac{v - b}{\beta}, & \text{if } a \le v \le b + \beta \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

and we denote F as $F = (a, b, \alpha, \beta)$. In this study, the turnover rate of the stocks i is defined by the trapezoidal fuzzy number $\hat{l}_i = (la_i, lb_i, \alpha_i, \beta_i)$. So, the turnover rate of the portfolio y is $\sum_{i=1}^{n} \hat{l}_i y_i$. The crisp possibilistic mean value of the turnover rate of the portfolio y is represented by (Fang et al., 2005)

$$E\left(\hat{l}(\boldsymbol{y})\right) = E\left(\sum_{i=1}^{n} \hat{l}_i y_i\right) = \sum_{i=1}^{n} E\left(\hat{l}_i y_i\right) = \sum_{i=1}^{n}\left(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) y_i \quad (4)$$

The equation (4) is used to measure the portfolio liquidity.

To accommodate the investor's desire, the S shape membership function is used to proclaim the aim of investment of an investor. The S shape membership function itself is given by

$$f(v) = \frac{1}{1 + \exp(-\tau v)} \quad (5)$$

With equation (5), the membership function for expected return, risk and liquidity are given as follows:

1) Membership function for expected return of portfolio

$$\mu_r(\boldsymbol{y}) = \frac{1}{1 + \exp\left(-\alpha_r\left(E\left(r(\boldsymbol{y})\right) - r_M\right)\right)} \quad (6)$$

with α_r is investor's satisfaction level about the expected return and r_M is the mid-point where the membership function value is 0.5. This r_M be regarded as the middle goal level for the portfolio return.

2) Membership function for portfolio risk

$$\mu_w(\boldsymbol{y}) = \frac{1}{1 + \exp\big(\alpha_w(w(\boldsymbol{y}) - w_M)\big)} \tag{7}$$

With α_w is the investor's satisfaction level about the risk portfolio and w_M is the mid-point where the membership function value is 0.5. This w_M be regarded as the middle goal level for the portfolio risk.

3) Membership function for portfolio liquidity

$$\mu_l(\boldsymbol{y}) = \frac{1}{1 + \exp\left(-\alpha_l\left(E\left(\hat{l}(\boldsymbol{y})\right) - l_M\right)\right)} \tag{8}$$

With α_l is the investor's satisfaction level about the liquidity and l_M is the mid-point where the membership function value is 0.5. This l_M be regarded as the middle goal level for the portfolio liquidity.

Using the semi-absolute deviation, trapezoidal fuzzy number and the S shape membership function, the model of selection portfolio is given as (Fang et al., 2005)

$$\max \theta \tag{9}$$

subject to

$$\alpha_r\left(\sum_{i=1}^{n} r_i y_i - \sum_{i=1}^{n} p(y_i^+ + y_i^-)\right) - \theta \geq \alpha_r r_M \tag{10}$$

$$\theta + \frac{\alpha_w}{T}\sum_{j=1}^{T} u_j \leq \alpha_w w_M \tag{11}$$

$$\alpha_l \sum_{i=1}^{n}\left(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) y_i - \theta \geq \alpha_l l_M \tag{12}$$

$$u_j + \sum_{i=1}^{n}(r_{ij} - r_i) y_i \geq 0, j = 1,2,\cdots,T \tag{13}$$

$$\sum_{i=1}^{n}(y_i^0 + y_i^+ - y_i^-) + \sum_{i=1}^{n} p(y_i^+ + y_i^-) = 1 \tag{14}$$

$$y_i = y_i^0 + y_i^+ - y_i^-, i = 1,2,\cdots,n \tag{15}$$

$$0 \leq y_i^+ \leq w_i, i = 1,2,\cdots,n \tag{16}$$

$$0 \leq y_i^- \leq y_i^0, i = 1,2,\cdots,n \tag{17}$$

$$u_j \geq 0, j = 1,2,\cdots,T \tag{18}$$

$$\theta \geq 0 \tag{19}$$

where
p is the rate of transaction cost;
y_i^0 is the proportion of the amount of investment for the asset-i before portfolio rebalancing;
y_i^+, y_i^- are the proportion of the amount of investment for an asset-i bought and sold by the investor, respectively;
w_i is upper bound of the proportion of the amount of investment to buy an asset-i;
$la_i, lb_i, \beta_i, \alpha_i$ is trapezoidal fuzzy number.
The problem (9) with constraints (10)-(19) is a linear programming problem. This problem can be solved using the simplex method, for an instance. If the investor has not the portfolio yet, the y_i^0's is set to zero so we only get the y_i^+'s. It is clear because the short selling is not allowed to form the portfolio.

