Investigation of Wave Orbital Velocity Estimation under Non-breaking Irregular Waves

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Abstract: Wave orbital velocity plays an important roles in many analysis of sediment transport calculation and hydrodynamic model. The accuracy should be ensured to represent the actual conditions. This paper reviewed and compared several approaches for estimating wave orbital velocity under non-breaking irregular waves. There are four methods reviewed in this paper such as Stretching method, Local Fourier Approximation (LFA) method, Fourier decomposition method, and a new proposed method based on the Kaczmarek and Ostrowski (Kaczmarek and Ostrowski, 1996) by adding the correction coefficient factor (α_c) with value of 4.35. Those method has been examined and compared through both experimental data and the estimation model. The proposed method gave the best agreement among other methods with smallest RMSE value. The proposed method can be used to estimate wave orbital velocity under non-breaking irregular waves with free surface elevation datas as an input in practical application.

1 INTRODUCTION

Wave orbital velocity plays an important roles in many analysis of sediment transport and hydrodynamics model in the case of coastal, channel, and estuaries models. It should be ensured that the estimation in accurate, so it can represent the actual conditions in the field (Soulsby, 1987).

Time-varying of wave orbital velocities can be calculated in several ways depending on the data availability. Some researchers used spectrum approach or wave by wave parameters. If measuring devices (i.e. micro-ADV, LDV, or PIV) is available, which is quite expensive tools for laboratory equipment's, then the wave orbital velocities can obtained directly. It will be difficult when only the wave surface elevation data obtained due to an absence of measuring devices. Then the estimation method for calculating the wave orbital velocity is necessary.

The calculation methods of the wave orbital velocity have been studied by many researchers, but most of them are for regular and non-linear waves e.g. (Sobey, 1992; Soulsby and Smallman, 1986; Soulsby, 2006; Abreu et al., 2010; Suntoyo et al., 2008; Suntoyo and Tanaka, 2009). However, it is very rare to review wave orbital velocity for time varying under non-breaking irregular wave's conditions. Several studies related to irregular waves have been carried out by researchers (Soulsby, 1987; Elfrink et al., 2006; Wiberg and Sherwood, 2008; Malarkey and Davies, 2012; Suntoyo et al., 2016; Wijaya et al., 2016; Fattah et al., 2018). However, some of those methods have limitations on certain conditions and a simple formulation due to non-breaking irregular waves has not been proposed, yet.

Therefore, the objective of this study is to examine and compare several formulation of wave orbital velocity with experimental data under non-breaking irregular waves motion (Ruiz, 2014). The four calculation methods evaluated and examend with a new proposed method based on the evaluation of Fourier decomposition method (Kaczmarek and Ostrowski, 1996), namely, Stretching method (Wheeler, 1969), LFA method (Soulsby, 1987) and Fourier decomposition (Kaczmarek and Ostrowski, 1996). The best agreement method obtained based on the smallest value of RMSE (root-mean-squared-error) as performance indicator. The best approach will be further used to estimate the wave orbital velocity under irregular wave motion.

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2 WAVE PARAMETERIZATION

Wave orbital velocity can be calculated in several ways depending on the available data. Some common rules using wave by wave method or spectrum parameterizations analysis to obtain wave period and wave height from time series or spectra of surface elevation. In this present paper zero-down crossing analysis method was used as as shown at Figure 1 as given (Holthuijsen, 2007).

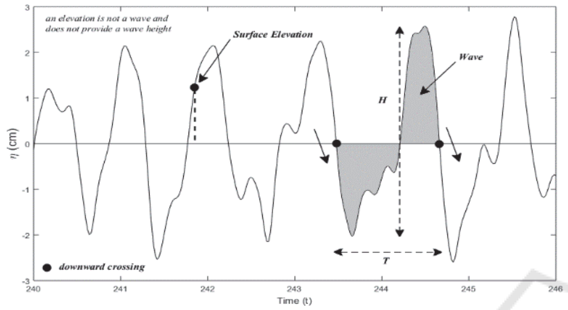

Figure 1: Definition of a “wave” with zero-down crossing analysis in time records of surface elevation. (Holthuijsen, 2007).

Surface-wave elevation and velocities data in this paper was obtained by digitation from Ruiz (Ruiz, 2014) with totally 15 of wave cycles taken from 19s until 48s of measured data. The velocity was measured at z = -0.33 m with significant wave height (Hs) = 0.14 m and wave peak period (Tp) = 1.98 s in 2.97m water depth as described in Table 1.

Table 1: Experimental data conditions (Ruiz, 2014).

