Two-dimensional Numerical Study of Scour beneath Subsea Pipeline under Regular Wave Condition

Titis Julaikha Atikasari¹, Suntoyo¹, Wahyudi¹, Umboro Lasminto² and Agro Wisudawan³

¹Ocean Engineering Department, Sepuluh Nopember Institute of Technology, Surabaya, Indonesia
²Civil Engineering Department, Sepuluh Nopember Institute of Technology, Surabaya, Indonesia
³Civil and Environmental Engineering Department, University College Cork, Cork T12K8AF, Ireland

Abstract: The interaction of pipe and soil under hydrodynamic flow becomes one of the concern in coastal engineering. Scoured seabed beneath sub-marine pipeline could develop the pipe instability and affect to its safety. The pressure difference between upstream and downstream of pipe will induce a seepage flow in the seabed underneath the pipe. When the velocity increased then the critical point reached, a mixture of sand and water divert to the gap below the pipe and leading to a large velocities in the gap. It generates a larger shear stress on the bed and increase the amount of sediment transport. Due to the complexity of the scour problem, there are many factors affecting the scour depth. Some of them are the pipe position and the wave angle of attack. This study evaluates the bed morphological evolution around pipeline, performed with variation of pipe positions and wave angles of attack.

1 INTRODUCTION

This research aims to investigate the bed evolution during scour process until it reaches a steady state (equilibrium stage). The stage reached when the bed shear stress along the bed below pipe becomes constant and equal to its undisturbed value. A computational fluid dynamics used to calculate flow and sediment transport subjected to regular wave and steady currents. The model performed the sediment erosion around pipeline that laid on erodible seabed, based on time dependent scour processes using Volume of Fluid (VoF) method with standard k-ω turbulence closure. Beside the bed deformation, the scour propagation rate discussed with respect of the pipe position.

Submarine pipelines have become a concern in marine and geotechnical engineering. The presence of pipeline though it lay out, buried or trenched in the seabed, it will change the marine environment. Wave and current action change the flow pattern when exposing the marine pipelines installed on seabed in coastal or subsea area. This change generally cause the increase of bed shear stress and the turbulence level. Both terms lead the sediment transport around pipe. Therefore interaction between pipeline, seabed, and flow causes the occurrence of local scouring. The developing scour along the pipe will generate gap between seabed and pipe, this commonly referred to pipeline span which become a threat to the stability of the pipeline structures.

Wave-induced scour around pipelines occurs in three stages, they are onset scour, tunnel erosion, and lee wake erosion. Onset scour driven by the pressure difference between the upstream and downstream of the pipe. When the flow velocity exceeds the critical point, a mixture of water and sand break through underneath the pipe. The stage followed by tunnel erosion stage, during this stage a substantial amount of water is diverted to the gap and leading large velocities, this resulting a large shear stresses below the pipe and increasing the sediment transport. In this stage, the scour occurs extremely fast at the beginning. As a result a dune begins to form at the downstream side of pipe. However this dunes gradually migrates downstream and there will be more scour at the downstream side of the pipe than at the upstream side. This stages is lee-wake erosion (Sumer and Fredsoe, 2002).

Over decades researchers have studied the local scour around pipeline and proposed different empirical equations to describe the equilibrium scour depth, such as Mao (1986), Sumer and
Fredsoe (1990), Chiew (1991), Cevik and Yuksel (1999), Bakhtiary (2011), and Dogan (2018). While numerical methods have been developed to scour around pipeline as well as Brors (1999), Liang and Cheng (2004), Shen et al. (2013), Fuhrman et al. (2014), Shen et al. (2015), Zhang and Shi (2016). Sumer and Fredsoe (1990) established scour below pipelines exposed by waves and found that lee-wake of the pipe is the key element in the scour process also. Keulegan-Carpenter number is the main parameter that governs the equilibrium scour depth. The larger the KC-number, the larger the stream-wise extent of the area affected by the lee-wake during half period of the flow. Hansen (1992) investigated combined wave and current parameter on local scour and identified that velocity ratio (m) as the governing equation for scour in combined wave and current. Velocity ratio developed from composed Uc, flow velocity due to current, and Uw, wave orbital velocity. However numerical model predictions in developing combined wave and current condition are limited. Hansen (1992) and Larsen et al. (2016) presented modelling of the scour under combined wave and current, but the study is limited to the value of combined velocity and assume the fluid as a rigid lid not free surfaces area. Ahmad (2019) investigated pipeline scour under combined wave and current, the results showed the scour depth and horizontal extent of the scour increase with Ucm (combined velocity of current and wave) for a given KC number. While the experiments of Cheng et al. (2014) showed that the scour propagation under wave only condition increases with KC number and Shield parameter.