Besides short selling is not allowed, stocks are traded in lots in regular market in Indonesia Stock Exchange, which 1 lot equal to 100 shares. So, if z_i is the number of lots of shares traded, we have the relationship between z_i and y_i as follow

$$y_i = 100\frac{z_i P_i}{M} \tag{20}$$

where
P_i is price of stock i and M is an investor's capital. In similar way, we have the relation between y_i^0, y_i^+, y_i^- and z_i^0, z_i^+, z_i^- respectively as follow

$$y_i^0 = 100\frac{z_i^0 P_i^0}{M} \tag{21}$$

$$y_i^+ = 100\frac{z_i^+ P_i^+}{M} \tag{22}$$

$$y_i^- = 100\frac{z_i^- P_i^-}{M} \tag{23}$$

where
z_i^0 is the number of lots for the stock-i before portfolio rebalancing;
z_i^+, z_i^- are the number of lots for the stock-i bought and sold by the investor, respectively;
P_i^0 is the price of stock i before rebalancing;
P_i^+, P_i^- are the price of stock i bought and sold by the investor, respectively.
Next, substitute equation (20)-(23) to equation (15)

$$z_i = \frac{z_i^0 P_i^0 + z_i^+ P_i^+ - z_i^- P_i^-}{P_i} \tag{24}$$

If the investor has not the portfolio yet, the z_i^0's is set to zero so we only get the z_i^+'s (because short selling is not allowed). If the investor wants to rebalance his portfolio, then we assume that the purchase and selling price of stock is same, that is $P_i^+ = P_i^- = P_i$.

Finally, we can get the model for selection portfolio with decision variable is a number of lots for stock to buy or sell, that is

$$\max \theta \tag{25}$$

subject to

$$\frac{100}{M}\alpha_r\left(\sum_{i=1}^{n} r_i z_i P_i - \sum_{i=1}^{n} p(z_i^+ P_i^+ + z_i^- P_i^-)\right) - \theta \ge \alpha_r r_M \tag{26}$$

$$\theta + \frac{\alpha_w}{T}\sum_{j=1}^{T} u_j \le \alpha_w w_M \tag{27}$$

$$\frac{100}{M}\alpha_l \sum_{i=1}^{n}\left(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) z_i P_i - \theta \ge \alpha_l l_M \tag{28}$$

$$u_j + \frac{100}{M}\sum_{i=1}^{n}(r_{ij} - r_i) z_i P_i \ge 0, \quad j = 1,2,\cdots,T \tag{29}$$

$$\frac{100}{M}\sum_{i=1}^{n}(z_i^0 P_i^0 + z_i^+ P_i^+ - z_i^- P_i^-) + \sum_{i=1}^{n} p(z_i^+ P_i^+ + z_i^- P_i^-) = 1 \tag{30}$$

$$z_i = \frac{z_i^0 P_i^0 + z_i^+ P_i^+ - z_i^- P_i^-}{P_i}, \quad i = 1,2,\cdots,n \tag{31}$$

$$0 \le z_i^+ \le \frac{w_i M}{100 P_i^+}, i = 1,2,\cdots,n \tag{32}$$

$$0 \le z_i^- \le \frac{z_i^0 P_i^0}{P_i^-}, i = 1,2,\cdots,n \tag{33}$$

$$u_j \ge 0, j = 1,2,\cdots,T \tag{34}$$

$$\theta \ge 0 \tag{35}$$

Actually, equation (31) is no longer needed because we can substitute that to equation (26), (28) and (29). Therefore, there are only nine constraints for the model. The z_i^+'s and z_i^-'s are non-negative integer because they are number of lots for the stock-i bought and sold by the investor. So, the objective function in (25) with its constraints (26)-(35) is a mixed-integer linear programming problem rather than a linear programming problem. Here is the complete model

$$\max \theta \tag{36}$$

subject to

$$\frac{100}{M}\alpha_r\left(\sum_{i=1}^{n} r_i z_i P_i - \sum_{i=1}^{n} p(z_i^+ P_i^+ + z_i^- P_i^-)\right) - \theta \ge \alpha_r r_M \tag{37}$$

$$\theta + \frac{\alpha_w}{T}\sum_{j=1}^{T} u_j \le \alpha_w w_M \tag{38}$$

$$\frac{100}{M}\alpha_l \sum_{i=1}^{n}\left(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) z_i P_i - \theta \ge \alpha_l l_M \tag{39}$$

$$u_j + \frac{100}{M}\sum_{i=1}^{n}(r_{ij} - r_i) z_i P_i \ge 0, \quad j = 1,2,\cdots,T \tag{40}$$

$$\frac{100}{M}\sum_{i=1}^{n}(z_i^0 P_i^0 + z_i^+ P_i^+ - z_i^- P_i^-) + \sum_{i=1}^{n} p(z_i^+ P_i^+ + z_i^- P_i^-) = 1 \tag{41}$$

$$0 \le z_i^+ \le \frac{w_i M}{100 P_i^+}, i = 1,2,\cdots,n \tag{42}$$

$$0 \le z_i^- \le \frac{z_i^0 P_i^0}{P_i^-}, i = 1,2,\cdots,n \tag{43}$$

$$u_j \ge 0, j = 1,2,\cdots,T \tag{44}$$

$$\theta \ge 0 \tag{45}$$

$$z_i^+, z_i^- \in \mathbb{Z}^+ \tag{46}$$

where z_i is given by equation (24) and Z^+ represent non-negative integer. The optimization problem above is solved using branch and bound algorithm. This algorithm is available in Matlab 2016 which syntax is `intlinprog`.