ID	Hs (m)	Depth (m)	Tp (s)
Plymouth A1	0.14	2.97	1.98
Plymooth B4	0.38	2.97	2.63

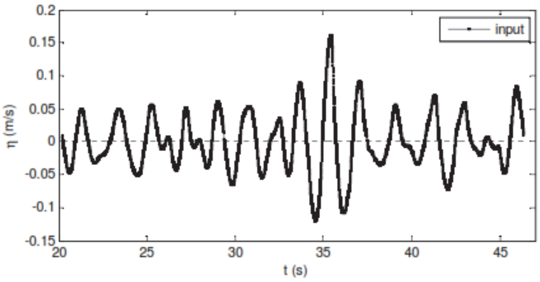

Figure 2: Surface wave elevation data Plymouth A1 obtained by digitation in Ruiz (Ruiz, 2014).

3 WAVE ORBITAL VELOCITY CALCULATION METHODS

In this section, the four of existing wave orbital velocity calculation methods under irregular waves motion are presented. In this present paper, there are four formula that evaluated and compared, namely, Stretching method (Wheeler, 1969), LFA method (Soulsby, 1987), Fourier decomposition (Kaczmarek and Ostrowski, 1996) examined by a new proposed method based on the evaluation of Fourier decomposition method by Kaczmarek and Ostrowski, (Kaczmarek and Ostrowski, 1996).

3.1 Method 1 (Stretching Method)

Wheeler (Wheeler, 1969) presented a method, the so-called stretching method of Wheeler, which is departed to calculate kinematic velocity from a measured free surface time history ($\eta_i \; with \; i = 1,2,\ldots,I$) trough Airy kinematics solution. Then transformed into Fourier-series transformation.

Therefore, the horizontal velocity is predicted as:

$$u_i = C_E + \sum\nolimits_0^{F/2} f\omega_0 \frac{\cosh(\alpha_i k_f h)}{\sinh(k_f h)} \dot{\eta}_f(t_i) \qquad i = 1,2,\ldots,I \quad (1)$$

$$\dot{\eta}_f = A_f \cos(f\omega_0 t) + B_f \sin(f\omega_0 t) \qquad f = 0,1,\ldots,F/2 \quad (2)$$

$$\eta_i = \sum\nolimits_0^{F/2} \dot{\eta}_f(t_i) \qquad i = 1,2,\ldots,I \quad (3)$$

Where C_E is Eulerian current (if current is present), $\alpha_i = (h+z)/(h+\eta_i)$, $\omega_0 = 2\pi/(t_I - t_1)$ and k_f is calculated from the linear dispersion relation:

$$gktanh(kh) - \omega_0 = 0 \quad (4)$$

3.2 Method 2 (Local Fourier Approximation)

Local Fourier (LF) approximation is kinematics calculation method presented by Soulsby (Soulsby, 1987) which addressed to calculate kinematics from a measured free surface time-varying history by means of a local approximation of the velocity potential as a truncated Fourier series.

LF Approximation method based on an approximation of the stream function (global) and the velocity potential (local) as a truncated Fourier series.

$$u_i = \frac{\partial\phi}{\partial x}(x,\eta_i,t_i) = C_E + \sum_{j=1}^{J} jkA_j \frac{\cosh(jk(\eta_i + h))}{\cosh(jkh)} \cos(j(kx - \omega_0 t_i)) \quad (5)$$

$$A_1 = \sqrt{\left(\frac{(\partial_\eta/\partial_t)}{k\tanh(kh)}\right)^2 + \left(\frac{g\eta_c}{\omega_0}\right)^2} \qquad (6)$$

$$A_j = \frac{A_1}{10^{j-1}} \qquad for\ j = 1, \dots, J \qquad (7)$$

Where ω_0 is estimated from the local zero-down crossing period (T_z) as $\omega_0 = 2\pi/T_z$.

3.3 Method 3 (Fourier Decomposition)

Kaczmarek and Ostrowski (Kaczmarek and Ostrowski, 1996) proposed the simple method to compute time series of wave orbital velocity based on Fourier decomposition of the water surface elevation as describes follow:

$$U(t) = \sum_n \eta_n \frac{\omega_n}{\sinh(k_n h)} \sin(\omega_n t + \varphi_n) + \frac{1}{2} U_0 \qquad (8)$$

In which ω_n and k_n are angular frequency and wave number respectively, related to each other by linear dispersion relationship. φ_n is phase and U_0 is the average initial velocity.

3.4 Method 4 (Proposed Method)

A proposed method is simple method based on the modification of Fourier decomposition method (Kaczmarek and Ostrowski, 1996) by adding the correction coefficient (α_c) factor to the measurement results of the experiment in the laboratory as follows:

$$U(t) = \sum_n \alpha_c \eta_n \frac{\omega_n}{\sinh(k_n h)} \sin(\omega_n t + \varphi_n) + \frac{1}{2} U_0 \qquad (9)$$

In which α_c is the correction coefficient with value of 4.35.