Based on the background above, the further discussion on pipeline scour under the combined action of wave and current still needed. Over more the occurrence of the actual scour beneath subsea pipeline occurs in the combined wave and current. This paper primarily concerned with numerical simulation of a single pipe that laid on sandy bed and subjected to combined wave and current loadings. The simulation was done under free surface dynamic analysis. The study discusses the generation of regular waves and steady current condition in the numerical wave tank. Wave only model and combined wave-current model validated with Airy wave theory, then the validated model used to validate pipeline scour.

### 2 NUMERICAL METHOD

In this study, three-dimensional wave hydrodynamics and sediment transport calculated by CFD numerical modelling.

#### 2.1 Flow Model

The hydrodynamics module based on the continuity and motion equation. The motion equation of the fluid velocity components has been done in the three-dimensional Navier-Stokes equation which for incompressible flow. The equation of continuity written as follow.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0
\]

The Navier-Stokes equations are as follows.

\[
p \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \rho g_x + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]

\[
p \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \rho g_y + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)
\]

\[
p \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \rho g_z + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\]

Where u, v, and z are the velocity components, p is the pressure, \( \rho \) is the fluid density, \( \mu \) is the turbulent eddy viscosity, and g is the gravitational acceleration.

The wall boundary conditions evaluated based on turbulence closure. A turbulence model is required to calculate the turbulent viscosity \( \mu \). In this study, the turbulent viscosity calculated by the standard two-equation \( \kappa-\omega \) of (Wilcox, 1988).

\[
\frac{\partial \kappa}{\partial t} = \frac{\partial}{\partial z} \left[ \left( v + \nu_t \sigma_{k-w} \right) \frac{\partial \kappa}{\partial z} \right] + v_t \left( \frac{\partial u}{\partial z} \right)^2 - \beta'_w \kappa
\]

\[
\frac{\partial \omega}{\partial t} = \frac{\partial}{\partial z} \left[ \left( v + \nu_t \sigma_{k-w} \right) \frac{\partial \omega}{\partial z} \right] + \gamma_1 \omega \left( \frac{\partial u}{\partial z} \right)^2 - \beta'_w \omega^2
\]

The standard \( \kappa-\omega \) closure has advantages to perform low Reynolds number for near-wall turbulence. It is numerically stable in flows with adverse pressure gradient. So the advanced model of the standard closure continue to be developed, i.e. Menter (1994), Suntoyo et al. (2008), and Suntoyo and Tanaka (2009).

Using CFD model, the free surface (fluid-fluid interface) between water and air is calculated using
Eularian methods that characterized by meshes to evolve the shape of interface, i.e., Shen et al. (2013), Putri and Suntoyo (2020). VOF method proposed by Hirt and Nichols (1981) is used to track the free surface interface.

2.2 Sediment Transport Model

The physical mechanism of sediment in this paper defined as sediment scour model. It estimates the effect of the flow mechanism on the transport sediment and the erosion of surfaces. It estimates the sediment motion by approximating the sediment erosion, advection, and deposition. The numerical model estimates the sediment transportation by computing the suspended sediment transport, computing the sediment settling due to gravity, also computing the bed-load transport and motion.

Bed-load transport determined by Shield number, where bed shear stress (τ) is the important part which becomes the basis of sediment movement under flow model. Bed shear stress (τ) is determined by profiling logarithmic velocity profile near bottom.

\[ \theta = \frac{\tau}{\rho g (s - 1)d_{50}} \]

The sediment transport occurs when shield number exceeds the critical value. The Soulsby-Whitehouse equation used to predict the critical Shield number parameter.

\[ \theta_{cr} = \frac{0.3}{1 + 1.2D_e} + 0.055[1 - \exp(-0.02D_e)] \]

Where \( D_e \) is the non-dimensional grain size and can be found by \( D_e = \left[ \frac{D}{\sqrt{g(z-1)/v}} \right]^{1/3} \). According to the critical Shield number parameter, the bed-load transport rate equation calculated by Van Rijn (1984) as follow.

\[ q_b = \frac{[s - 1]g}{(s - 1)g}^{0.5}D_{50}^{2.1} = 0.053 \frac{\tau^{2.1}}{D_{50}^{1.5}} \]

3 MODEL SETUP

The scheme of this numerical simulation based on the experimental input from Suntoyo (2017). It conducted a series of experiments to study the two dimensional scour phenomenon due to combined wave and current flow over cylindrical pipe that laid on the erodible bed. This study developed the investigation to the two dimensional scour analysis. An open top flume with a hydraulically smooth pipe made brought to the model with diameter 3,81cm of pipe and placed over the sand bed which has median grain size diameter (\( d_{50} \)) 0,55 mm. The model simulated by the regular wave flow. Wave only simulation and combined wave and current given as the variation to define the difference.