3 RESULTS

In this section, we give a numerical example to illustrate the proposed portfolio rebalancing model. There are 45 stocks in LQ45 index. We neglect six stocks because five among of them have a negative expected return rate and one stock just offered the shares in 2016 through an initial public offering (IPO). So, we use 39 stocks in total and collect historical data from 1 July 2014 to 30 June 2019. The data are downloaded from the web-site www.finance.yahoo.com. Then, we use one month as a period to obtain the historical return rate for 60 periods. Assume that the rate of transaction costs for purchase and sell stock is 0.002.

Next, we give an example of the estimation method for the fuzzy turnover rates for TLKM (PT Telekomunikasi Indonesia Tbk.). Since the future turnover rates of the stocks is assumed trapezoidal fuzzy numbers, so tolerance interval, left width and right width need to estimate. These parameters are estimated using frequency statistic method. In this study, we used the historical data of the stocks turnover rates. First, the frequency of historical turnover rates is calculated via daily turnover rates from 1 July 2014 to 30 June 2019. Figure 1 expresses the frequency distribution of historical turnover rates for stock TLKM. Consider that the most of the historical turnover rates fall into the intervals $[0.0004,0.0006]$, $[0.0006,0.0008]$, $[0.0008,0.0010]$ and $[0.0010,0.0012]$. We regard that the left and right endpoints of the tolerance interval, respectively, as the mid-points of the intervals $[0.0004,0.0006]$ and $[0.0010,0.0012]$. So, the tolerance interval of the fuzzy turnover rate is $[0.0005,0.0011]$. By observing all the historical data, the minimum and the maximum possible values of uncertain turnover rates in the future are 0.00014 and 0.00383, respectively. Assume that the rate of transaction costs for purchase and sell stock is 0.002.

Therefore, the left width is 0.00036 and the right width is 0.002733.

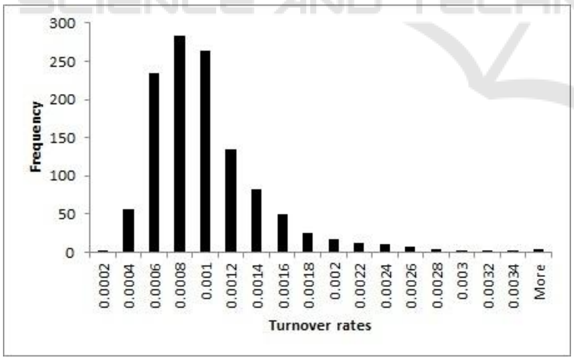

Figure 1: Frequency of turnover rates of TLKM.

Thus, the fuzzy turnover rate of stock TLKM is $(0.0005,0.0011,0.00036,0.002733)$.

In general, there are two kinds of investor, i.e. conservative and aggressive. So, in the following, we give two kinds of computational results. For conservative investor, the value of r_M, w_M and l_M is given by $0.02, 0.024$ and 0.016 whereas for aggressive investor, the value of r_M, w_M and l_M is given by $0.05, 0.06$ and 0.04. For each case, we propose two portfolios. The one, investor has not a portfolio yet (so $z_i^0 = 0$ for all i) and the other, investor has already a portfolio contain of seven stocks with the number of lots as shown in Table 1. Stock's prices for portfolio rebalancing lots are shown in Table 2 (prices appear only for stocks that are bought or sold to rebalance the portfolios in Table 3-6). For all cases, the value of α_r, α_w and α_l are given by $600, 800$ and 600, respectively. The investor's fund to form portfolios Table 3 and Table 4 is Rp 100,000,000.

Table 1: Number of lots of stocks.

Stock	Lots	Stock's Price (Rp)
AKRA	75	4,000
BBCA	10	29,400
HMSP	50	3,360
JSMR	60	5,700
PTBA	100	2,940
TLKM	20	4,040
UNVR	20	44,650

Table 2: Stock's price for portfolio rebalancing.