4 RESULTS AND DISCUSSIONS

Evaluating the comparison of those different methodologies presented above, it needs to be validated using laboratory measurement data. Comparison result evaluated trough the root-mean-squared error (RMSE) defined as follow:

$$RMSE(u) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(u_i^{cal} - u_i^{exp}\right)^2} \qquad (10)$$

Where, u_i^{cal}: the wave orbital velocity from calculation methods, u_i^{exp}: the wave orbital velocity from experimental results, *N*: total number of data and *i*: index.

Table 2: Summary of calculation method performance of wave orbital velocities.

No	Method	RMSE Value
1.	Method 1	0.0668
2.	Method 2	0.0289
3.	Method 3	0.0791
4.	Method 4	0.0265

If the calculation method is perfect, it can be indicated that RMSE results should be zero. So, the smaller RMSE is the better performance results of the calculation methods. The summary of those calculation method performance is presented in Table-2.

Comparison results among the experimental data and the calculation methods are given in Figure 2. It can be seen that the proposed method (Method 4) has highest performance with the lowest value of RMSE among others methods with *RMSE = 0.0265* then followed by Local Fourier (Method 2) (Soulsby, 1987) with *RMSE = 0.0307*, Method 1(Wheeler, 1969) with *RMSE=0.0668* and Method 3 (Kaczmarek and Ostrowski, 1996) with *RMSE=0.0781*, respectively.

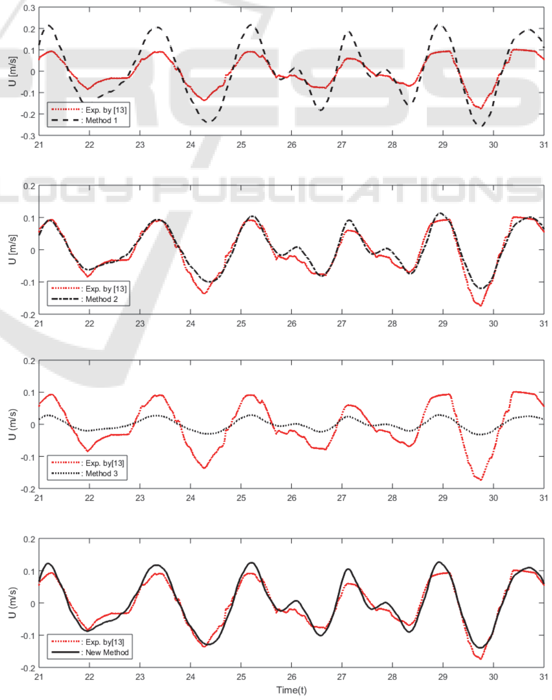

Figure 3: Comparison of the experimental data (Ruiz, 2014) and the calculation methods of wave orbital velocity under non-breaking irregular waves.

Method 1 gives overestimation both in the crest and trough of the waves, while Method 3 gives significant different result against experimental data.

However, Method 3 is the simplest formula among others while the Method 1 and Method 2 need advance mathematical calculation to compute approximation. Method 3 has a similar in line trend with the experimental data, so it has an opportunity to review further.

The proposed method (Method 4) that based on the evaluation of the Method 3, with the addition of a correction factor, gave smallest RMSE value indicating that it has best agreement with the wave orbital velocity of experimental result provided (Ruiz, 2014). It can be concluded that Method 4 can be used to estimate wave orbital velocities under irregular waves with time-varying free surface elevation as an input. Furthermore, the proposed method can be further used to an input calculation of bottom shear stress and sediment transport model under non-breaking irregular waves in practical application.

5 CONCLUSIONS

The calculation method of wave orbital velocity under non-breaking irregular waves has been examine and compare through both experimental data and the estimation model. Method 4 as proposed method gave best agreement with lowest RMSE value and simplest formulation that indicating the best performance among other method then followed by Method 2, Method 1 and Method 3. Method 1 gave over estimation both in the crest and trough condition of the waves. Method 3 gave significant different results, but it has a similar in line trend with the experimental data. Beside that, Method 2 gave almost the same results with Method 4, but need an advance mathematical method to estimate wave orbital velocity.Moreover, the proposed method (Method 4) based on the evaluation of the Method 3 by adding the correction coefficient factor (α_c) with value of 4.35 gave the best agreement with the measured experimental data than other estimation methods. It can be concluded that proposed method can be used further to estimate wave orbital velocity under non-breaking irregular waves with free surface elevation data as an input in practical application.

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