![Figure 1: Domain and Boundary Scheme.](image1)

![Figure 2: Top View Sketch of Computational Domain.](image2)

The governing equation of flow simulation discretized by finite difference method. A mesh of fixed rectangular cells used to subdivide the flow region, with each cells have local average values of all dependent variables.

Good meshing quality of computational domain put up the better understanding convergence of the numerical model so the better accuracy will resulted. Figure 1 (upside) illustrates the perspective view of the 3D mesh on the seabed and pipe. Figure 2 (downside) is the cross sectional view of the mesh of the model. Three different mesh sizes adopted to define flow, sediment, and pipe. Considering the flow behaviour near bed and pipe, finer mesh blocks used in respect where the velocity boundary layer near bed develop and the erosion around pipe expected. Smooth grid mesh applied near sediment around the pipe, this made the finer convergence and accuracy which dealing of the contacts between pipe, sediment and flow. The computational domain contain 7,5 million cells of mesh which define the boundary condition that aimed to get best estimation of the flow characteristic around seabed.

The numerical models were set up with assumption of hydrostatic pressure distribution from the inlet to the outlet boundaries. The top boundary of the domain is symmetry, where the air pressure assumed does not affect the numerical wave tank. At the side surfaces, the wall boundaries used with no-slip velocity condition. The inlet is set to be regular wave condition with wave height and angle variation. The outlet set to be outflow boundary with allowing flow to enter at outflow boundary condition.
Table 1: Input Data.

<table>
<thead>
<tr>
<th>Module</th>
<th>Parameter</th>
<th>Notation</th>
<th>Magnitude</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave</td>
<td>Depth</td>
<td>d</td>
<td>0.75</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Height</td>
<td>H</td>
<td>13</td>
<td>cm</td>
</tr>
<tr>
<td></td>
<td>Period</td>
<td>T</td>
<td>1</td>
<td>s</td>
</tr>
<tr>
<td>Angle</td>
<td></td>
<td>α</td>
<td>0; 45; 60</td>
<td>deg</td>
</tr>
<tr>
<td>Current</td>
<td>Velocity</td>
<td>v</td>
<td>0; 1; 1.3</td>
<td>cm/s</td>
</tr>
</tbody>
</table>

4 RESULTS AND DISCUSSIONS

4.1 Model Verification

This section discusses the validation of the proposed numerical model to predict wave propagation and scour induced wave to understand the accuracy of the model. The numerical results of the surface elevation compared with theoretical solution. The validation consists of the linear wave propagation using linear Airy wave consideration.

\[ \eta = -\frac{H}{2} \cos(kx - \omega t) \]

Where \( H \) is the water depth, \( k \) is the wave number that derived from \( \frac{2\pi}{L} \), and \( \omega \) is wave frequency from \( \frac{2\pi}{T} \). The formula shows that the water level fluctuations are periodic in respect to \( x \) and \( t \). It shows a sinusoidal and progressive wave that propagate to the positive \( x \) direction.

Based on Liu et al. (2016) to compare the numerical result of the wave propagation with the theoretical solution that formed from potential flow theory, the viscosity is set to zero. To compare the numerical results of the wave propagation with theoretical solution, one case with inviscid fluid simulated to examine the mesh resolution in the computation. In simulation with inviscid fluid, it is assumed that the fluid is in ideal fluid mode so the viscosity and turbulent boundary ignored and set to zero. In contrast to the ideal fluid, the real fluid which has viscosity and turbulent boundary also proposed. The viscous fluid will be used as reference fluid in this study. Then the results of Airy wave theory, simulation in inviscid fluid, and simulation in viscous fluid are compared. Figure 3 shows the comparisons of numerical result of the time series of free surface elevation and the theoretical solution. The comparison of free surface elevation monitored at cross section \( x=75D \) of time series \( t=748s-752s \).

![Comparison between Airy Formula and CFD Simulation in Wave only Condition.](image1)

Figure 3: Comparison between Airy Formula and CFD Simulation in Wave only Condition.

The comparison graph of the simulation results show the surface elevation between airy formula and the CFD simulation results. The simulation of wave under ideal fluid mode, where viscosity and turbulent are set to be zero, shows the similar trend-line with empirical Airy Formula. The comparison of viscous fluid that simulated under viscosity and turbulent mode also shows a good agreement. The error results from both results comparison is 0.3%.