Stock	Stock's Price (Rp)
AKRA	4,090
ANTM	845
BBCA	29,975
BBTN	2,460
BRPT	3,210
ELSA	378
ERAA	1,920
HMSP	3,140
INDY	1,680
INKP	9,375
JSMR	5,725
PTBA	2,960
PWON	730
SRIL	338
TKIM	12,575
TLKM	4,140
TPIA	4,970
UNVR	45,000

Table 3: Portfolio rebalancing lots with $r_M = 0.02, w_M = 0.024, l_M = 0.016$ and $z_i^0 = 0$ for all i.

Stock	Buy
AKRA	1
ANTM	1
BBTN	1
BRPT	35
ELSA	607
ERAA	51
INDY	30
PWON	213
SRIL	739
TLKM	23

Table 4: Portfolio rebalancing lots with $r_M = 0.05, w_M = 0.06, l_M = 0.04$ and $z_i^0 = 0$ for all i.

Stock	Buy
BRPT	40
ELSA	320
ERAA	122
HMSP	1
INDY	148
SRIL	739

Table 5: Portfolio rebalancing lots with $r_M = 0.02, w_M = 0.024, l_M = 0.016$ and contain seven stocks as in Table 1.

Stock	BR	Buy	Sell	AR
AKRA	75	0	72	3
BBCA	10	0	10	0
BBTN	0	2	0	2
BRPT	0	100	0	100
ELSA	0	1378	0	1378
ERAA	0	126	0	126
HMSP	50	0	50	0
INDY	0	68	0	68
JSMR	60	0	60	0
PTBA	100	0	100	0
PWON	0	535	0	535
SRIL	0	1754	0	1754
TLKM	20	23	0	43
UNVR	20	0	20	0

Table 6: Portfolio rebalancing lots with $r_M = 0.05, w_M = 0.06, l_M = 0.04$ and contain seven stocks as in Table 1.

Stock	BR	Buy	Sell	AR
AKRA	75	0	75	0
BBCA	10	0	10	0
BBTN	0	1	0	1
BRPT	0	120	0	120
ELSA	0	589	0	589
ERAA	0	303	0	303
HMSP	50	0	50	0
INDY	0	352	0	352
JSMR	60	0	60	0
PTBA	100	0	100	0
SRIL	0	1754	0	1754
TLKM	20	0	20	0
UNVR	20	0	20	0

BR and AR in Table 5 and Table 6 refer to number of lots of stocks before and after portfolio rebalancing, respectively. From Table 3-6, it can be seen that the portfolios owned by a conservative investor (Table 3 and Table 5) are more numerous of stocks than those of an aggressive investor (Table 4 and Table 6). On Table 3, it can be seen that almost a half of investor's fund is invested to buy SRIL and ELSA whereas on Table 4, that fund is invested to buy SRIL and ERAA.

If we assume that an investor already has the portfolio in Table 1, then the investor's fund is Rp 237.18 million. The results are slightly different from the previous portfolios. Almost a half of investor's fund is invested to buy SRIL and ELSA for a conservative investor (Table 5) and that fund is invested to buy SRIL and INDY (Table 6).

4 CONCLUSIONS

Liquidity factor plays an important role besides expected return and risk. Liquidity is measured using the turnover rates of stocks. The levels of investor's goals are appraised to be fuzzy numbers with a non-linear S shape membership function. These goals are expected return, risk and liquidity. Considering all these factors together with fuzzy decision theory, transaction costs and a number of lots for stocks as the decision variable, a mixed-integer linear programming model for portfolio rebalancing is proposed. The computation results show that the portfolio rebalancing model can form a portfolio based on the level of satisfaction of investor.

ACKNOWLEDGEMENTS

This research is supported by Lembaga Penelitian dan Pengabdian kepada Masyarakat (LPPM) Parahyangan Catholic University.

REFERENCES

Chen, D. S., Batson, R. G., & Dang, Y. (2010). *Applied integer programming: modeling and solution*. John Wiley & Sons, Inc.

Chin, L., Chendra, E., & Sukmana, A. (2018). Analysis of portfolio optimization with lot of stocks amount constraint: case study index LQ45. *IOP Conf. Series: Materials Science and Engineering*, *300*.

Fang, Y., Lai, K. K., & Wang, S. Y. (2005). Portfolio rebalancing model with transaction costs based on fuzzy decision theory. *European Journal of Operational Research*, *175*(2), 879–893.

Indonesia Stock Exchange (ISE). (2010). *Buku panduan indeks harga saham Bursa Efek Indonesia*. https://idx.co.id/media/1481/buku-panduan-indeks-2010.pdf

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*(1), 91.

Sukmana, A., Chin, L., & Chendra, E. (2019). Analysis of

portfolio optimization with inequality constraints. *IOP Conf. Series: Journal of Physics*, *1218*.
Yahoo Finance. (2019). *No Title*. Www.Finance.Yahoo.Com. 5 July 2019