![Comparison between Airy Formula and CFD Simulation in Wave and Current Condition.](image2)

Figure 4: Comparison between Airy Formula and CFD Simulation in Wave and Current Condition.

The trend of simulation study on its surface elevation shows a slight shift to the empirical Airy trend-line. The comparison between theory and results of wave and current simulation shows that the CFD simulation results under wave and current have the upward trend shift compared to wave only simulation and theory. The greater of the current value that given, the more it leads to a shift in surface elevation.

4.2 Scour around Pipe

Calculation divided into three parts based on the wave angle of response, 0°, 45°, and 60°. The three sections are carried out on a pipe laying on the seabed with \( e/D=0 \). Calculations performed under wave only condition and combined wave and current of 1cm/s and 1.3cm/s. Predicted time of equilibrium scour depth and scour deposition is about 25 minutes, this...
is based on the experiments of Cheng et al. (2009) and Suntoyo et al. (2017). Maximum scour depth and deposition after 25 minutes simulation obtained. However, the calculation results in CFD for stages of scour development not as expected. The equilibrium scour depth and deposition not reached yet at the expected given time.

Figure 5: Bed Profile after 25 Minutes on $\alpha=0^\circ$.

Although the results have not shown the equilibrium condition at the expected given time (25 minutes), but the results show good agreement for how scour around pipe formed. The CFD calculation show the deformation movements of seabed which in accordance with Sumer et al. (2002) research. The simulation results in wave only condition shows the result of less sediment erosion beneath the pipe compared to the combination of wave and current results. However for the wave only simulation, the sand dune around pipeline that deposed is higher than the combined results.

In Figure 5, it can be seen if the flow still carry out sediment downstream the pipe, so that erosion occurs around the downstream of the pipe. Based on Sumer et al. (2002) wake pattern in oscillatory motion in wave are govern by Keulegan-Carpenter (KC) number. Larger KC number means that the water particles travel quite large distances, and resulting in separation and probably vortex shedding. The statement is compatible with the calculation above. In wave only condition, the generated erosion at the downstream pipe is greater than the combined conditions, despite the scour depth in the combined of wave and current simulations show the deeper results.

Typical time-dependent scour on figure 6 represents that current velocity magnitude influence the scour depth. It is clear if the greater current velocity magnitude, the deeper and wider scour produced. Different wave angle which lead to the pipe also have an influence on the width and depth of scour. The difference is quite significant. But it cannot yet be concluded whether the smaller the angle of wave attack will reduce the scour depth.

Figure 7: Bed Profile under Combined Wave and Current $v = 1$ cm/s in Different Angle of Attack.

CFD simulation on different wave angle of attack indicates that the scour depth and width trend-line of $45^\circ$ wave angle has the widest and deepest scour profile. However, although wave angle $60^\circ$ does not have the deepest and widest trend, the simulation results show that there is a slight difference in depth between angles $45^\circ$ and $60^\circ$. Where angle $60^\circ$ has the deepest scour depth compared to the other results.

Figure 8: Typical Time-Dependent Scour in Different Wave Angle.

The bed profile deformation during scour shows if the initial stage of scour started from onset scour then followed by tunnel erosion. Initial flow that hits the pipe lead pressure difference between upstream and downstream the pipe and caused piping. Later on the gap between pipe and the bed formed as shown in Figure 9 ($t=30$ s). The initial gap continuously exposed to flow, and the gap evolves during the time. As seen in Figure 9, the scour occurs faster at the beginning ($t=0$ s until $t=180$ s).

Just as described in Lee-Wake erosion process, a dune begins to form and gradually resettle to the reverse side and deposed as the scour progressed. In this stage scour hole formed and the distance between
pipe and the bed seen as a gap in two-dimensional scour approach. If the scour hole traced and observed in three-dimensional view, it will be a free span between pipe and the bed that can disturb the stability of the pipe.

![Figure 9: Bed Profile Deformation during Scour Process.](image)

**5 CONCLUSIONS**

1. It should be noted that viscosity and turbulence affect flow condition. Verifying flow between CFD and empirical formula, should be done by set both conditions as zero.

2. The angle of attack is one of influencing factors of scour depth. The scour depth is reduced by decreasing of wave angle of attack.

**ACKNOWLEDGEMENTS**

The authors would like to acknowledge the support from Indonesian Research and Technology Council, through PMDSU research grant.

**REFERENCES**


Shen, W., et al., 2015. Shear Stress Amplification around Subsea Pipelines: Part 3, 3D Study of Spanning
Pipelines. 7th International Conference on Scour and Erosion, ISCE. 325-335.